Uncertainty Aversion and Business Condition

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May 2007

Abstract

This thesis focuses on uncertainty which begins with Knightian Uncertainty. First we introduce a concept of time-varying uncertainty aversion. We find that uncertainty aversion is increasing before the crash and is resolved after the crash, and tends to move together with S&P 500. Second we present a relationship between uncertainty aversion and business condition. We construct a VECM regression and Granger Causality tests. Using credit spread and term spread as indicators of business conditions, we find some interesting results: (1) Uncertainty aversion has significant positive relationship with credit spreads in United States. (2) Uncertainty aversion has no significant relationship with term-spreads. (3) Uncertainty aversion granger causes both credit spreads and term spreads. This implies that with uncertainty aversion we can explain the credit spread puzzle as well as we can predict future business conditions. If today's uncertainty increases, tomorrow's business condition will be worse, and if today's uncertainty decreases or is resolved, tomorrow's business condition will be better.

Keywords: *Model Uncertainty; Robust Control; Bayesian Learning; Uncertainty Aversion; Business Conditions;*
JEL Classification: *D81, G10, G11*

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1. Introduction

Investors fear unexpected shocks such as Black Monday (1987/10), 9.11 Terror (2001/9). Economists have tried to measure the aversive attitude about these unexpected shocks. They disentangle the risk aversion from uncertainty aversion. Risk aversion means aversive attitude about the known distribution. But uncertainty aversion assumes that investors do not know the reference distribution. So, economists define the uncertainty aversion to be the aversive attitude about the unknown distribution. This Uncertainty aversion is based on the Knightian Uncertainty of Ellsberg Paradox[1]. The Ellsberg Paradox is a paradox in decision theory in which investor's choice violates the Von-Neumann expected utility hypothesis. It is generally considered to be an evidence for ambiguity aversion.

This paper focuses on uncertainty which begins with Knightian Uncertainty. According to latest papers, uncertainty aversion can be measured by robust control theory, developed by Anderson Hansen Sargent (2000). Recently Maenhout (2004, 2006) presented a new method of the dynamic portfolio and consumption rules based on AHS (2000). Assuming the uncertainty about the return process, they found a closed form solution of the optimal portfolio rules and estimated constant uncertainty aversion in the style of Lucas' General Equilibrium (1978). Maenhout showed that robustness dramatically decreases the demand for risky assets and is equivalent to recursive

[1] Ellsberg Paradox violates dominance axiom of Von-Neumann expected utility.

preferences. This means that equity premium puzzle and risk-free rate puzzle can be explained by robustness.

In this paper, first we introduce a concept of time-varying uncertainty aversion. Above recent studies assumed constant uncertainty aversion. But, sometimes unexpected shocks like 9.11(2001) terror, Black-Monday (1987) happened and small shocks in a year also happened even now. So, to capture this premium of unexpected shocks, we set up a basic model of time-varying uncertainty aversion. Extending Maenhout (2004)'s work, we estimate time varying uncertainty aversion in United States. With this time varying uncertainty aversion, we find that uncertainty aversion is increasing before the crash and is resolved after the crash, and tends to move together stock return.

Second we suggest a relationship between uncertainty aversion and business condition. We construct a regression of uncertainty aversion and indicators of business cycle. As indicators of explanatory variables, we use both credit spreads and term spreads that were used in Fama & French (1989). Applying VECM models and Granger Causality test, we find some interesting results: Uncertainty aversion has significant positive relationship with credit spreads in United States. And uncertainty aversion has no significant relationship with term-spreads. And finally we find that uncertainty Aversion Granger causes both credit spreads and term spreads. This implies that we can explain the credit spread puzzle with uncertainty aversion as well as we can predict business conditions with uncertainty aversion,

The organization of this paper is as follows. Chapter2, we introduce a brief overview of related papers. Chapter3, we suggest the time varying uncertainty aversion. Chapter4, we have some regression models about uncertainty aversion. And we present the relationship between the uncertainty aversion and indicators of business cycle. Chapter5, we give a conclusion.

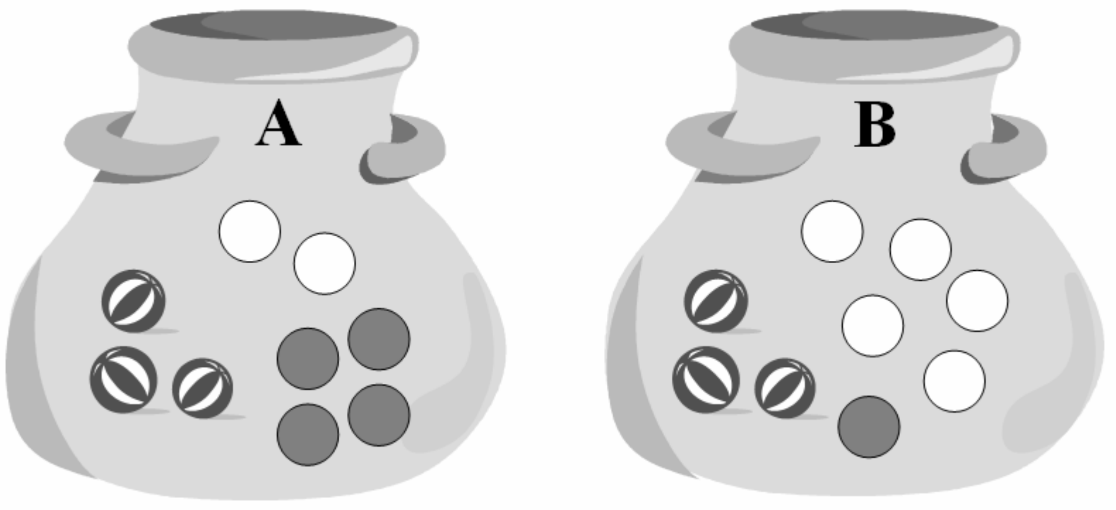
2. Literature Review

2.1 Knightian Uncertainty & Ellsberg Paradox.

This paper is based on Knightian uncertainty. In economics, Knightian uncertainty is considered to be a kind of risk that is impossible to calculate. As an aversion of unknown distribution, economists usually disentangle uncertainty aversion from risk aversion. Also, Ellsberg paradox is generally taken to be an evidence of uncertainty. To understand our framework, let us give a simple example.

Figure 1

Urn for uncertainty aversion



Suppose we have an urn containing nine balls. We know the certainty three stripe balls, but we don't know the distribution of white balls or black balls. As seen above, we will consider two cases. First urn (case A) has two white balls and four black balls, and Second urn (case B) has five white balls and one black balls. Besides on two cases, there will be another distributions, for example all six may be black or all six may be white. In this setting, if one play two types of game, famous Ellsberg Paradox happens. Two types of games are as follows. As shown below table, each game has two states. In game I, if one draws a stripe ball, he or she will get a one hundred dollar (case A). And if one draws a black ball, he or she will get a one hundred dollar (case B). Considering two states in game I, people usually prefer A to B. That's because game participants want to avoid the unknown distributions. Similar to case game I, in game II, if one draws a stripe ball or white ball, he or she will get a one dollar (case C). And if one draws a black or white ball, he or she will get a one dollar (case D). Considering two states in game II, people tends to prefer D to C. That's because the probability of having either black or white ball is certain and the probability of having either stripe or black ball is uncertain.

Table 1
Ellsberg Paradox

Game (I)

	State A: Drawing a stripe ball	State B: Drawing a black ball
Prize	$100	$100

Game (II)

	State C: Drawing a stripe or white ball	State D: Drawing a black or white ball
Prize	$100	$100

Let us define the estimated probability of stripe balls, white balls, and black balls as P_s, P_w, P_b, we can easily find contradiction in the world that Von-Neumann's expected utility theorem holds.

$$\begin{aligned}
&Game\ I: \ P_s \cdot U(\$100) + (1-P_s)\cdot U(\$0) \succ P_b \cdot U(\$100) + (1-P_b)\cdot U(\$0) \\
&Game\ II: \ P_b \cdot U(\$100) + P_w \cdot U(\$100) + P_s \cdot U(\$0) \succ P_s \cdot U(\$100) + P_w \cdot U(\$100) + P_b \cdot U(\$0)
\end{aligned} \tag{1}$$

Simplifying two games, we can express preference orders as equations (1). Rearranging equation (1), we can derive contradict results[2]. This is a famous Ellsberg Paradox.

$$\begin{aligned}
&Game\ I: \ P_s \succ P_b \\
&Game\ II: \ P_b \succ P_s
\end{aligned} \tag{2}$$

Based on this Ellsberg Paradox, we map these unknown balls on unknown asset's returns. Especially we are interested in game which provides increasing number of balls.

2.2 Robust Control Problem

Uncertainty aversion is related with min-max utility and recursive preferences. Anderson, Hansen, Sargent (2000) developed robust control theory considering this kind of recursive preferences and Maenhout (2004) applied robust control theory to

2 $A \succ B$ implies that investors prefer A to B.

asset pricing and find some interesting results.

Anderson, Hansen, Sargent (2000) assumes that investors has a reference model and other alternative models. Alternative models are associated with the idea that investors worry about pessimistic situation such as a 9.11 terror (2001), Black Monday (1987). Anderson, Hansen, Sargent (2000), Hansen and Sargent (2001) introduced robust control and model uncertainty. First we will review their frameworks.

Let $\{B_t\}$ be the standard Brownian motion on the probability space (Ω, F, P). And investors want to maximize his objective utility U , where C is a set of control process such as a consumption plan, portfolio rules and x_t is a state process such as wealth dynamics.

$$\begin{aligned} &\sup_{c \in C} E\left[\int_0^\infty \exp(-\delta t) U(c_t, x_t) dt\right] \\ &s.t\ dx_t = \mu(c_t, x_t) dt + \sigma(c_t, x_t) dB_t \end{aligned} \tag{3}$$

Equation (3)'s constraint equation is a kind of reference model. Usually investors believe this reference model, but suspect it to be miscalculated. So, fearing unexpected shocks, investors consider alternative models.

$$dx_t = \mu(c_t, x_t) dt + \sigma(c_t, x_t)(h_t dt + d\hat{B}_t) \tag{4}$$

To build a perturbed model, Anderson, Hansen, Sargent (2000), Hansen and Sargent (2001) and Maenhout (2004, 2006) replaces B_t in (3) into $\widehat{B_t} + \int_0^t h_s ds$ where h is measurable and $\widehat{B_t}$ is a Brownian motion. Anderson, Hansen, Sargent (2000) says that potential distortions in the state evolution indexed by $\{h_t\}$ induces a model family of the state evolution. Also this family plays the role of the multiple priors in Gilboa and Schmeidler (1989). Equation (4) assumes drift uncertainty rather than second moment's uncertainty. It is reasonable because for second moments, large number of sampling could remove estimation risk.

To measure the discrepancy between reference model and alternative models, Anderson, Hansen, Sargent (2000) uses relative entropy. Relative entropy can be considered as an expectation of log likelihood ratio. Explicitly, they define the relative entropy be the expected log Radon-Nikodym derivative.

Let $P^* = P \times M$ [3] where P is a probability measure and M is exponentially distributed with density $\delta \exp(-\delta t)$. And expectation $E^*(g)$ can be defined like below where $\{g_t\}$ is a stochastic process.

$$E^*(g) = \delta \int_0^\infty \exp(-\delta t) E(g_t) dt \tag{5}$$

Similarly let $Q^* = Q \times M$ where Q is a probability measure and M is above

[3] $P \times M = \{(p,m) | p \in P, m \in M\}$

defined. And the process $\{q_t\}$ is a Radon-Nikodym derivative for Q^* with respect to P^*.

$$E_Q^*(g) = \delta \int_0^\infty \exp(-\delta t) E(q_t g_t) dt \tag{6}$$

The Relative Entropy $R(Q)$ is defined as follows.

$$R(Q) = \int_0^\infty \exp(-\delta u) E_Q \left(\frac{|h_\tau|^2}{2} \right) du^{4} \tag{7}$$

Hansen, Sargent (2001) introduces two robust control problems, multiplier robust control problem and constraint robust control problem. Multiplier robust control problem[5] is

$$\begin{aligned} &\sup_{c \in C} \inf_Q E_Q \left[\int_0^\infty \exp(-\delta t) U(c_t, x_t) dt \right] + \theta R(Q) \\ &s.t\ dx_t = \mu(c_t, x_t) dt + \sigma(c_t, x_t) dB_t \end{aligned} \tag{8}$$

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[4] when q_t is Radon-Nikodym derivative,

$$\log q_t = \int_0^t h_\tau \cdot d\hat{B}_\tau - \int_0^t \frac{|h_\tau|^2}{2} d\tau$$

[5] Constraint robust control problem is

$$\begin{aligned} &\sup_{c \in C} \inf_Q E_Q \left[\int_0^\infty \exp(-\delta t) U(c_t, x_t) dt \right] \\ &s.t\ R(Q) \le \eta \text{ and } dx_t = \mu(c_t, x_t) dt + \sigma(c_t, x_t) dB_t \end{aligned}$$

With the parameter θ^*, we can measure the robust preferences. This means that more robust investor who has large θ^* has less belief in the reference model. And they show that multiplier robust control problem has the same solution as a recursive risk sensitive control problem, where θ^{-1} is the risk sensitivity parameter[6].

2.3 Dynamic Portfolio Problem with model uncertainty

Merton (1969, 1971) pioneered dynamic portfolio problem. But he assumed non stochastic investment opportunity set, no market friction, i.i.d stock returns, and complete market. Kim & Omberg (1996) solved optimal portfolio rules incorporating stochastic opportunity set. With market friction and non-Gaussian based approaches including jump process, stochastic volatility and so on, have been applied to dynamic portfolio selection problem. Recent Maenhout (2004, 2006)[7] presented model uncertainty to the dynamic portfolio selection problem. In this section, we will briefly summarize his paper.

A representative agent considers another wealth dynamics which is related to worry about pessimistic scenarios. Maenhout (2004, 2006) used the same equation to originally referred one by Anderson, Hansen, Sargent (2000). This alternative model contains the endogenous drift distortion $u(W_t)$ as below.

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[6] Duffie and Epstein (1992) call θ^{-1} the variance multiplier in Stochastic Differential Utility.

[7] He assumes incomplete market, since he suggested one risky asset and more than two Brownian Motions

$$dW_t = \mu(W_t)dt + \sigma(W_t)\left[\sigma(W_t)u(W_t) + dB_t\right] \tag{9}$$

In this model uncertainty world, a representative agent chooses drift adjustment $u(W_t)$ to minimize the sum of expected differential payoff equation based on equation (9) and the penalty entropy. The entropy penalty happens when choosing the drift distortions $u(W_t)$ and moving away from the reference model, and is weighted by ψ^{-1}. Thus, Maenhout (2004) derived robust Hamilton-Jacobi-Bellman equation which contains both distorted drift distortion and entropy penaly. Alternative models with low entropy are statistically hard to distinguish from the reference model[8]. If an agent has no robustness, i.e. he or she has strong belief in reference model, then $\psi = 0$, which is same to expected utility maximization case.

So, robust Bellman equation for optimality is

$$0 = \sup_{\alpha,C} \inf_u \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \delta V(W,t) + D^{(\alpha,C)}V(W,t) + V_w \alpha^2 \sigma^2 W^2 u + \frac{1}{2\psi} \alpha^2 \sigma^2 W^2 u^2 \right] \tag{10}$$

Where

[8] The reference model is

$dW_t = \mu(W_t)dt + \sigma(W_t)dB_t$

$$D^{(\alpha,C)}V(W,t)=V_w\left[W(r+\alpha(\mu-r))-C\right]+V_t+\frac{1}{2}V_{ww}\alpha^2\sigma^2W^2 \tag{11}$$

In equation (10), third term reflects distorted drift distortion and fourth term is entropy penalty.

Also Assuming that the dividend process is geometric Brownian motion like below.

$$dD_t=\mu_D D_t dt+\sigma_D D_t dB_t \tag{12}$$

In Lucas (1978) style Equilibrium, let the price of risky asset be $S_t=a^{-1}D_t$. Then the excess return[9] on the risky asset is

$$\frac{dS_t+D_t dt}{S_t}-rdt=[\gamma+\theta]\sigma_{cs}dt+\sigma_s dB_t \text{ }^{10} \tag{13}$$

And equilibrium risk free rate is

$$r=\delta+\psi\mu_d-\frac{1}{2}\left[1+\psi\right][\gamma+\theta]\sigma_d^2 \tag{14}$$

Using equation (13) and (14), Maenhout (2004) suggests that robustness parameter

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[9] $\sigma_{cs}=\rho\sigma_c\sigma_s$

[10] Proof : see the appendix of Maenhout (2004)

increases the equity premium and decreases the risk free rate. In other worlds, he presents a solution for both equity premium puzzle and low risk free rate puzzle. In addition, equation (13) shows that the discrepancy between pessimistic equity premium[11] and equity premium puzzle can be expressed as a function of robustness parameter i.e. uncertainty aversion.

[11] Pessimistic equity premium supporting equilibrium is $\gamma\sigma_{cs}$ in Maenhout (2004).

3. Time Varying Uncertainty Aversion

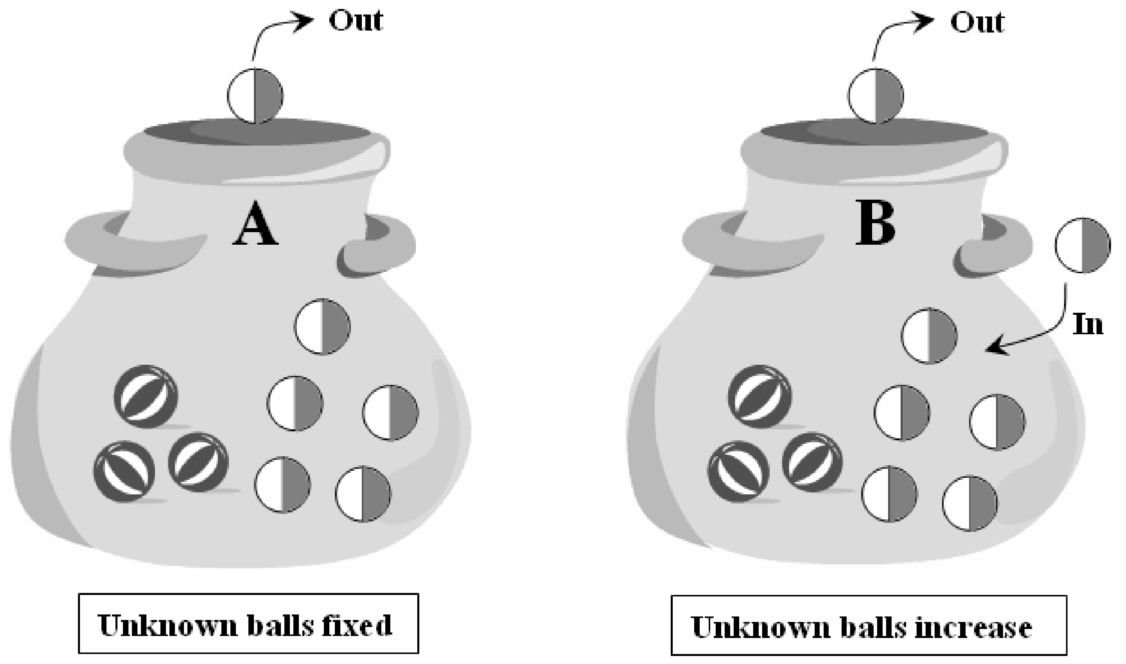
To incorporate the dynamics of discrepancy between reference model and alternative models, we present time varying uncertainty aversion, i.e. a kind of uncertainty aversion time series. Based on model uncertainty, we assume that investors consider both reference model and alternative model, and each reference model and alternative models evolve as time passes. Similar to Anderson, Hansen, Sargent (2000), Maenhout (2004, 2006), in our framework, investors worry about the pessimistic situation due to a sudden shock and so investors consider alternative models that have drift distortions away from reference model. However, using the learning property, we extend fixed uncertainty aversion into time varying uncertainty aversion. Our agents update models similar to generalized Bayesian[12] Learning of Epstein and Schneider (2005). But they assume that using memoryless mechanism, learning can cease without all uncertainty having been resolved. However, we don't fix the information set; rather consider information set is expanding as time passes. Hence our agents update reference model through updating, simultaneously considering worst case alternative model at each period.

[12] Garlappi, Uppal, Wang (2007) refers that in Bayesian approach, unknown parameters were treated as random variables, and assumed to have only single prior i.e. to be neutral to uncertainty.

3.1 Drawing Balls in an urn

Figure 2

Urn for time varying uncertainty aversion



Our agents take part in drawing balls in an urn. Epstein & Schneider (2005) suggested the multiple priors' model but they assume as the number of draws increases, uncertainty will be resolved in the long run. As seen by the left part of Figure 2, since they fix the total number of unknown balls, it is plausible that uncertainty will be resolved in the long run. In contrast, as seen by the right part of the Figure 2, we don't fix the total number of unknown balls, rather assumes the number of balls are increasing. This scenario is associated with the information set is expanding as time passes. So, our agent draws and updates her model, but at each period she considers worst case alternative model.

3.2 Basic Setup

We consider one risky asset with two models and one risk free asset with constant interest rate. Let $\{B_t\}$ be a standard Brownian motion on a probability space (Ω, F, P) and F_t is a filtration generated by this Brownian motion. Then F_t is increasing set[13].

Given risky asset process is

$$dS_t = \mu S_t dt + \sigma S_t dB_t \tag{15}$$

The reference model of state (or wealth) dynamics is

$$dW_t = \mu(W_t)dt + \sigma(W_t)dB_t = \left[W_t(r + \alpha_t(\mu - r) - C_t\right]dt + \alpha_t \sigma W_t dB_t \tag{16}$$

And alternative model of state (or wealth) dynamics is

$$dW_t = \mu(W_t)dt + \sigma(W_t)\left[\sigma(W_t)u(W_t) + dB_t\right] \tag{17}$$

Also, our agent updates her reference model with maximizing her log likelihood.

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13 $F_1 \subset F_2 \ldots \subset F_t \subset F_{t+1} \subset F_{t+2}$

$$L_t\left(\hat{\mu}_t,\hat{\sigma}_t\right)=\prod_{i=1}^{t} f\left(\mu_1,\mu_2,...,\mu_t\,\middle|\,\hat{\mu}_t,\hat{\sigma}_t\right)=\left(\frac{1}{2\pi\hat{\sigma}_t^{\,2}}\right)^{t/2}\exp\left(-\frac{\sum_{i=1}^{t}\left(\mu_i-\hat{\mu}_t\right)^2}{2\hat{\sigma}_t^{\,2}}\right) \qquad (18)$$

Where μ_i is an observed risky asset return on i-period, $\hat{\mu}_t$ is an updated drift of risky asset on t-period, $\hat{\sigma}_t$ is an updated standard deviation of risk asset on t-period. With estimated her reference model, she worries about pessimistic situation considering worst case alternative model. Hence, updating her reference model, we can derive the time varying discrepancy between reference model and alternative model. We call this time varying discrepancy a time varying uncertainty aversion.

3.3 Time Varying Uncertainty Aversion

Substituting θ in equation (13) into θ_t, we can rewrite equation (13) as following equation (19).

$$\frac{dS_t+D_t dt}{S_t}-rdt=[\gamma+\theta_t]\rho\sigma_{cs}dt+\sigma_s dB_t \qquad (19)$$

Let the i-period's excess return on risk asset be ζ_i, and the drift of excess return on risky asset be $\hat{\zeta}_t$. If we assume $S_t = a^{-1}D_t$, then $\hat{\zeta}_t = \hat{\mu}_t + \frac{1}{a}\hat{\mu}_t - r$.

So, we can induce a simplified equation (20).

$$\hat{\zeta}_t = [\gamma + \theta_t]\sigma_{cs} \tag{20}$$

Finally, using equation (20), we can measure the time varying uncertainty θ_t with assuming time varying standard deviation of consumption increase and stock return.

$$\theta_t = \frac{\hat{\zeta}_t}{\sigma_{cs}} - \gamma = \frac{\hat{\zeta}_t}{\rho\hat{\sigma}_{c,t}\hat{\sigma}_{s,t}} - \gamma \tag{21}$$

Where $\hat{\zeta}_t, \hat{\sigma}_{c,t}, \hat{\sigma}_{s,t}$ [14] can be estimated by updating log likelihood maximization of similar approach to equation (18).

3.4 Empirical Tests

3.4.1 Data

To measure the time varying uncertainty aversion, we used equation (21) which

[14] $\hat{\sigma}_{c,t}$ is estimated standard deviation of consumption increase on t-period, $\hat{\sigma}_{s,t}$ is estimated standard deviation of stock return on t-period.

contains the drift of excess return on risky asset and its standard deviation, standard deviation of consumption increase, correlation between consumption increase and return on risky asset, and finally risk aversion. Using the quarterly S&P 500 index, its dividend yield and 10-year U.S.A government benchmark bond as a risk free rate from 1954/1Q through 2006/2Q, we estimated the drift of excess return on risky asset and its standard deviation from 1978/1Q through 2006/2Q. Also as a proxy of consumption data, we used the seasonal adjusted quarterly Gross Domestic Product in U.S.A and we estimated standard deviation of consumption data. And we assume that constant correlation between consumption increase and return on risky asset[15]. Finally we assume that pure risk aversion is constant and used its value is less than 10, which is usually estimated between 0~10 in other empirical papers[16].

3.4.2 Method (I): long memory mechanism

As we referred, our agents take part in drawing balls in an urn and each ball has a number representing the excess return of risky asset. In our framework, they don't know the distribution of balls and number of balls in an urn is increasing. In this scenario, we try to measure the uncertainty aversion at each period.

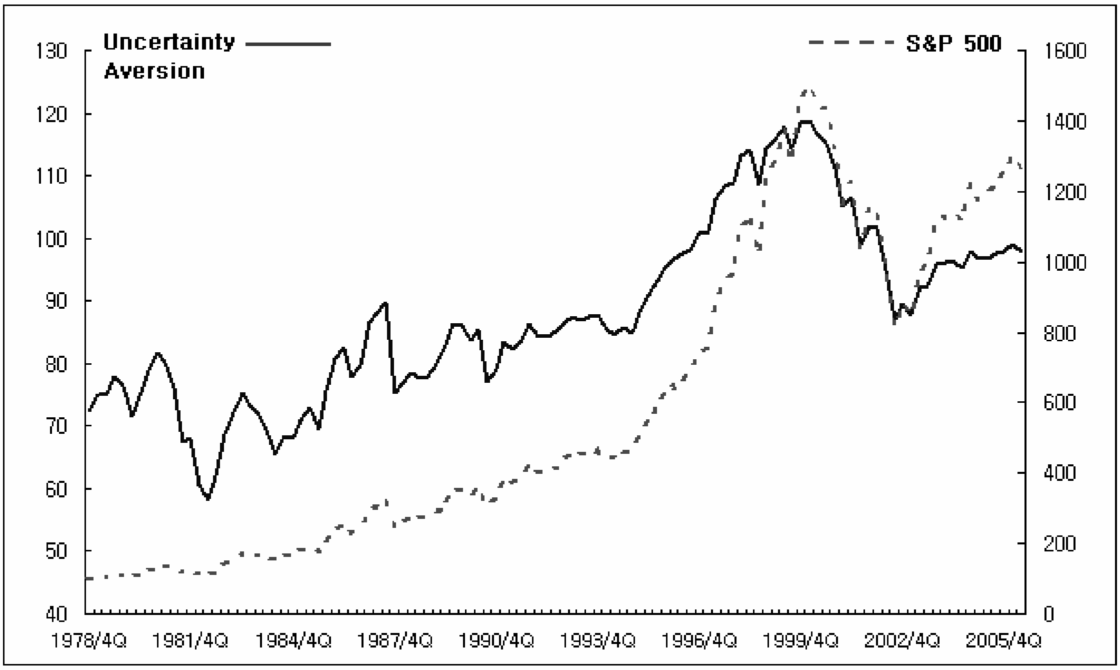
Specifically using the data between 1954/1Q and 1978/4Q, first we estimated the drift

[15] We use 0.193 which is same to Maenhout (2004).

[16] We assume that constant risk aversion is 5.

of excess return of the model at 1978/4Q and measured the uncertainty aversion at 1978/4Q. Next, using the data between 1954/1Q and 1979/1Q we estimated the drift of excess return of the model at 1979/1Q and measured the uncertainty aversion at 1979/1Q. Lastly, using the data between 1954/1Q and 2006/2Q we estimated the drift of excess return of the model at 2006/2Q and measured the uncertainty aversion at 2006/2Q. Following this step, we extracted the uncertainty aversion time series from January 1978/4Q through 2006/2Q. Estimated time series can be shown at Figure 3.

Figure 3
Uncertainty aversion for method (I)



Notes : The correlation of uncertainty aversion and S&P 500 index is 0.90. However applying Johanson Cointegration tests for two non-staionary time series, we find that no cointegration exists. The reason why the correlation is high, when the stock increases sharply, unceratiny aversion tends to increase, and when the stock decreases sharply, uncertainty aversion tends to decrease.

Figure 3 shows that uncertainty aversion goes with stock return and has increased as stock increases. Interestingly, before Black Monday in 1987/4Q, uncertainty aversion has increased and, after Black Monday in 1987/4Q uncertainty aversion fell down more sharply. The uncertainty aversion of 1987/3Q is around 90, which is peak before 1995, even S&P 500 at 1995/4Q is two times more than S&P 500 at 1987/3Q. It is associated with the investor's preferences to worry about the pessimistic situation. Actually when a sudden shock happens, investors' uncertainty can be resolved partly. Another interesting thing is that before IT Bubble period around 1999/4Q, the uncertainty aversion was peaked and even on 2006/2Q, S&P 500 index sustained similar to 1999/4Q, uncertainty aversion is less than the IT Bubble period 1999/4Q. Considering these results, we are interested in the relationship between uncertainty aversion and business condition.

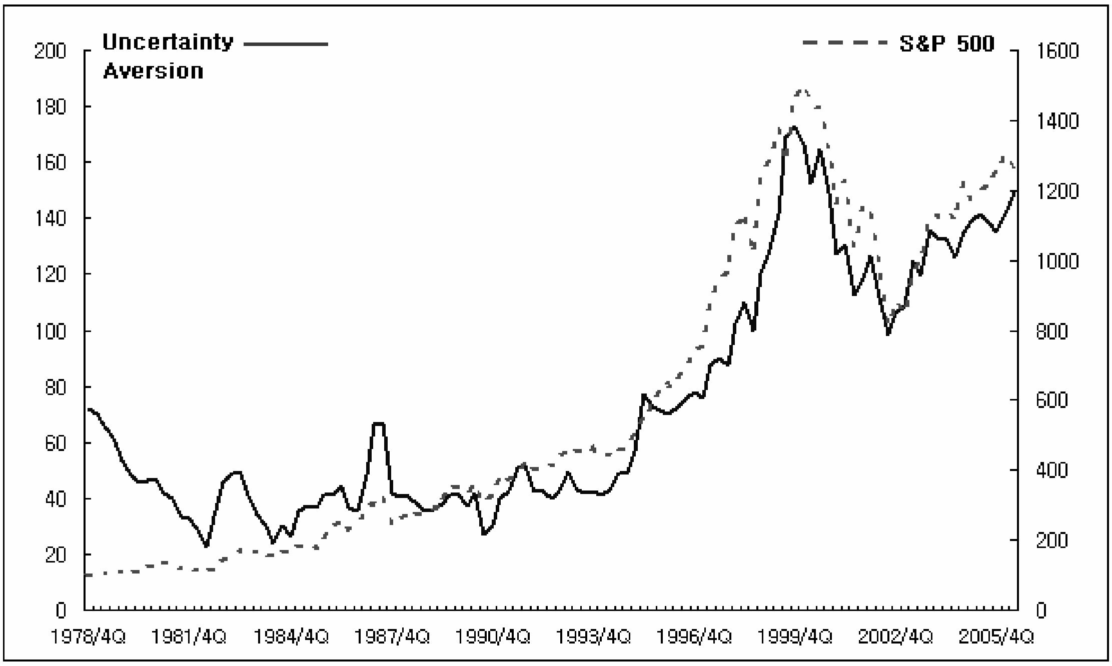
3.4.3 Method II: memoryless mechanism

Unlike the method (I), we assume that number of balls is fixed and one unknown ball in an urn is changing at every period. This case is similar to the scenario of Epstein and Schneider (2005). They presented memoryless mechanism and assume that learning may cease without all uncertainty having been resolved.

Similarly using the data between 1954/1Q and 1978/4Q, first we estimated the drift of excess return of the model at 1978/4Q and measured the uncertainty aversion at

1978/4Q. Next, using the data between 1954/2Q and 1979/1Q we estimated the drift of excess return of the model at 1979/1Q and measured the uncertainty aversion at 1979/1Q. Lastly, using the data between 1981/3Q and 2006/2Q, we estimated the drift of excess return of the model at 2006/2Q and measured the uncertainty aversion at 2006/2Q. Following this step, we extracted the memoryless uncertainty aversion time series from January 1978/4Q through 2005/4Q.

Figure 4
Memoryless uncertainty aversion for method (II)



Notes : The correlation of memoryless uncertainty aversion and S&P 500 index is 0.94. However, focusing on around 1987/4Q, this tendancy does not alwas hold. In addition, If the information uncertainy will increase continously, we can predict stock will go up in the very long run as time passes.

Similar to the figure 3, memoryless uncertainty aversion tend to move together S&P 500. However, compared to the figure 3, memoryless uncertainty aversion is more correlated with stock return than the uncertainty aversion suggested in chapter 3.4.2. Interestingly, we can capture the peak in memoryless uncertainty aversion at 1987/3Q where is ahead of crash on 1987/4Q's Black Monday[17]. Also we found that the average of memoryless uncertainty aversion is less than the average of uncertainty aversion. So we can say that investors with long memory mechanism much more worry about the pessimistic situation than the investors with memoryless mechanism. Because it is natural that the more do investor has information, the more uncertainty exists.

[17] Black Monday is Monday, Oct, 19, 1987, when the Dow Jones fell 22.6%.

4. Uncertainty Aversion and Business Condition

As seen in both figure 3 and figure 4, we suggest uncertainty aversion time series. This time varying uncertainty aversion can be a proxy of an aversive attitude about worst case pessimistic situation such as a Black Monday, a 9.11 terror. Thus it’s natural to relate business condition and uncertainty aversion. As an indicator of business cycle, many papers used credit spread, term spread, dividend yield, and risk free rate. Fama and French (1989) show that risk premia are lower when business conditions are strong and higher when business conditions are weak. Also Rosenberg and Engle (2002) measure the relation between time varying risk aversion and business cycle supporting Fama and French (1989). Along the lines of this research, we extend the time varying risk aversion into time varying uncertainty aversion based on Knightian Uncertainty. We offer evidences that aversive attitude about pessimistic situation is related with business conditions.

4.1 Regression Model

To measure the relation between uncertainty aversion and business conditions, we used uncertainty aversion time series estimated on chapter 3. As indicators of business condition, we used both credit spread and term spread. To avoid multi collinearity we didn't include risk free rate and dividend yield as indicators of business conditions in measuring the relation between uncertainty aversion and business condition, because both risk free rate and dividend yield were already used in measuring time varying uncertainty aversion.

Considering cointegrating and relationships among the variables, we used vector error correction model (VECM)[18] instead of vector autoregressive model (VAR). We construct two VECM models: one is to relate uncertainty aversion and credit spread, the other is to relate uncertainty aversion and term spread.

$$\begin{aligned} d(dUnc) &= \beta_{11} \cdot d(dUnc(-1)) + \beta_{12} \cdot d(dUnc(-2)) + \beta_{13} \cdot d(CS(-1)) + \beta_{14} \cdot d(CS(-2)) \\ d(CS) &= \beta_{15} \cdot d(dUnc(-1)) + \beta_{16} \cdot d(dUnc(-2)) + \beta_{17} \cdot d(CS(-1)) + \beta_{18} \cdot d(CS(-2)) \end{aligned} \tag{22}$$

$$\begin{aligned} d(dUnc) &= \beta_{21} \cdot d(dUnc(-1)) + \beta_{22} \cdot d(dUnc(-2)) + \beta_{23} \cdot d(TERM(-1)) + \beta_{24} \cdot d(TERM(-2)) \\ d(TERM) &= \beta_{25} \cdot d(dUnc(-1)) + \beta_{26} \cdot d(dUnc(-2)) + \beta_{27} \cdot d(TERM(-1)) + \beta_{28} \cdot d(TERM(-2)) \end{aligned} \tag{23}$$

[18] After applying ADF Unit Root test, we found that uncertainty aversion time series is non-stationary, so we made stationary time series by log differentiation.

Where $dUnc$ is a 1st differentiated uncertainty aversion, CS is a credit spread, $TERM$ is a term spread.

4.2 Regression Results

We calculate credit spread as a difference between U.S.A 10-year government benchmark bond yield and U.S.A 10-year corporate bond yield[19]. And we calculate term spread as a difference between U.S.A 10-year government benchmark bond yield and U.S.A 3-month T-bill yield. Sample is quarterly based from 1954/1Q through 2006/2Q.

Applying VECM tests of equation (22) and (23), we found a very interesting result. Table 2 shows that t period's uncertainty aversion has significant positive relationship with t+1 period's credit spread. This means that the more worrying about pessimistic situation on t period, the more credit spreads causes on t+1 period. In other words, business conditions will be poor when uncertainty aversion increases and business conditions will be better when uncertainty aversion decreases. This result supports the Fama and French (1989), and Rosenberg and Engle (2002). Also, with this interesting result we can explain the credit spread puzzle[20]. Of course many studies have tried to find the source of credit spread including liquidity risk, jump risk, and so on. However, besides those factors, we present the uncertainty aversion as a source of credit spread.

[19] We used the database of Reuters Ecowin.
[20] Credit spread puzzle is that observed credit spread is more than model based credit spread.

As table 2 shows, if investors have large uncertainty aversion on today, i.e. if investors worry about more pessimistic situation on today, tomorrow's default probability of a firm will increase.

Table 2
Uncertainty Aversion and Credit Spread

	β_{11}	β_{12}	β_{13}	β_{14}	β_{15}	β_{16}	β_{17}	β_{18}
estimation	0,059	0,073	0,013	0,016	2,385	0,095	−0,275	−0,09
t-statistics	[0,422]	[0,716]	[1,233]	[1,478]	[1,982]*	[0,107]	[−2,883]*	[−0,979]

Notes : Table 2 shows that just two coefficients are statistically significant. Estimated β_{15} is 2.385 and estimated β_{17} is -0.275. Especailly we are interested in β_{15}, which implies the relationship between t-period uncertaionty aversion and t+1 period credit spread. With this positive relationship, we can say that today's uncertainy increase will make a bad effect on tomorrow's business conditions, and today's uncertainty resolution will make a good effect on tomorrow's business conditions.

* : 95% confidence interval

Table 3
Uncertainty Aversion and Term Spread

	β_{21}	β_{22}	β_{23}	β_{24}	β_{25}	β_{26}	β_{27}	β_{28}
estimation	0,150	0,120	−0,298	−0,638	0,013	−0,015	−0,280	−0,290
t-statistics	[1,085]	[1,202]	[−0,663]	[−1,427]	[0,451]	[−0,732]	[−2,936]*	[−3,060]*

Notes : Table 3 shows that just two coefficients are statistically significant. Both estimated β_{27} and estimated β_{27} is related to autoregressive effects of quarterly based term spread. In our research, we are not interested in this value.

* : 95% confidence interval

In addition, to examine the specific relationship between uncertainty aversion and business cycle, we provided granger causality tests[21]. Applying three time series (uncertainty aversion, credit spread, term spread), we find some interesting results. First we reject the null hypothesis that uncertainty aversion does not granger cause credit spread. So, we can say that uncertainty aversion is useful in predicting credit spread. Secondly we reject the null hypothesis that uncertainty aversion does not granger cause credit spread. Similarly, we can say that uncertainty aversion is useful in predicting term spread. Considering these two results, we can conclude that uncertainty aversion is very effective in predicting future business conditions, which can be represented by indicators of credit spread and term spread. More specifically, if today's uncertainty increase, tomorrow's business conditions will be weak, and if today's uncertainty decrease or resolved, tomorrow's business conditions will be strong. This supports Fama and French (1989).

[21] In applying Granger Causality test, it is possible to include more than two time series.

Table 4
Granger Causality on Uncertainty Aversion, Credit Spread, Term Spread

Null Hypothesis	F-statistics	P-value
Credit Spread does not granger cause Uncertainty Aversion	0.109	0.896
Uncertainty Aversion does not granger cause Credit Spread	4.224*	0.017
Term Spread does not granger cause Uncertainty Aversion	0.637	0.530
Uncertainty Aversion does not granger cause Term Spread	4.376*	0.014
Term Spread does not granger cause Credit Spread	0.683	0.507
Credit Spread does not granger cause Term Spread	2.188	0.117

Notes : For three time series, we have six null hypothesis. Table 4 shows that we can reject two hypothesis; second and fourth. With these P-value, we find granger causality between uncertainty aversion and credit spread, and between uncertainty aversion and term spread.

* : 95% confidence interval

5. Conclusion

In conclusion, this paper focuses on time-varying uncertainty aversion. With robust control theory, originally developed by Anderson, Hansen, Sargent (2000), we extend the concept of Maenhout (2004, 2006) framework which assumes constant uncertainty. Using the example of drawing balls in an urn, we simplified the model. In this simplification, we measured the time varying uncertainty aversion and we found some interesting results. First, uncertainty aversion tends to go together with S&P 500. As stock increases, uncertainty aversion also increases, and vice versa. Especially before the crash, uncertainty is increasing and after the crash, uncertainty is resolved. In addition, comparing to memoryless uncertainty aversion, we found that investors with memoryless mechanism less worry about pessimistic situation than the opposite. Second, we found the relationship between uncertainty aversion and business cycle. Using credit spread and term spread as indicators of business cycle, we found that credit spread is highly associated with uncertainty aversion. When uncertainty aversion is high, the business condition is weak, and when uncertainty aversion is low, the business condition is strong. In addition, uncertainty aversion granger causes both credit spread and term spread. This result implies that uncertainty aversion can be the source of credit spread as well as we can predict business conditions with uncertainty aversion.

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