Optimum Quantum Sequence Control of Quantum Series Resonant Converter for Minimum Output Voltage Ripple

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Abstract—A new control scheme named optimum quantum sequence control (OQSC) which always minimizes the output voltage ripple of the quantum series resonant converter (QSRC) for all possible sequences is proposed. This control scheme is so general that it is irrelevant to all circuit conditions such as magnitudes of circuit elements as well as input/output voltage so far as it is operating in the continuous conduction mode (CCM). Further more the dynamic range of QSRC is much extended by the OQSC. This feature is verified by simulations and experiments with good agreements.

1. INTRODUCTION

Numerous circuit topologies of high frequency DC-DC converters can be categorized as the PWM converter and the resonant converter. The former has been used widely because of its simple structure and easy control. However, the switching frequency of the PWM converter is highly limited by the switching stress and switching loss of the circuit elements.

The need for high power density converter yields the advent of the resonant converter which operates at soft switching conditions. Conventional resonant converters excluding quasi-resonant [1] ones can be classified into two categories according to their control methods. One is frequency domain control [2] scheme and the other is phase domain control scheme [3]. Since the output voltage of the former depends largely on the condition as well as the switching frequency and the current switching stress of the latter becomes large as phase difference becomes large, those control schemes have very limited practical applications.

Recently Joung [4] and Rim [5]-[6] proposed a new time domain control method named as quantum resonant converters which operate in the optimum frequency and phase. The switching frequency and phase of this scheme are always fixed to the resonant frequency and zero phase respectively so that zero voltage switching conditions can be always guaranteed. The output voltage is controlled by the number of powering modes which results in the quantized output voltage level. This scheme provides the maximum utilization of the filters and magnetic components as well as the applicability of linear control theories. However the output voltage ripple of this scheme is relatively large due to the limit cycle phenomenon because the converter output voltage is controlled by the ratio of successive powering modes and free resonant modes.

This large voltage ripple highly impedes the practical application of the converter. One of the best way to minimize the ripple is to decentralize the successive powering modes of the integral cycle mode control scheme [4].

In this paper, optimum control sequences of powering mode and free resonant mode which minimize the output voltage ripple are proposed by ripple analyses and simulations. Furthermore these are verified by appropriate experiments. In this way the output voltage ripple is greatly reduced (about 1/2–1/5) by the proposed control scheme compare with the integral cycle mode control one.

II. REVIEW OF QUANTUM SERIES RESONANT CONVERTER (QSRC)

A. Operating Mode of QSRC

The buck type SRC whose static and dynamic behavior is identical with the PWM buck converter is shown in Fig. 1.

There are four distinct operating modes in the QSRC [4]: powering, free resonant, regeneration and discontinuous modes. The output voltage can be controlled by the appropriate combinations of the powering mode and one of the other modes. In this paper, however, the regeneration and discontinuous modes are excluded from the discussion for the higher input power factor and easier control. The combination of the powering and free resonant mode is though to be the best one for the DC-DC quantum converters.

It is assumed that all circuit elements including switches are ideal and the QSRC operates in the CCM. And that the output
capacitor \( C_o \) is sufficiently large so that the output voltage variation during one resonant cycle is negligible.

B. Dynamic Equation of QSRC

An equivalent circuit for \( k \)-th quantum interval (half of a resonant cycle) with equivalent series registor \( R \), is shown in Fig. 2(a). And the voltage and current waveforms are shown in Fig. 2(b) where \( v_i(k), v_o(k), i_L(k) \), and \( i_o(k) \) indicate the resonant capacitor voltage, output voltage, resonant inductor current and the rectified output current, respectively.

From the equivalent circuit, the resonant inductor current \( i_L(t) \) at \( k \)-th quantum interval is found as
\[
i_L(t) = \frac{v_i(k-1) + M_k V_s - v_o(k-1)}{Z} \left( e^{-\frac{\alpha}{2} t} \sin(\omega_r t - k\pi) \right)
\]

where,
\[
\omega_r = \frac{1}{\sqrt{L C}}, \quad Z = \sqrt{L / C}.
\]

From (1), the resonant capacitor peak voltage \( v_c(k) \) and output voltage \( v_o(k) \) can be evaluated as follows:
\[
v_c(k) = \frac{1}{C} \int_{(k-1)T/2}^{kT/2} i_L(t) dt - v_c(k-1)
\]
\[
= \left( 2 - \frac{\pi R_s}{2Z} \right) \left( M_k V_s - v_o(k-1) + v_c(k-1) \right) - v_c(k-1)
\]

(2)

\[
v_o(k) = \frac{1}{C_o} \int_{(k-1)T/2}^{kT/2} i_L(t) dt - v_o(k-1)
\]
\[
= \frac{1}{C_o R_s} \int_{(k-1)T/2}^{kT/2} v_o(k-1) dt
\]
\[
= \alpha v_o(k-1) + \frac{(1 - \alpha - \beta) v_o(k-1) + \alpha M_k V_s}{4Z} - \frac{\pi R_s}{4Z} \alpha (M_k V_s - v_o(k-1) + v_c(k-1))
\]

(3)

where,
\[
T = \frac{2\pi}{\omega_r}, \quad \alpha = 2 \left( \frac{C}{C_o} \right), \quad \beta = \alpha Q, \quad \text{and} \quad Q = \left( \frac{\pi}{2} \right) \left( \frac{Z}{R_s} \right)
\]

and assuming that \( (\pi R_s)/(2Z) \ll 1 \), then
\[
e^{-\frac{\alpha}{2} t} \approx 1 - \frac{\pi R_s}{2Z}
\]
\[
\sin \left( \pi \sqrt{1 - \left( \frac{R_s}{2Z} \right)^2} \right) \approx \sin \pi = 0
\]
\[
\cos \left( \pi \sqrt{1 - \left( \frac{R_s}{2Z} \right)^2} \right) \approx \cos \pi = -1
\]

C. Steady State Analysis

In the QSRC, the DC voltage transfer ratio is given by \( m/n \) where \( m \) is the number of powering modes and \( n \) is the total half resonant cycles of one control period. The terminology quantum sequence is defined as a set of periodic \( M_k \)'s, that is, \( \{M_1, M_2, \cdots, M_n\} \) where \( M_{n+k} = M_k \) and \( k = 1, 2, 3, \cdots, n \).

The steady state tank capacitor voltages from \((k+1)\)-th half resonant period to \((k+n)\)-th half resonant period are given by
\[
v_c(k+1) = \left( 2 - \frac{\pi R_s}{2Z} \right) (M_1 V_s - v_o(k) + v_c(k)) - v_c(k)
\]
\[
\vdots
\]
\[
v_c(k+n) = \left( 2 - \frac{\pi R_s}{2Z} \right) (M_n V_s - v_o(k+n-1) + v_c(k+n-1)) - v_c(k+n-1) - v_c(k) \quad (4)
\]

From (4), summation of the tank capacitor voltage is expressed by
\[
\sum_{i=0}^{n-1} v_c(k+i) = \left( 2 - \frac{\pi R_s}{2Z} \right) \left( \sum_{i=1}^{n} M_i V_s - \sum_{i=0}^{n-1} v_o(k+i) \right)
\]
\[
+ \sum_{i=0}^{n-1} v_c(k+i) - \sum_{i=0}^{n-1} v_o(k+i) \quad (5)
\]

From (5), the average output voltage in the steady state is expressed by
\[
V_o = \frac{V_s}{n} [M_1 + M_2 + \cdots + M_n] - \frac{\pi R_s}{4Z} \frac{V_c}{V_s}
\]
\[
= \frac{m}{n} V_s - \frac{\pi R_s}{4Z} V_c \quad (6)
\]

where, \( V_c \) is the average magnitude of \( v_c(\cdot) \).

The steady state output voltages from \((k+1)\)-th half resonant period to \((k+n)\)-th half resonant period are given by
\[
v_o(k+1) = \alpha v_c(k) + (1 - \alpha - \beta) v_o(k) + \alpha M_1 V_s
\]
\[
- \frac{\pi R_s}{4Z} \alpha (M_1 V_s - v_o(k) + v_c(k))
\]
\[
\vdots
\]
\[
v_o(k+n) = \alpha v_c(k+n-1) + (1 - \alpha - \beta) v_o(k+n-1)
\]
\[
+ \alpha M_n V_s - \frac{\pi R_s}{4Z} \alpha (M_n V_s - v_o(k+n-1) + v_c(k+n-1)) \quad (7)
\]
From (7), summation of the output voltage is expressed by

\[
\sum_{i=0}^{n} v_o(k+i) = \alpha \sum_{i=0}^{n-1} v_o(k+i) + (1-\alpha-\beta) \sum_{i=0}^{n-1} v_o(k+i) \\
+ \alpha \sum_{i=1}^{n} M_i V_o - \frac{\pi R_s}{4Z} \left( \sum_{i=1}^{n} M_i V_o - \sum_{i=0}^{n-1} v_o(k+i) \right) \\
+ \sum_{i=0}^{n-1} v_o(k+i)
\]  

(8)

From (6) and (8), the average magnitude of \( v_o(\cdot) \) is expressed by

\[
V_{cp} = \frac{1}{n} \sum_{i=0}^{n-1} v_o(k+i)
\]

= \( QV_o \)  

(9)

From (6) and (9), the average output voltage in the steady state is expressed by

\[
V_o = \frac{1}{1 + \frac{\pi R_s}{4Z}} \left( \frac{m}{n} V_o \right)
\]

= \( \frac{m}{n} V_o \), if \( R_s = 0 \)  

(10)

It is found that \( V_o \) is irrelevant to the quantum sequence type so far as the values \( m, n \) are same. Assuming \( R_s \) is zero, \( V_o \) is directly proportional to the ratio \( m/n \) and irrelevant to the circuit parameters such as load resistor, \( C \), \( C_o \) and \( L \). The very linear and robust output characteristics of the quantum converter provide simple and easy controller design. And the average magnitudes of \( i_L(\cdot) \) and \( i_o(\cdot) \) are evaluated as follows:

\[
i_L(k) = \frac{V_{cp}}{Z}
\]

\[
i_o = \frac{2}{\pi} i_Lp = \frac{V_o}{R_o}
\]

III. BOUNDARY CONDITION BETWEEN CONTINUOUS AND DISCONTINUOUS CONDUCTION MODE

It is assumed that the QSRC operates in the continuous conduction mode. And that the output capacitor \( C_o \) is sufficiently large so that the output voltage variation during one resonant cycle is negligible.

From (2), the resonant capacitor peak voltages \( v_c(\cdot) \) are evaluated as follows:

\[
v_c(k+1) = E(M_1 V_o - V_o) + F v_{cmin}(\cdot)
\]

\[
v_c(k+2) = E[(F M_1 + M_2)V_o - (1+F)V_o] + F^2 v_{cmin}(\cdot)
\]

\[
\vdots
\]

\[
v_c(k+n) = E[(F^{n-1} M_1 + F^{n-2} M_2 + \cdots + M_k)V_o - (1+F+\cdots+F^{n-1})V_o] + F^n v_{cmin}(\cdot)
\]

where, \( E = 2 - (\pi R_s)/(2Z) \), \( F = E - 1 \), \( v_{cmin}(\cdot) = v_c(k) \).

From (13), summation of the tank capacitor voltage is given by

\[
\sum_{i=1}^{n} v_c(k+i) = E\left[ (1+F+\cdots+F^{n-1}) M_1 \right. \\
+ (1+F+\cdots+F^{n-2}) M_2 + \cdots \\
+ (1+F) M_{n-1} + M_n] V_o \\
- E\left[ (1+F+\cdots+F^{n-1}) V_o \right. \\
+ (1+F+\cdots+F^n) v_{cmin}(\cdot) \\
+ (1+F+\cdots+F^n) v_{cmin}(\cdot)
\]

(14)

First, in case of \( R_s \neq 0 \), summation of the tank capacitor voltage is evaluated as follows:

\[
\sum_{i=1}^{n} v_c(k+i) = \frac{E}{1-F} \left[ m - (F^n M_1 + F^{n-1} M_2 + \cdots \\
+F M_n] V_o - \frac{E}{F(1-F^n)} \left[ m - F(1-F^n) \right] V_o \\
+ \frac{E(1-F^n)}{1-F} v_{cmin}(\cdot)
\]

(15)

From (15), the average magnitude of \( v_c(\cdot) \) is given by

\[
V_{cp} = \frac{E}{n(1-F)} \left[ m - (F^n M_1 + F^{n-1} M_2 + \cdots + F M_n] V_o \\
- \frac{E}{n(1-F)} \left[ m - F(1-F^n) \right] V_o + \frac{E(1-F^n)}{n(1-F)} v_{cmin}(\cdot)
\]

(16)

Fig. 3 represents the waveform of \( v_c(\cdot) \) in case of quantum sequence 1000. From (16), the minimum magnitude of \( v_c(\cdot) \) is expressed by

\[
v_{cmin}(\cdot) = \frac{n(1-F)}{F(1-F^n)} QV_o + \frac{E}{1-F^n} V_o - \frac{E}{1-F^n} V_o \\
- \frac{E}{F(1-F^n)} V_o + \frac{E}{1-F^n} V_o \\
+ \frac{E(1-F^n)}{F(1-F^n)} V_o + \frac{E}{F(1-F^n)} v_{cmin}(\cdot)
\]

(17)

Being at the boundary condition between the continuous and the discontinuous conduction mode, it is determined by free resonant mode as final quantum sequence. In order to operate the free resonant mode at final quantum sequence, \( V_{CD}(\cdot) \) must be larger than output voltage \( V_o \). \( V_{CD}(\cdot) \) is determined just in front of final quantum sequence. From (4), (9) and (17),
V_{CD}() is expressed by

\[
V_{CD}() = \frac{1}{F}v_{\text{min}}() - \frac{E}{F}(M_nV_o - V_o)
\]

\[
= \frac{n(1 - F)}{F^2(1 - F^n)}QV_o + \frac{E_n}{F^2(1 - F^n)}V_o - \frac{E}{1 - F}V_o
\]

\[
- \frac{E_n}{F^2(1 - F^n)}V_o + \frac{E}{F^2(1 - F^n)}(F^nM_1
\]

\[
+ F^{n-1}M_2 + \cdots + FM_n)V_o - \frac{E}{F}M_nV_o
\]

(18)

where, \(n > 2\).

If the QSRC operates in the CCM, then \(V_{CD}() \geq V_o\).

From (10) and (18), the magnitude of \(R_o\) in the CCM can be calculated as shown at the bottom of the page in (19).

As can be seen from (19), the magnitude of \(R_o\) in the CCM is relevant to the ratio \(m/n\), the equivalent series resistor \(R_o\), and the circuit parameter. For example, in case of quantum sequence 1000, \(Z = 60(L = 80 \, \mu\text{H} \text{ and } C = 22 \, \text{nF})\) and \(R_o = 2.5 \, \Omega\), the magnitude of \(R_o\) is 44 \(\Omega\) by calculation but 44 \(\Omega\) by simulation.

IV. ANALYSIS OF OUTPUT VOLTAGE RIPPLE

The output current and its envelope are illustrated in Fig. 5(a), and the output voltage ripple is depicted in Fig. 5(b) for a quantum sequence \(\{M_k\} = \{1111000\}\).

As shown in Fig. 5(b) the output voltage ripple consists of the sinusoidal ripple induced by the harmonic component of \(i_o\) and the envelope ripple induced by the envelope variation of \(v_o\). The output voltage ripple is then evaluated by summing up the two ripples.

\[
R_o \leq \frac{n \frac{\pi^2 R_o}{2}}{2F^2(1-F^n)} \left[ (F^nM_1 + F^{n-1}M_2 + \cdots + FM_n) - F(1 - F^n)M_n \right]
\]

(19)
The sinusoidal ripple is induced from the difference of the rectified sinusoidal current and its average. The relation between the output current waveform and its average is shown in Fig. 6. The dashed area represents the source current of this ripple.

Since the output current is of the form, $I_{op} \sin(\omega_t t)$, the average output current is given by

$$I_o = \frac{1}{\pi} \int_0^{\pi} I_{op} \sin \theta d\theta = \frac{2}{\pi} I_{op} \tag{21}$$

From the assumption the harmonic current flows mainly into the output capacitor which is selected as very large. Then the sinusoidal ripple voltage is given by

$$\Delta V_{os} = \frac{2I_{op}}{C_0 \omega_r} \int_{\theta_1}^{\theta_2} \left( \frac{2}{\pi} - \sin \theta \right) d\theta$$

$$= \frac{2I_{op}}{C_0 \omega_r} \int_{\theta_1}^{\theta_2} \left( \frac{2}{\pi} - \sin \theta \right) d\theta$$

$$= \frac{I_{op}}{C_0 \omega_r} \int_{\theta_1}^{\theta_2} \left( \sin \theta - \frac{2}{\pi} \right) d\theta$$

$$= 0.66130 \frac{I_{op}}{C_0 \omega_r} \tag{22}$$

It is assumed that the variation of $i_o$ is minimal, hence $i_o$ is kept constant in the calculation. Note that the harmonic ripple is not sensitive to the quantum sequence so far as $m$ and $n$ are same, because $I_o$ is nearly kept constant for all quantum sequences if only their $m$ and $n$ are same. This feature is deduced from the fact that the quantum converter is a linear system [7].

B. Envelope Ripple

The envelope ripple is produced by the fluctuation in the envelope of $i_o$. The fluctuation is generated from the different operating modes, powering and free resonant modes. Hence the envelope ripple is strictly determined by the quantum sequences. The envelope ripple waveform for the control sequence, $\{1111000\}$ is illustrated in Fig. 7.

From Fig. 7, the envelope of $\Delta i_o(t)$ for a duty control sequence number $n$ is given by

$$\Delta i_o(t) = \left[ i_o(k + 1) - i_o(k) \right] \frac{(n \pi + k \pi)}{\pi} + i_o(k) - I_o \left[ (n \pi + k \pi) \right] \tag{23}$$

where, $k \pi \leq n \pi + k \pi \leq (k + 1) \pi$, $k = 0, 1, 2, \ldots, n - 1$.

In (23), $i_o(k)$ and $I_o$ are determined from (1) and (12) when the quantum sequence, $\{M_k\}$ is given.

Hence the envelope voltage ripple can be calculated as follows:

$$\Delta V_{en} = \left[ \frac{1}{C_0} \left( \max \left( \int_0^{\pi} \Delta i_o(x) dx \right) \right) - \min \left( \int_0^{\pi} \Delta i_o(x) dx \right) \right] \tag{24}$$

for $0 < t < nT/2$

Since (24) is no longer analytical, this should be calculated numerically for a given quantum sequence $\{M_k\}$. Obviously $\Delta V_{en}$ may be different from each other for different quantum sequences even though their $n$ and $m$ are identical, for example $\{11100\}$ and $\{10110\}$.

C. Total Output Voltage Ripple

Assuming that the interaction between the two ripples is negligible; that is the fluctuation in the envelope of $i_o$ doesn’t affect the sinusoidal ripple or vice versa, then the total output voltage ripple can be expressed by just summing up the two ones as follows:

$$\Delta V_o = \Delta V_{os} + \Delta V_{en} \tag{25}$$
The proposed model is valid for in the transient state as well as steady state. Note that this model neglects the harmonic ripple effect; however, this model can be used to verify whether the OQS listed in Table I is general or not. Since the harmonic ripple is mostly irrelevant to the quantum sequence as mentioned in the previous section.

Assume that the voltage source $v_s$ of Fig. 8 excluding dc component is expressed of the Fourier series form as,

$$v_s = \sum_{k=\infty}^{\infty} V_{sk}e^{jk\omega_0 t}$$  \hspace{1cm} (26)

Then the output voltage ripple $\Delta V_o$ is given as follows considering $C_o$ is sufficiently large so that the resonant frequency of the equivalent circuit is much higher than the switching frequency:

$$\Delta V_o = \text{Max} \{ \Delta V_o(t) \} - \text{Min} \{ \Delta V_o(t) \}$$  \hspace{1cm} (27)

where,

$$\Delta V_o(t) = \frac{2}{n^2 L_{eq} C_o \omega_0^2} \text{Re} \left\{ \sum_{k=1}^{\infty} (V_{sk}/k^2) e^{jk\omega_0 t} \right\}$$  \hspace{1cm} (28)

Since only $V_{sk}$ of (28) is varied by the quantum sequence if $n$, $m$ is same, $\Delta V_o$ of (27) is always minimized by the OQS if other conditions such as $n$, $C_o$, $\omega_0$, and $L$ are unchanged. This fact implies that the OQS is so general that an OQS obtained by a specific circuit may be also valid for arbitrary circuits. In other words, OQS listed in Table I is applicable to any QSRC previously defined.

### Table I

<table>
<thead>
<tr>
<th>Integral Cycle Mode Control</th>
<th>Optimum Quantum Sequence Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>Sequence</td>
</tr>
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<td>5</td>
<td>1 100000</td>
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</tbody>
</table>

As can be seen from (25) it is interesting that the output voltage ripple, $\Delta V_o$, is directly proportional to the inverse of $C_o$ and $\omega_0$, which is just the same as that of conventional PWM converters. Now the optimum quantum sequence which minimizes the value of $\Delta V_o$ can be determined from (22), (24) and (25) by applying all possible $\{M_k\}$'s for given $m$ and $n$. By numerical simulations these are calculated as shown in Table I for a typical condition.

In Table I, the output voltage ripples of the integral cycle mode control (ICMC) and the optimum quantum sequence control (OQSC) are compared for different control sequences for non-trivial cases. As shown in Table I, the OQSC is a few times superior to the ICMC.

Furthermore, it is found that the optimum quantum sequence (OQS) is obtained when the powering modes are most evenly distributed. Since (24) is not of analytical form, the validity of the OQS shown in Table I can not be guaranteed in general. This generality problem will be clarified in the next section.

### D. Equivalent Circuit Using Quantum Transformation

Conventional resonant converters are known as non-linear switching system described by complex and implicit equations. But a simple equivalent circuit model is newly suggested by Rim [5], [6] as shown in Fig. 8.

The proposed model is valid for in the transient state as well as steady state. Note that this model neglects the harmonic ripple effect, however, this model can be used to verify whether the OQS listed in Tabel I is general or not. Since the harmonic ripple is mostly irrelevant to the quantum sequence as mentioned in the previous section.

Assume that the voltage source $v_s$ of Fig. 8 excluding dc component is expressed of the Fourier series form as,

$$v_s = \sum_{k=-\infty}^{\infty} V_{sk}e^{jk\omega_0 t}$$  \hspace{1cm} (26)

Then the output voltage ripple $\Delta V_o$ is given as follows considering $C_o$ is sufficiently large so that the resonant frequency of the equivalent circuit is much higher than the switching frequency:

$$\Delta V_o = \text{Max} \{ \Delta V_o(t) \} - \text{Min} \{ \Delta V_o(t) \}$$  \hspace{1cm} (27)

where,

$$\Delta V_o(t) = \frac{2}{n^2 L_{eq} C_o \omega_0^2} \text{Re} \left\{ \sum_{k=1}^{\infty} (V_{sk}/k^2) e^{jk\omega_0 t} \right\}$$  \hspace{1cm} (28)

Since only $V_{sk}$ of (28) is varied by the quantum sequence if $n$, $m$ is same, $\Delta V_o$ of (27) is always minimized by the OQS if other conditions such as $n$, $C_o$, $\omega_0$, and $L$ are unchanged. This fact implies that the OQS is so general that an OQS obtained by a specific circuit may be also valid for arbitrary circuits. In other words, OQS listed in Table I is applicable to any QSRC previously defined.
V. CLOSED LOOP OUTPUT VOLTAGE CONTROL OF QSRC

A. State Equations

From Fig. 8, the state variables are defined as
\[ x_1(t) = v_0(t) \]
\[ x_2(t) = \frac{dv_0(t)}{dt} \]

Then the state equations are represented as
\[ \frac{d}{dt} x(t) = Ax(t) + Bu(t) \]  
(30)

where,
\[ A = \begin{bmatrix} -\frac{1}{L_v C_v} & 0 \\ \frac{1}{L_v C_v} & -\left( \frac{R_{esat}}{L_v} + \frac{1}{R_s C_s} \right) \end{bmatrix}, \]
\[ B = \begin{bmatrix} 0 \\ \frac{R_{esat}}{L_v C_v} \end{bmatrix}, \quad u(t) = M_x V_s, \]
\[ M_x = \begin{cases} 1 & : \text{powering mode} \\ 0 & : \text{free resonant mode} \end{cases} \]

For the control of output voltage of QSRC, the closed-loop system is formed by feeding back the state variables through a constant matrix \( G \). In this case, the state variables are newly defined as
\[ \dot{x}_1(t) = V_{\text{ref}} - v_0(t) \]
\[ \dot{x}_2(t) = \frac{d\dot{x}_1(t)}{dt} = \frac{dV_{\text{ref}}}{dt} - \frac{dv_0(t)}{dt} \]  
(31)

where, the state equations are written as
\[ \frac{d}{dt} \dot{x}(t) = A\dot{x}(t) - Bu(t) + W \]  
(32)

where,
\[ W = \left[ \frac{1}{L_v C_v} \left( 1 + \frac{R_{esat}}{R_s} \right) V_{\text{ref}} + \left( \frac{R_{esat}}{L_v C_v} + \frac{1}{R_s C_s} \right) \frac{dV_{\text{ref}}}{dt} + \frac{d^2V_{\text{ref}}}{dt^2} \right] \]

The closed-loop transfer function has no zeros at the origin, then constant command input can be uniquely determined.

The state equations have a constant input \( W \). In this case, if we use state feedback to stabilize the original system, then the presence of constant input will yield a nonzero steady-state value. This can be reduced by increasing feedback gain \( G \), but this has limits, because of saturation and noise effects.

Constant input \( W \) can often be eliminated by using the so-called "integral-error" feedback. Thus, introduce an additional state variable
\[ \frac{d}{dt} q(t) = V_{\text{ref}} - v_0(t) \]  
(33)

and use the feedback
\[ u(t) = G\ddot{x}(t) + Kq(t) \]  
(34)

and use the feedback
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(34)

then, the closed-loop system is
\[ \frac{d}{dt} \begin{bmatrix} \ddot{x}(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} A - BG & -BK \\ C & 0 \end{bmatrix} \begin{bmatrix} \ddot{x}(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} W \\ 0 \end{bmatrix} \]  
(35)

and it \( \{G, K\} \) are chosen to make this system stable, then the steady-state value of \( q(t) \) will be zero.
A. Sliding Mode Voltage Control

Sliding mode control is obtained by means of the following feedback control strategy, which relates the input control sequence $M_k$ with the value of $u(t)$:

$$M_k = \begin{cases} 
1 & : \ u(t) > 0 \\
0 & : \ u(t) < 0 
\end{cases} \quad (36)$$

When a sliding mode exists locally on switching surface, Utkin [8] defines the average motion as the response of the system to a smooth control function called the equivalent control.

VI. SIMULATION AND EXPERIMENTAL RESULTS

The simulated steady state waveforms of the resonant inductor, output current and the output voltage ripple are shown in Fig. 10(a) and (b) when the quantum sequence $M_k$s are given as \{1 1 1 1 1 0 0\} and \{1 1 1 0 1 1 1 0\}, respectively.

![Experimental waveforms of QSRC for (a) \{1 1 1 1 1 0 0\} (b) \{1 1 1 0 1 1 1 0\}](image)

And the experimental waveforms are shown in Fig. 11. The output voltage ripple versus voltage gain for various quantum sequences is shown in Fig. 12. A little discrepancy between the simulated and the theoretically calculated value arises due to the DCM at low voltage gain.

![Output voltage ripple vs voltage gain at n = 15 for L = 80 \mu H, C = 0.2 \mu F, C_o = 150 \mu F, R_o = 3 \Omega.](image)
It can be concluded that the OQSC is much superior to the ICMC. Furthermore, the output voltage ripple for the OQS case is nearly independent of the output range only if the QSRC is in the CCM. And the operating range of the QSRC (CMC range) is much extended by the OQSC as depicted in Fig. 12.

The output voltage ripple for different load resistances is simulated as shown in Fig. 13 for an example case. The control range of the output voltage is extended when the OQSC is utilized because it can be widely operated in the CCM for large range of load resistance variation. On the other hand, it is also shown that OQSC always provides minimum output voltage ripple even if the load resistance changes.

The simulated transient response when the quantum control sequence is (a) \{1 1 1 1 1 0 0\} and (b) \{1 1 1 0 1 1 0\} respectively are shown in Fig. 14. And experimental waveforms are shown in Fig. 15. The output voltage ripple is greatly reduced by the OQSC as depicted.
It is identified that the quantum sequences do not affect to the transient response so far as \( m, n \) are same but affect to the output voltage ripple.

The generality of the OQSC is extensively tested by simulations and experiments for more than 50 different circuit parameters, \( m \) and \( n \) including several extreme cases. Through this procedure it is verified that the OQSC provides always the minimum output voltage ripple among all possible quantum sequence (QS) whose \( m \) and \( n \) are identical. A part of this test is shown in Fig. 13 as a simulational verification and in Fig. 16 as an experimental verification.

And the closed loop responses are shown in Fig. 17 and Fig. 18 when \( R_o \) and \( V_o \) are changed. Fig. 17 shows the simulation results of step response and the experimental results are shown in Fig. 18.

The results shown that the quantum control concept is usefulness of the QSRC.

VII. CONCLUSION

Throughout the time-domain analysis, the boundary condition between continuous and discontinuous conduction mode in QSRC is obtained and the expression for the output voltage ripple is determined and utilized for determining the OQS.

The output voltage ripple is found to be inversely proportional to the \( C_o \) and \( \omega_r \). And the OQS is verified to be so general that it is irrelevant to circuit parameters. The output ripple is reduced in a few times for the OQSC case in comparison with the ICMC case.

It is also found that the output voltage ripple of the OQS is nearly independent of the output voltage ratio only if the QSRC is in the CCM. Furthermore the dynamic ranges of the output voltage and load resistance, are maximized for the OQSC case widly.

REFERENCES


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