Modeling and Analysis of Static and Dynamic Characteristics for Buck-Type Three-Phase PWM Rectifier by Circuit DQ Transformation

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Abstract—The static and dynamic characteristics of buck-type three-phase pulse-width-modulation (PWM) rectifier are fully analyzed based on the dc and ac circuit models developed by the circuit DQ transformation. Various static converter characteristics such as gain, real and reactive power, power factor, and unity power factor conditions are completely analyzed. Transition characteristics are also analyzed by both exact small-signal model with full set of equations and simplified output model in explicit form. The usefulness of the models is verified through computer simulations and experiments with good agreements.

Index Terms—Circuit DQ transformation, current-source rectifier, PWM rectifier.

I. INTRODUCTION

PULSE-WIDTH-modulation (PWM) rectifiers have been widely used in recent years because of the increased demand of high power factor and low harmonics in utilities. Buck type has unique advantages compared to the boost one in high-power applications due to its low-voltage stress and wide output voltage dynamic range [1]. A clear understanding of the characteristics of the practical buck-type rectifier where input LC filters are contained is, however, not easy due to the increased dimension of system equation. The filter is used to suppress the switching harmonics, but it causes the unwanted phase shift and degrades the power factor. It is found that the phase shift depends highly on the modulation index and the phase angle of PWM switching pattern. This is why the control of the converter is so complicated for wide range operation.

There have been proposed a few control methods of the buck PWM rectifier, however most of them are based on simple models only considering the steady-state properties [2], ignoring input filter characteristics [3] or using per-phase circuit model [4]. It is quite evident that the applications of such simple models can guarantee neither high performance nor good stability because the dynamics of real converter varies widely with the operating point. A model with a full set of differential equations [5] describes the system precisely, but the dimensions of equations are of eighth order. Such an equational model requires the cumbersome and tedious matrix manipulation which results in very complex functions giving little physical intuition about the rectifier operation.

In this paper, the complete models of the buck-type PWM rectifier are presented using the circuit DQ transform method that is very effective in analyzing the multiphase ac circuits [6]–[8]. Equivalent dc and ac circuits which are linear and exact are obtained for the static and dynamic analyses, respectively. Various steady-state characteristics such as dc transfer function, real power, reactive power and power factor as functions of modulation index, phase angle, and load variations are investigated based on the dc circuits. Transition characteristics with respect to control inputs and disturbances are also analyzed based on the ac circuit of both state-space form and the transfer function form. Especially a simplified output model is supposed for practical control. Finally, various experimental results are included to verify the usefulness of the proposed models.

II. CIRCUIT DQ TRANSFORMATION OF THE RECTIFIER

A. Process of Circuit DQ Transformation

The buck-type rectifier shown in Fig. 1 is basically a time-varying system due to the switch set although the other circuit elements are linear time invariant (LTI). This impedes the application of any well-established LTI circuit analysis techniques, but the circuit DQ transformation method enables us to develop the complete circuit models by the following three steps [6], [7].

1) Partition the circuit into basic subcircuits.
2) Transform each subcircuits into DQ equivalent circuits based on the DQ transformation equations to eliminate the time-varying nature of the switching system. The three-phase-balanced voltage sources, switching functions, power invariant DQ transformation matrix, and a related equation are given as follows:

\[
V_{dc} = \begin{bmatrix}
V_{d} \\
V_{s} \\
V_{c}
\end{bmatrix} = \begin{bmatrix}
\sin(\omega t + \phi_1) \\
\sin(\omega t - 2\pi/3 + \phi_2) \\
\sin(\omega t + 2\pi/3 + \phi_3)
\end{bmatrix}
\]

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Fig. 1. Partitioned circuit of the buck-type PWM rectifier.

Fig. 2. DQ transformed equivalent circuit of the rectifier.

\[
S_{abc} = \begin{bmatrix} s_a \\ s_b \\ s_c \end{bmatrix} = m \cdot \begin{bmatrix} \sin(\omega t + \phi_2) \\ \sin(\omega t - 2\pi/3 + \phi_2) \\ \sin(\omega t + 2\pi/3 + \phi_2) \end{bmatrix}
\]  \hspace{1cm} (2)

and (3), shown at the bottom of page, and

\[
V_{qdo} = \begin{bmatrix} v_q \\ v_d \\ v_o \end{bmatrix} = K \cdot V_{abc}; \quad V_{abc} = K^{-1} \cdot V_{qdo}
\]  \hspace{1cm} (4)

where the fundamental components of switching functions are only considered by ignoring harmonics, and 
\( m \) represents the modulation index of PWM switching function. These relations are applied to each subcircuits to produce time invariant circuits.

3) Reconstruct the transformed subcircuits by connecting the nodes of adjacent subcircuits.

According to the above rule, the rectifier whose circuit elements and switches are assumed to be ideal is divided by six basic subcircuits as shown in Fig. 1. The equivalent LTI circuit model of the rectifier is obtained as shown in Fig. 2 by rejoining transformed subcircuits.

\[
K = (K^{-1})^T = \sqrt{\frac{2}{3}} \times \begin{bmatrix} \cos(\omega t + \phi) & \cos(\omega t - \frac{2\pi}{3} + \phi) & \cos(\omega t + \frac{2\pi}{3} + \phi) \\ \sin(\omega t + \phi) & \sin(\omega t - \frac{2\pi}{3} + \phi) & \sin(\omega t + \frac{2\pi}{3} + \phi) \\ \sqrt{1/2} & \sqrt{1/2} & \sqrt{1/2} \end{bmatrix}
\]  \hspace{1cm} (3)
B. Simplification of Equivalent Circuit

The equivalent circuit can be simplified further without sacrificing generality by selecting \( \phi \) with \( \phi_2 \) as shown in Fig. 3. This is justified that the \( \phi \) of the DQ transformation matrix \( K \) in (3) can be chosen arbitrarily independent of the system operation. Another way of simplification is \( \phi = \phi_1 \) case, however, this is not so much helpful for easy analysis of the rectifier.

The circuit shown in Fig. 3 represents the whole dynamic characteristics of the rectifier, hence, this can be used for dynamic performance simulations where the nonlinear control variables, \( \theta \) and \( m \), are not fixed to some constant values. It is found from Fig. 3 that the system order is reduced to six and the system stability is changed only by the modulation index \( m \) so far as all passive circuits elements are kept constant. Note that the power angle \( \phi \) does not change the system poles, but changes the zeros. These characteristics could also be understood through complicated equational analysis or experience, but none of the previous works shows so unified and simple physical views as this does. Now this circuit of Fig. 3 needs to be simplified further for the specific analyses purposes such as following dc and ac analyses.

III. Steady-State Analysis

A dc equivalent circuit representing the steady-state characteristics of the converter is obtained from Fig. 3 by shortening all the inductors and opening all the capacitors as shown in Fig. 4. In the dc circuit, instantaneous circuit variables and varying parameters \( \{i, v, m, \theta, \omega_0\} \) have stationary values \( \{I, V, m_0, \theta_0, \omega_0\} \). We represent the electrical dc variables as capital letter and denote the steady-state parameters by adding subscript “\( \alpha \),” and the expression of various circuit variables is followed by the representation of Fig. 1. The ac-side inductor resistance is ignored in the dc analysis since it is normally very small, and this simplifies the analysis for the main features of the system greatly. Now the analysis is straightforward.

Since the input currents of two gyrators are identical as shown in Fig. 4

\[
I_{\alpha_1} = \omega_0 C V_{\alpha_1} = I_{\alpha_2} = \frac{(V_{\alpha_1} - V_{\alpha_2})}{\omega_0 L}
\]

for

\[
V_{\alpha_1} = \frac{V_{ds}}{1 - \omega_0^2 LC}
\]

so

\[
V_o = D V_{\alpha_1} = 3 \frac{m_0 \tan \theta_0}{2 (1 - \omega_0^2 LC)} V_s.
\]

The other dc variables are as follows:

\[
V_{\alpha_2} = \sqrt{\frac{3}{2}} \frac{V_s}{1 - \omega_0^2 LC} \left( \sin \theta_0 - \frac{3 \omega_0 L m_0^2 \cos \theta_0}{2 (1 - \omega_0^2 LC)} R_0 \right)
\]

\[
I_{\alpha_2} = \frac{V_{\alpha_2}}{\omega_0 L} = \sqrt{\frac{3}{2}} \frac{V_s}{1 - \omega_0^2 LC} \left( \frac{3 m_0^2 \cos \theta_0}{2 (1 - \omega_0^2 LC)} R_0 - \omega_0 C \sin \theta_0 \right)
\]

\[
I_{\alpha_2} = \omega_0 C V_{\alpha_2} = \sqrt{\frac{3}{2}} \frac{\omega_0 C \cos \theta_0}{2 (1 - \omega_0^2 LC)} V_s.
\]

A. DC Output Characteristic

It is noted from (7) that the rectified output voltage is independent of load resistance \( R_0 \). This indicates the output voltage of the converter can be regarded as an ideal voltage source controlled by phase difference \( \theta \) and modulation index \( m_0 \), which is just a dual characteristic of the boost-type converter where the rectified output current is equivalent to a controlled ideal current source [7].

It is also noted that the dc gain ranges from zero to some maximum. It can be much higher than unity if the resonance frequency of the ac-side LC filter \( (1/\sqrt{LC}) \) is set to be near the line frequency \( \omega_0 \). But it is limited to a certain value in the real system, where the resonant frequency of the filter is designed much higher than the line frequency, which results in a little increase in the dc gain. Fig. 5 shows that the voltage gain can be controlled by switching function parameters such as phase angle \( \theta_0 \) and modulation index \( m_0 \). The circuit parameters used in the analysis are given in Appendix A.
B. Real and Reactive Powers

The real power $P$ generated from ac source is obtained as

$$P = V_{qs}I_{qs} + V_{ds}I_{ds} = \frac{3}{2} \frac{V^2_o}{1 - \omega_o^2 LC} \cdot Y \cos^2 \theta_o \cdot \frac{1}{R_o},$$

where

$$Y = \frac{3}{2(1 - \omega_o^2 LC)} \cdot \frac{1}{R_o}. \quad (12)$$

The first term of (13) represents the variable reactive power term generated from the converter, which is controlled by $\theta_o$ for a specific $P$. This indicates that reactive power can be controlled lagging or leading only by the phase difference $\theta_o$, where $m_o$ is predetermined to produce a certain output power. The second term means the constant leading VAR generated by the input LC filter. Hence, the converter must generate the same amount of lagging reactive power as this second term to obtain unity power factor at the source side. The reactive and real powers normalized by $V_o^2/R_o$ are shown in Fig. 6 as functions of phase and modulation index of the switching function.

C. Power Factor and Unity PF Operation

Power factor is given by

$$\text{PF} = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{\cos^2 \theta_o}{\sqrt{(\omega_o C)^2 - (\omega_o C) \sin(2\theta_o) + \cos^2 \theta_o}}, \quad (14)$$

The power factor can be unity when the reactive power $Q$ is controlled to be zero. From (13), the unity PF condition is given as

$$Y \cos \theta_o \sin \theta_o - \omega_o C = 0 \quad \text{or} \quad \theta_o = \frac{1}{2} \sin^{-1}(2\omega_o/C) \quad \text{for} \quad Y \geq 2\omega_o C, \quad (15)$$

If the parameter $Y$ of (12) is less than $2\omega_o C$, the power factor cannot be unity. This condition can occur when the load resistance $R_o$ is large, which is also a dual property of the boost-type converter when the load resistance is small [7]. Fig. 7(a) shows that power factor can be controlled as functions of control parameters, $\theta_o$ and $m_o$, but there are restricted ranges where power factor cannot be unity. Although there are two points for the power factor to be unity, only one of them, placed between $0^\circ$ and $45^\circ$, can be used in practice since usable output voltage is obtained. In this region, the rectifier can supply the maximum power with high power factor, but in the region of between $45^\circ$ and $90^\circ$, power utilization is not efficient and the power factor greatly changes with the phase difference. Fig. 7(b) shows the unity PF condition varies with load, and the range is narrower for lighter load.

Under the unity power factor condition of (15), the rectifier has some interesting characteristics. For example, the output
inductor current becomes

\[
I_o = \frac{3 V_s \cdot \cos \theta_0}{2(1 - \omega_s^2 L C R'_o)} \times \cos \left(\frac{1}{2} \sin^{-1} \left(\frac{2 \omega_s C}{Y}\right)\right)
\]

It is noted that the dc-side voltage and the real power as well as the current can be controlled only by the modulation index nearly independent of phase difference under the unity PF condition.

D. Operation Characteristics over All Range

The knowledge of operation over whole range is useful for the design and control of the rectifier. Fig. 8 shows how the gain, real power, reactive power, and power factor operate with the various operating points. The gain and the real power are higher as the phase difference goes to zero and modulation index is higher, but the reactive power has maximum leading power at \(45^\circ\) and maximum lagging power at \(-45^\circ\). The internal resistance, of course, changes the magnitude of the electrical quantities, but the general tendency is not affected greatly. For an analysis including the internal resistance, the relations of steady state in Appendix B must be used.

IV. DYNAMIC CHARACTERISTICS ANALYSIS

A. Small-Signal Perturbation Analysis

In this section, the dynamic characteristics of the PWM rectifier, i.e., the small-signal perturbation responses with respect to the control parameters such as phase difference \(\theta\) and modulation index \(m_o\) are analyzed. The circuit of Fig. 3 is perturbed since the system is nonlinear with respect to the control variables [6]. A small-signal perturbed circuit is shown in Fig. 9 by excluding the dc term and higher order perturbation terms. A complete state-space equation is obtained from this small-signal model as follows:

\[
\frac{d}{dt} \begin{bmatrix}
I_{es} \\
I_{eb} \\
\psi_{eq} \\
\psi_{cd} \\
i_o \\
V_{cd}
\end{bmatrix} = \begin{bmatrix}
-L & -\omega_o & -\frac{1}{L} & 0 & 0 & 0 \\
\omega_o & -\frac{1}{L} & 0 & -\frac{1}{T} & 0 & 0 \\
\frac{1}{C} & 0 & 0 & -\omega_o & 0 & 0 \\
0 & \frac{1}{C} & \omega_o & 0 & -\frac{\sqrt{3} m_o C}{2} & 0 \\
0 & 0 & 0 & \sqrt{\frac{3}{2} \frac{m_o}{T_o}} & -\frac{1}{T_o} & -\frac{1}{T_o} \\
0 & 0 & 0 & 0 & \frac{1}{T_o} & -\frac{1}{T_o}
\end{bmatrix} \times \begin{bmatrix}
\frac{\sin \theta_o}{L} \\
\frac{\cos \theta_o}{L} \\
-V_o \frac{\cos \theta_o}{L} \\
-V_o \frac{\sin \theta_o}{L} \\
0 \\
0
\end{bmatrix} \cdot \begin{bmatrix}
\dot{\theta} \\
\dot{\alpha}
\end{bmatrix}
\]

\[
(17)
\]
However, it is hardly possible to analyze manually either the circuit of Fig. 9 or the equation of (17), hence, the signal flow graph is used as shown in Fig. 10. The analysis results are summarized in Appendix B, where various transfer functions with respect to the inputs of the control variables and the source disturbance are derived, respectively. Still it is not easy to analyze the signal flow graph of Fig. 10, however, this is the most feasible way of analyzing such a sixth-order converter system among the analysis techniques ever found. This analytical result can be used for the controller design, which guarantees stable and fast responses for wide operating range. It is found from (17) or Appendix B that the dynamic characteristics of the buck-type PWM rectifier are summarized as follows.

1) System poles are dependent only on the modulation index when the circuit parameters are fixed. In the real system, however, the load $R_o$ varies and the rectifier poles approach to the origin as the load resistance increases.

2) Output transfer functions with respect to the modulation index are independent of the phase difference, but vary by the modulation index, output voltage, and output current.

3) Zeros of the output transfer functions with respect to the phase difference or source disturbance are independent of the modulation index while dc gains are dependent.

4) Fig. 11 shows the dynamics change greatly with the operating point. The system property is changed from the minimum phase system at $m_o = 0.2$ to nonminimum phase system with positive zero in the right-half plane at $m_o = 0.8$. 

Fig. 8. Operation characteristics as functions of phase difference and modulation index.

Fig. 9. Perturbed ac circuit for small-signal analysis [$v_1 = \sqrt{2}(\dot{\theta}_e \sin \theta_o + \dot{\theta} V_o \cos \theta_o)$ and $v_2 = \sqrt{2}(\dot{\theta}_e \cos \theta_o - \dot{\theta} V_o \sin \theta_o)$].
Fig. 10. Perturbed signal flow graph with respect to variations of control parameters $\theta$ and $m$.

Fig. 11. Bode diagram of transfer function $\frac{V_{cc}}{m}$.

B. Simplified Dynamic Characteristic

Analytically, the previous exact ac models are quite preferable, however, in the engineering sense they are still too complicated to apply in practice. Hence, it is necessary to develop more simplified model for practical application purpose. Fortunately, the converter has two distinct characteristics. One is the fast response dominated by the ac-side LC filters, and another is the slow response dominated by the dc-side LC filter. They are coupled with each other through the rectifier switch set, equivalently a transformer.

If the control objectives are power factor correction and stable and fast output voltage/current regulations, a slow response circuit is more dominant over a fast response one. In this case, the ac-side circuits are regarded as ideal voltage sources at a specific operating point, and the fast response characteristic is ignored. Thus, a simplified output model is obtained as shown in Fig. 12, where the ac-side filters are assumed to be in the steady state while the dc-side filter is assumed to be in the transient state. There are three perturbed voltage sources as

$$\theta(s) = -\frac{\sqrt{3}}{21-\omega_c^2 LC} \hat{V}(s)$$ (18)
The simplified model of Fig. 12 is quite different from the simple output model ignoring ac input characteristics because this model has full information about input ac filters and current operating point. The accuracy of this simplified model is determined by the degree of separation between the poles of ac filters and those of dc filters. This simplified ac circuit model is quite useful for a fast and simple controller design with plenty of physical insights.

V. VERIFICATION OF MODELS BY SIMULATION

Since the purpose of this paper is to provide exact and easy understandable models, the verifications are done first by the computer simulations to avoid erroneous disturbances imposed by many practical situations in the experiment. The computer simulations enable us to simulate various considerations with great ease, very low cost, and little time consumption.

The simulations are done by the numerical calculation of the eighth-order time-varying differential equations of (21), given at the bottom of the next page, for the original rectifier of Fig. 1, and a fourth/fifth-order Runge–Kutta method is used as a numerical algorithm. In these equations, source voltages $V_s$, $V_{a_s}$, and $V_{c_s}$ of (1) are used with $V_s = 220\sqrt{2}$ and $s_{a_s}$, $s_{b_s}$, and $s_{c}$ can be any switching functions from various PWM

\[
\begin{align*}
\hat{v}_m(s) & = \frac{3}{2} \frac{V_s \cos \theta_o}{1 - \omega^2_{2LC}} \hat{n}(s) \\
\hat{v}_d(s) & = \sqrt{3} \frac{V_s \cos \theta_o}{2} \hat{v}_b(s) 
\end{align*}
\]
methods. For general performance comparison, we use both real switching functions of the modified PWM [9] as shown in Fig. 13 and ideal switching functions defined in (2) with the switching pattern harmonics excluded.

There are three time steps in simulation. The first is for source voltage perturbation, and the second and third are for modulation index and phase difference perturbations as shown in Fig. 14. The system is started at \( t = 0 \) with \( m_o = 0.8 \) and

\[
\begin{bmatrix}
\frac{di_1}{dt} \\
\frac{di_2}{dt} \\
\frac{di_3}{dt} \\
\frac{di_4}{dt}
\end{bmatrix} =
\begin{bmatrix}
-\frac{1}{L} & 0 & 0 & -\frac{1}{L} \\
0 & -\frac{1}{L} & 0 & 0 \\
0 & 0 & -\frac{1}{L} & 0 \\
0 & 0 & 0 & -\frac{1}{L}
\end{bmatrix}
\begin{bmatrix}
i_{s1} \\
i_{s2} \\
i_{s3} \\
i_{s4}
\end{bmatrix} + \begin{bmatrix}
\frac{1}{L} \\
\frac{1}{C} \\
\frac{1}{C} \\
\frac{1}{C}
\end{bmatrix}
\begin{bmatrix}
v_o \\
v_{o1} \\
v_{o2} \\
v_{o3} \\
v_{o4}
\end{bmatrix}.
\]
\( \theta_o = 10^\circ \), all initial conditions are zeros. When the system reaches to a steady-state condition, the modulation index is abruptly increased by 10\% at \( t = 60 \) ms. After reaching another steady-state condition, the phase difference is now increased by 5\% at \( t = 90 \) ms.

It is noted that when source voltages are applied abruptly at the start, high inrush current flows due to the resonance with the ac-side LC filter and this also drive the dc-side current and voltage to have high peak values. So careful process is needed at the starting point. In the real PWM case, it is observed that much ripples in output current \( i_o \) exist by the harmonics of switching function, but the average trajectory of the ripple is almost identical with the ideal switching function case. This shows that the circuit DQ transformed model describes averaging behavior of the rectifier and it is identical to the ideal switching function model. Although the switching harmonics are reflected in the output current, the line-side current \( i_{ds} \) and output dc voltage \( v_{dc} \) have little harmonics due to the filtering effect of ac- and dc-side LC filters, respectively. The small-signal responses of Fig. 14 verify that the numerical calculation method and the small-signal model show almost the same transient response.
characteristics. Especially the response of the system with ideal switching function is almost identical to the small-signal response. Only a negligible error occurs in the case of the phase perturbation due to the exclusion of the higher-order terms during the small-signal model.

The simplified output model of Fig. 12 is also verified through the simulations as shown in Fig. 15. It is identified from Fig. 15 that there are same mismatches in the response, however, the simplified output model could be useful for practical situations. This model does not guarantee wide application areas which include some extreme cases. The role of it could be understood that gives physical insight and usefulness for rough design.

An example for an extreme case where the input ac filters have lower poles than the output dc filters is shown in Fig. 16 with the case of the small-signal model for comparison. Here,
the outputs $L_o$, $C_o$, and $R_o$ are changed to 2 mH, 50 $\mu$F, and 15 $\Omega$, respectively. Note that the small-signal model is still valid for this extreme case while the simplified model is not.

There is some uncertainty in the parameter of the converter. The values of input-side inductors are not easily determined since they include the line and source inductors that may vary with the structure changes in power lines or power facilities. Considering this uncertainty the exactness of the model is relatively not so much important than the simplified one in the real situation. This is another reason why the simplified output model is useful even though it has some amount of error.

VI. EXPERIMENT

By comparison of the solution of numerical full-order differential equation, it has been shown that the model based on the circuit DQ transformation is sufficiently accurate and more effective to estimate the behavior of the converter. Now, to confirm the effectiveness of the model in real system, various experiments of a scaled-down 1-kVA converter of Fig. 1 are performed by using IGBT as main switch. Converter is designed as the values of Appendix A and $V_s = 40\sqrt{2}$. Converter waveforms and various electrical quantities are measured by Lecroy’s DSO scope 9354M and Voltech’s power analyzer PM3000A, respectively.

A. Experiment Results of Steady-State Characteristics

Fig. 17 shows the whole steady-state waveforms of the buck PWM rectifier at the operation condition of $m_o = 0.8$ and $\theta_h = 5^\circ$. Detected signal of source voltage $v_{sa}$ of phase $a$ and gate signal of switch $s_1$, converter unfiltered output voltage $v_c$, converter input current $i_{1i}$, converter output current $i_o$, converter filtered output voltage $v_{co}$, phase current $i_{as}$ of phase $a$ and ac-side capacitor voltage between phase $a$ and phase $b$, $v_{c1} - v_{c2}$, are shown sequentially at identical trigger position. Detected signal of source voltage $v_{as}$, which operates as a reference signal of the system, is synthesized from the sensing signal of line to line voltages by the relation $v_{as} = (v_{ab} - v_{ca})/3$. Fig. 17 also includes simulated waveforms of $i_{as} \cdot i_{co}$, $v_{co}$, and $v_{c1} - v_{c2}$ by inverse transformation from the values of dc model, and this shows all fundamental components waveforms both the ac and dc side can be exactly calculated. From these, the waveform of unfiltered output voltage $v_c$ can be easily calculated by using instantaneous switching function [9].

Various steady-state characteristics of Figs. 5 and 6 in the real system are verified as shown in Fig. 18. The values calculated from the steady-state relations in Appendix B including the internal resistance are nearly identical to the measured values. Around the phases of 90° and −90°, some mismatch is occurred due to the effect of used snubbers, but it is negligible in the main operation of rectifier.

Fig. 19 demonstrates the analysis of Fig. 7(b) which shows that unity power factor condition changes by the load variation. When the load varies from $R_o = 10 \Omega$ to $R_o = 20 \Omega$, unity power factor condition moves from $\theta_h = 5^\circ$ to $\theta_h = 25^\circ$ at $m_o = 0.8$.

B. Experiment Results of Transient Characteristics

Fig. 20 shows the dynamic characteristics of the case where the operating point at $m_o = 0.8$, $\theta_h = 10^\circ$ changes to the point of $m_o = 0.88$, and after 30 ms, the operating point changes again to the point of $\theta_h = 15^\circ$ whose operation was estimated by Fig. 14(b). From Fig. 20, It is known that the dynamics between the rising step response and the falling step response is different, the falling step response has some longer delay before the falling process.

It is expected that the dynamics of the converter can be operated as nonminimum phase system in Fig. 11. To show this clearly, a step response experiment from $m_o = 0.9$ to $m_o = 0.6$ is performed in stiff condition of load $R_o = 2 \Omega$ and $V_s = 17\sqrt{2}$. Fig. 21 shows the result that at the instance of step change, negative direction of change is happened before the voltage drop, which shows the phenomena of the nonminimum phase system where some zeros are in the right-half plane. In high-order system such as this rectifier, the effect of positive zero is occurred as a simple delay in dynamic response or the
negative initial slope of the response for a command. Anyway, such a delay in the start of the response may impede the successful design of the high performance controller, so careful analysis should be performed for the fast and wide range operated control.

VII. CONCLUSION

The buck-type PWM rectifier is completely analyzed using circuit DQ transformation. Exact LTI dc and ac equivalent circuits in a unified form are induced and full set of equations are derived in explicit form. From the dc circuit, various characteristics such as gain, powers, power factor, unity power factor condition, and the practical behavior including component and load variations are analyzed clearly. From the ac perturbation circuits, the system dynamic characteristic equations of small-signal dynamic model and simplified output model are fully derived. The circuit models are verified through computer simulations and experiments with good agreement, so it is very useful to estimate all the electrical values of the designed converter and to design the controller.

APPENDIX A

PWM RECTIFIER PARAMETERS

The parameters used for the buck-type PWM rectifier are as follows unless otherwise stated:

\[ L = 3 \text{ mH},\ C = 50 \mu\text{F},\ r = 0.3 \text{ \Omega},\ r_{dc} = 0.8 \text{ \Omega},\ L_0 = 5 \text{ mH},\ C_0 = 800 \mu\text{F},\ \hat{R}_0 = 10 \Omega,\ \omega_0 = 120\pi \text{ rad/s}. \]

APPENDIX B

TABLE OF TRANSFER FUNCTIONS

A. Characteristic Equations

\[
D(s) = D_6 s^6 + D_5 s^5 + D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0
D_6 = L_2^2 C^2 L_0 C_0 \hat{R}_0
D_5 = L_2^2 C^2 (L_0 + r_{dc} R_0 C_0) + 2r LC^2 R_0 L_0 C_0
D_4 = L_2^2 C^2 (R_0 + r_{dc} C_0) + 2r LC^2 (L_0 + r_{dc} R_0 C_0)
D_3 = \frac{3}{2} L^2 C R_0 C_0 r^2 C + 2r^2 C^2 + 2 LC
D_2 = (2^\omega_0^2 r LC^2 + 2 r C)(L_0 + r_{dc} R_0 C_0)
D_1 = (2^\omega_0^2 r LC^2 + 2 r C)(R_0 + r_{dc} C_0)
D_0 = (\omega_0^2 r^2 C^2 + (1 - \omega_0^2 LC)^2) R_0 + r_{dc} + \frac{3}{2} r m_0^2 R_0 C_0
\]

B. Control Variable to Output Transfer Functions

\[
\frac{\hat{V}_co}{\hat{m}} = \frac{3 m_o N(s)}{2 D(s)},\ \frac{\hat{I}_o}{\hat{m}} = \frac{(C_o R_o S + 1)}{R_o},\ \frac{\hat{V}_co}{\hat{m}} \cdot \frac{\hat{m}}{\hat{m}}
N(s) = N_1 s^4 + N_2 s^3 + N_2 s^2 + N_1 s + N_0
N_1 = L_2^2 C^2 V_o
N_2 = (2^\omega_0^2 r LC^2 + 2 r C) V_o - 3r LC m_0 L_o
N_3 = 2r LC V_o - \frac{3}{2} L_2^2 C^2 m_0 L_o
N_4 = (2^\omega_0^2 r LC^2 + 2 r C) V_o
N_5 = (2^\omega_0^2 r LC^2 + 2 r C) V_o - 3r LC m_0 L_o
N_6 = (2^\omega_0^2 r LC^2 + (1 - \omega_0^2 LC)^2) V_o - \frac{3}{2} r m_0^2 L_o
[see (B-1) at the top of the page] and
\]

C. Source Perturbation to Output Transfer Functions

\[
\frac{\hat{I}_o}{\hat{v}_s} = \frac{(R_0 C_o S + 1)}{R_o},\ \frac{\hat{V}_co}{\hat{v}_s} \cdot \frac{\hat{v}_s}{\hat{v}_s}
\]

See (B-2) at the top of the page and
D. Steady-State Relations

\[
V_{cq} = \left(1 - \omega_o^2LC + \frac{3}{2} \frac{m_o^2r}{R_o + r_{dc}}\right) V_{gs} - \left(\omega_o r C + \frac{3}{2} \frac{m_o^2\omega_o L}{R_o + r_{dc}}\right) V_{ds}
\]

\[
V_{cd} = \frac{\omega_o r CV_{gs} + \left(1 - \omega_o^2LC\right) V_{ds}}{\Delta}
\]

\[
I_{qs} = \frac{\omega_o^2C^2r V_{gs} + \left(1 - \omega_o^2LC\right) \omega_o CV_{ds}}{\Delta}
\]

\[
I_{ds} = \frac{-\omega_o C^2(1 - \omega_o^2LC)}{\Delta} V_{gs} + \frac{\omega_o^2 C^2 r + \frac{3}{2} \frac{m_o^2}{R_o + r_{dc}}}{\Delta} V_{ds}
\]

\[
= \sqrt{3} m_o \cdot I_o
\]

\[
V_o = \sqrt{\frac{3}{2}} m_o \cdot V_{cd} = \left(r_{dc} + R_o\right) I_o
\]

where

\[
\Delta = \left(1 - \omega_o^2LC\right)^2 + \left(\omega_o r C\right)^2 + \frac{3}{2} \frac{m_o^2 r}{R_o + r_{dc}}
\]

\[
V_{gs} = \sqrt{\frac{3}{2}} V_s \cdot \sin \theta_o, \quad V_{ds} = \sqrt{\frac{3}{2}} V_s \cdot \cos \theta_o.
\]

REFERENCES


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