Technical Program

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Sensors and Smart Structures Technologies
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Nondestructive Evaluation
for Health Monitoring and Diagnostics

6–10 March 2005
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San Diego, California USA

Conferences • Courses • Poster Session • Exhibition

The International Society
for Optical Engineering
SESSION 10
California
Wed. 1:30 to 2:50 pm
SMA
1:30 pm: Pressurized shape-memory thin films and films, T. Shin, National Taiwan Univ. (Taiwan) ........................................ [576-54]
1:50 pm: Hybrid micro-macro-mechanical constitutive model for shape-memory alloys, E. Wang, Defence Research Development Canada/Valcartier (Canada); G. Bonetto, ETS-EMNI, ENSAM, Paris (France) ........................................ [576-55]
2:10 pm: Control of shape recovery force in SMA fiber-reinforced composite materials, K. Yamashita, A. Shimanishi, Saitama Institute of Technology (Japan) ........................................ [576-56]
2:30 pm: Structural applications of SMA/superelastic materials, E-I. Ruan, Wayne State Univ. ........................................ [576-58]
Coffee Break ........................................ 2:50 to 3:00 pm

SESSION 12
California
Wed. 3:40 to 6:00 pm
SMA and FMSM
3:40 pm: Dynamic buckling and recovery of shape-memory thin-cylindrical shells, H. R. Ammar, S. Nemati-Nasir, Univ. of California, San Diego ........................................ [576-59]
4:00 pm: Magnetic and conventional shape-memory characteristics of Co51Ni33Cu16 and Ni24Mn44In23 shape-memory alloys, H. E. Kajorn, B. Baburjan, K. Karamti, Texas AM Univ.; Y. I. Chumlyakov, Silvram Physical Institute (Russia); H. Maier, Univ. Paderborn (Germany) ........................................ [576-60]
4:20 pm: Modelling of the magnetic field induced martensitic variant reorientation in and the associated magnetic shape-memory effect in MSMMs, B. Kuehn, D. C. Lagoudas, Texas AM Univ. ........................................ [576-61]
4:40 pm: Multiscale constitutive model of magnetic shape-memory alloys, N. M. Stoliar, Univ. of Windsor (Canada) ........................................ [576-62]
5:00 pm: Characterization of piezoelectrically induced actuation of Ni-Mn-Ga single crystals, J. M. Chambers, S. R. Hall, R. C. O'Hanley, D. C. Bono, Massachusetts Institute of Technology ........................................ [576-63]
5:20 pm: Mechanical efficiency of acoustic-assisted, magnetic-induced strain in ferromagnetic shape-memory alloy actuators, B. W. Peterson, R. C. O'Hanley, S. M. Allen, Massachusetts Institute of Technology ........................................ [576-64]

SESSION 8
Royal Palm II
Wed. 1:15 to 2:30 pm
Microsensors, Actuators, and MEMS II
Chairs: Ananth S. Venkatasubramaniam, Indian Institute of Science (India); Vijay K. Varadan, The Pennsylvania State Univ.
1:30 pm: Analysis of a monolithic cantilever beam MOEMS accelerometer under closed-loop operation, S. S. Venkatasubramaniam, Indian Institute of Science (India); R. Srinivasan, Research Ct. (India); S. Balasubramaniam, Indian Institute of Science (India); V. K. Sathyaray, Research Ct. (India) ........................................ [576-75]
1:50 pm: Feedback controlled nano-positioner using the fiber optic EPI sensor with novel demodulation technique, S. W. Park, G. S. Kim, Korea Advanced Institute of Science and Technology (South Korea) ........................................ [576-76]
Coffee Break ........................................ 2:10 to 2:40 pm

SESSION 9
Royal Palm II
Wed. 3:40 to 5:40 pm
Biosensors and BioMEMS I
Chair: Tian-Bing Xu, National Institute of Aerospace
3:40 pm: Development of an SH-SAW sensor for detection of DNA immobilization and hybridization, X. Guo, Y. Hsu, Kyungpook National Univ. (South Korea); E. Park, Sungkyung Advanced Institute of Technology (South Korea) ........................................ [576-92]
4:00 pm: Piezoelectric polymer micro sensor for biomass detection, T. Xu, National Institute of Aerospace, N. M. Hakkak, J. Su, NASA Langley Research Ctr. ........................................ [576-93]
4:40 pm: An innovative all-polymeric drug delivery device, S. C.年, J. Ros, Pediatric, University of Pennsylvania, Philadelphia, PA ........................................ [576-95]
5:00 pm: Biosensor-based magnetocytostatic micromagnet, J. Li, Y. Li, F. Cheng, Shunyi University ........................................ [576-96]
5:20 pm: Synthesis and characterization of block copolymer microwolive integrated with energy transducing biomolecules, D. Ho, B. C. Wu, L. Lee, K. Kuo, C. D. Meng, University of California/Los Angeles ........................................ [576-97]

SESSION 10
Royal Palm II
Wed. 1:30 to 3:10 pm
Dynamic Systems and Control II
Chairs: Antonio Cercello, Italian Aerospace Research Ctr. (Italy); Mehrdad N. Ghaseemi-Nejad, Univ. of Khorasani (Iran)
1:30 pm: Piezoelectrically based vibration control optimization in nonlinear composite structures, N. J. John, J. C. Cao, T. M. Nguyen, RMIT Univ. (Australia) ........................................ [576-40]
1:50 pm: Active vibration suppression of a satellite frame using an adaptive composite thruster platform, N. Arum, M. N. Ghaseemi-Nejad, Univ. of Khorasani (Iran) ........................................ [576-41]
2:10 pm: The Fraunhofer MAVISAPS for smart space system design for automotive and mechanical engineering, J. Meis, M. Matthias, Fraunhofer-Institut fur Betriebsfahigkeit (Leu) (Germany) ........................................ [576-42]
2:30 pm: Design of smart actuator based on magnetoelectrical actuator sandwich beam, G. Zhou, Q. Wang, Univ. of Central Florida ........................................ [576-43]
2:50 pm: An MRF-based device able to control the torque stiffness of all movable vertical links, A. Behzadi, A. Cerrusio, A. Giannini, Italian Aerospace Research Ctr. (Italy) ........................................ [576-44]
Coffee Break ........................................ 3:10 to 3:30 pm

SESSION 11
Royal Palm II
Wed. 3:40 to 5:40 pm
SMA/FEM
Chair: Reginald Des Roches, Georgia Institute of Technology
4:00 pm: Design of composite structures with high-energy absorption, C. Zhao, M. E. D. Univ. of Washington ........................................ [576-46]
4:20 pm: Effect of ambient temperature on the performance of shape-memory alloy energy devices, G. A. Anderlid, R. Des Roches, Georgia Institute of Technology ........................................ [576-47]
4:40 pm: Finite element analysis of adaptive deployable structures with SMA strip actuator, J. Rob, J. Han, J. Lee, Korea Advanced Institute of Science and Technology (South Korea) ........................................ [576-48]
5:00 pm: Experimental and numerical characterization of multi-actuator piezoelectric polymer based topology optimization, R. C. Caronlier, E. C. Silva, G. C. N. Barretta, S. N. Nishi, Kyoto Univ. (Japan) ........................................ [576-49]
5:20 pm: Three-dimensional electromagnetic finite element models of actuator driven by a magnetoelectric iron-gallium nichrome, J. E. Lebenthal, A. B. Flattau, Univ. of Maryland/College Park ........................................ [576-50]
Finite Element Analysis of Adaptive Inflatable Structures with SMA Strip Actuator

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ABSTRACTION

The interactions between the inflatable structure and shape memory alloy (SMA) strip actuators are investigated using finite element simulation. The numerical algorithm of the 3-D SMA thermomechanical constitutive equations based on Lagoudas model is implemented to analyze the unique characteristics of SMA strip. For the numerical results presented in this paper, the ABAQUS finite element program has been utilized with an appropriate user supplied subroutine (UMAT) for the modeling SMA strip. In this model of SMA strip, the shape memory effect is restricted to one-way applications. The geometrically nonlinear, updated Lagrangian equilibrium formulation implemented in ABAQUS is used for the numerical model of inflated membrane structures.

Keywords: Inflatable structure, Shape memory alloy strip, Geometrically nonlinear, UMAT

1. INTRODUCTION

The inflatable structures are the specific applications of ultra-low-mass hybrid structures with extensive use of membranes. These membrane materials imply highly flexible, thin, low-modulus materials such as polymer films. Compared to traditional mechanical systems, inflatable structures have the advantage of a much lower cost, weight, and packaging volume, as well as more favorable thermal gradients and damping characteristics. These structures can be packaged into smaller volumes, and potentially reduce the overall space mission cost by reducing the launch vehicle size requirements. Reduction in total system mass and deployment complexity can also increase system reliability. Other advantages of an inflatable structural system include a significant reduction in weight by eliminating mechanical support mechanisms. However, surface distortion of the structures may be introduced by boundary conditions, thickness variations, wrinkling, thermal distortions, membrane inflation level, and surface roughness in the membrane material itself. Furthermore, during the operation, spacecraft is subject to variety of internal and external disturbances that would degrade the surface accuracy of inflatable structures. Therefore, it can be definitely said that reliable maintenance of the surface precision during the mission life is one of the most important technologies for the demonstration of the feasibility of inflatable structures.

The basic approach for maintaining the desired surface accuracy on orbit is to integrate actuator and sensor into the precision surface, as well as into the support structure of the inflatable structure system. Traditional methods to maintain the surface accuracy of inflatable structure either actively or passively will increase the weight and maintenance cost of the structures. However, by using the developed intelligent materials and embedded system hardware, the technology of maintaining the surface precision could be achieved for the potential development of inflatable structures. The concept using smart materials to maintain the surface accuracy of inflatable structure was proposed by Salama et al. [1]. They studied a potentially lightweight and simple technique to correct local aberrations in the shape of an inflated membrane by shrinking or stretching sections of the membrane or support structure. They employed piezoelectric elements in the construction of the membrane and support structure. Many researchers have also performed the vibration control of inflatable structures using smart materials. Wagner et al. [2] developed membrane mirror with PVDF film to maintain a stable imaging platform. Lewis and Inman [3] discussed the structural dynamics and vibration suppression via

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piezoelectric actuators of an inflatable torus. But there are many problems to control shape or vibration of inflatable structures using piezoelectric materials. One of the limitations of a piezoelectric actuator is the amount of force it can exert. In addition, active control (both position control and vibration suppression) loses authority because of the extreme flexibility of an inflatable structure’s skin. In addition, the choice of applicable sensing and actuation materials suitable to inflatable structures is somewhat limited because of the need for these material actuation and sensing systems to be able to fold up prior to deployment. More recently, efforts have been directed at demonstrating actuator architectures and approaches suitable for controlling very thin inflatable structures. Example work including active seam bonded onto membranes was proposed by Jenkins and Schur [4]. This concept mimicked the wing structure of bat just as the bones and tendons of a bat together deploy the wing and keep the skin aerodynamically configured. Mixed deployment/surface actuation approaches (mostly by the use of SMA and PZT) had also been presented [5]. Duvvuru et al. [6] reported on the possibility of active seams consisting integrated actuators (SMA) and sensors, which allows for controlled deployment of the inflatable structures. However, there remain many studies needed to demonstrate a great potential of the applications of smart materials into inflatable system. The interaction between the sensor/actuator and the inflated structures needs to be investigated with an improved modeling technique. Also, much more detailed analysis is required to truly understand and determine the effectiveness of position and vibration control of the inflatable structure.

In this paper, the interactions between the inflatable structure and shape memory alloy (SMA) strip actuators are investigated using finite element simulation. The numerical algorithm of the 3-D SMA thermomechanical constitutive equations based on Lagoudas model is implemented to analyze the unique characteristics of SMA strip. For the numerical results presented in this paper, the ABAQUS finite element program has been utilized with an appropriate user supplied subroutine (UMAT) for the modeling SMA strip. The geometrically nonlinear, updated Lagrangian description of equilibrium formulation implemented in ABAQUS is used for the numerical model of inflated membrane structures. In this model of SMA strip, the shape memory effect is restricted to one-way applications. The shape change of inflatable structure is caused by initially strained SMA strip bonded on the surface of the structure when thermally activated. The SMA strip starts transformation from the martensite to the austenite state upon actuation through heating, simultaneously recovering the initial strain, thus making inflatable structure to adjust the shape. In the numerical results, SMA strip actuator can generate enough recovery force to deform the inflatable structure. However, the initial shape of inflatable structure shifts to deformed state after temperature cycle due to the residual recovery stress of SMA strip actuators.

2. NUMERICAL ALGORITHM OF SHAPE MEMORY ALLOY CONSTITUTIVE EQUATIONS

Shape memory alloys (SMAs) are characterized by solid state transformation between austenite and martensite phases in response to mechanical and thermal loadings. This provides the SMAs with the capability for sustaining and recovering transformation strain up to 10% which imbues them with unique actuator and potential sensor capabilities in smart structural systems. Because SMAs can sustain large forces and displacements, alter their shape, and change their stiffness and damping characteristics with temperature or applied load, they have been excellent candidates for adaptive or smart structural systems. Roh and Kim [7] considered a low velocity impact for the hybrid smart plate. The hybrid smart plate using SMA actuators and piezoelectric sensors can enhance its global resistance to low velocity impact. Roh et al. [8] also investigated the thermomechanical responses of shape memory alloy hybrid composite shell panel. The numerical results showed that SMA actuator could enhance the structural stiffness and suppress thermally buckled deflection of the composite shell panel.

The inherent thermal-mechanical coupling and hysteresis associated with the phase transformations also pose significant modeling challenges which must be implemented to investigate the capabilities of SMAs as actuators or sensors. In particular, Liang and Rogers [9] have developed an empirically based cosine model to represent the martensite fraction as a function of stress and temperature during transformation. But this model has a limitation to investigate the behavior of SMAs below the some range of temperature. Brinson [10] modified the Liang model to predict the thermomechanical response of SMAs in more general cases. Some three dimensional models from these researches have been derived by the generalization of one-dimensional results. Boyd and Lagoudas [11] proposed the unified thermodynamic constitutive model for SMA materials based on the thermodynamic framework. By using a free energy function and dissipation potential, pseudelasticity and shape memory effect are modeled accounting for three dimensional transformation. Qidwai and Lagoudas [12] presented a consistent thermodynamical model based on the principle of maximum dissipation transformation considering generalized transformation function. A comprehensive was presented study on the numerical implementation of SMA thermomechanical constitutive response using elastic predictor-transformation corrector algorithms.
For the numerical analysis, the 3-D incremental formulation of the SMA constitutive model based on Lagoudas model [12] is used in this study to predict the thermomechanical responses of SMA. The model consists of three sets of equations: The constitutive equations, which describe the increment of strain, \( \dot{\varepsilon}_{ij} \), in terms of the increments of stress, \( \dot{\sigma}_{ij} \), temperature, \( \dot{T} \), and martensite fraction, \( \dot{\xi} \), i.e.,

\[
\dot{\varepsilon}_{ij} = S_{ijkl} \dot{\sigma}_{kl} + \alpha_j \dot{T} + Q_{ij} \dot{\xi}
\]

the transformation equations, which relate the increment of martensite fraction to transformation strain, \( \dot{\xi} \), i.e.,

\[
\dot{\varepsilon}_{ij} = \Lambda_{ij} \dot{\xi}
\]

and the transformation surface equation, which controls the start of the forward and reverse phase transformation, i.e.,

\[
\pi = \sigma_{ij}^M \Lambda_{ij} + \frac{1}{2} \sigma_{ij} \Delta S_{ijkl} \alpha_{ij} + \Delta \sigma_j \sigma_j (T - T_u) + \rho s_{ij} \Delta \ln \left( \frac{T}{T_u} \right) + \rho \Delta s_{ij} T - c_{ij} \dot{T} - \rho \Delta u_{ij} = \pm \Theta^o
\]

where \( \pi \) is the thermodynamic force conjugated to \( \dot{\xi} \). The terms that are defined with the prefix \( \Delta \) in Equation (3) indicate the difference of a quantity between the martensite and austenite phases as follows:

\[
\Delta S_{ijkl} = S_{ijkl}^M - S_{ijkl}^A, \quad \Delta \sigma_j = \sigma_j^M - \sigma_j^A, \quad \Delta c = c^M - c^A
\]

Also, \( \rho, c, s_j, \) and \( u_j \) are the mass density, the specific heat, the specific entropy, the specific internal energy at the reference state, respectively. The superscript \( A \) stands for austenite phase, and superscript \( M \) stands for the martensite phase. The plus sign on the right hand side in Equation (3) should be used for the forward phase transformation (austenite to martensite), while the minus sign should be used for the reverse transformation (martensite to austenite). Note that the material constant \( \Theta^o \) is the measure of internal dissipation due to phase transformation and can be interpreted as the threshold value of the transformation surface \( \pi \) for the start of the phase transformation. The transformation function can be defined in terms of the transformation surface equation as follows:

\[
\Phi = \begin{cases} 
\pi - \Theta^o, & \dot{\xi} > 0 \\
-\pi - \Theta^o, & \dot{\xi} < 0 
\end{cases}
\]

The transformation function \( \Phi \) takes a similar role to the yield function in plasticity theory, but in this case, an additional constraint for martensite fraction \( \dot{\xi} \) must also be satisfied. Constraints on the evolution of the martensite fraction are expressed as,

\[
\dot{\xi} \geq 0, \quad \Phi (\sigma, T, \dot{\xi}) \leq 0, \quad \Phi \dot{\xi} = 0
\]

\[
\dot{\xi} \leq 0, \quad \Phi (\sigma, T, \dot{\xi}) \leq 0, \quad \Phi \dot{\xi} = 0
\]
The inequality constraints on $\Phi(\sigma,T,\xi)$ is called as the transformation condition and regarded as a constraint on the state variables’ admissibility. For $\Phi < 0$, Equation (6) requires $\dot{\xi} = 0$ and elastic response is obtained. On the other hand, the forward-phase transformation (austenite to martensite) is characterized by $\Phi = 0$ and $\dot{\xi} > 0$, while the reverse-phase transformation (martensite to austenite) is characterized by $\Phi = 0$ and $\dot{\xi} < 0$. Finally, Equations (1), (2), and (6) can be composed to find unknown state variables and predict the thermomechanical responses of SMAs. In the formulation, Equation (1) is a generalized Hooke’s law in incremental form, Equation (2) is the flow rule for transformation strain and Equation (6) is the transformation function. Also, detail expressions for the parameters used in Equations (1) to (6) can be found in Qidwai and Lagoudas [12]. In total, there are five unknown state variables in three Equations (1), (2) and (6), i.e., total strain tensor $\epsilon$, the stress tensor $\sigma$, the transformation strain tensor $\eta$, the temperature $T$ and the martensite fraction $\xi$.

In the numerical implementing of the SMA constitutive model, the tangent stiffness tensor and the stress tensor at each integration point of all elements should be updated in each iteration for given increments of strain and temperature. To derive the tangent stiffness tensor, the consistency condition, $\dot{\Phi} = 0$, can be expressed by,

$$\frac{\partial \Phi}{\partial \sigma_i} \dot{\sigma}_i + \frac{\partial \Phi}{\partial T} \dot{T} + \frac{\partial \Phi}{\partial \xi} \dot{\xi} = 0 \tag{7}$$

Equations (1) and (7) can be used to eliminate $\dot{\xi}$ and obtain the relationship between stress increments and strain and temperature increments as,

$$\dot{\sigma}_i = L_{ijkl} \dot{\epsilon}_{kl} + l_{ij} \dot{T} \tag{8}$$

where the tangent stiffness tensor $L_{ijkl}$ and tangent thermal moduli $l_{ij}$ are defined by,

$$L_{ijkl} = \left( S_{ijkl} - \frac{Q}{\xi} \frac{\partial \Phi}{\partial \sigma_i} \left( S_{ijkl} \right)^{-1} \right), \quad l_{ij} = \left( S_{ijkl} \right)^{-1} \left( Q \frac{\partial \Phi}{\partial T} \frac{\partial \sigma_i}{\partial \sigma_j} \left( S_{ijkl} \right)^{-1} - \alpha_{ij} \right) \tag{9}$$

To calculate the increment of stress for the given strain and temperature increments, a return mapping integration algorithm [13] has been used. Equation (1) can be written in the following incremental form,

$$\Delta \sigma_i = \left( S_{ijkl} \right)^{-1} \left( \Delta \epsilon_{kl} - \alpha_{ij} \Delta T - Q \Delta \xi \right) \tag{10}$$

The elastic predictor is calculated in the first step by letting $\Delta \xi = 0$, i.e.,

$$\Delta \sigma_i^p = \left( S_{ijkl} \right)^{-1} \left( \Delta \epsilon_{kl} - \alpha_{ij} \Delta T \right), \quad \sigma_i^p = \sigma_i^0 + \Delta \sigma_i^p \tag{11}$$

An iterative scheme is then carried out to obtain the transformation corrector from equation (10) by assuming $\Delta \epsilon_i = 0$, and $\Delta T = 0$, i.e., during the $p$th iteration,

$$\Delta \sigma_i^{p+1} = -\left[ S_{ijkl} \left( S_{ijkl} \right)^{-1} Q \right]^{1} Q \Delta \xi^{p+1} \tag{12}$$

The transformation function, Equation (5), is expanded into a Taylor series about the current value of state variables, denoted by ‘$p’$, and is truncated at the linear part as shown below:
where temperature $T$ is fixed, and thus $\Delta T = 0$. By using equation (12), $\Delta \xi^{p+1}$ and $\Delta \xi_y^{p+1}$ can be obtained as follows:

$$\Delta \xi^{p+1} = \frac{\Phi^e}{\partial \sigma_y} (S_{\sigma \sigma})^{-1} Q_y^{e} \frac{\partial \Phi^e}{\partial \xi} \Delta \xi_y^{p+1}$$

Figure 1: Algorithm of the SMA constitutive equation for the ABAQUS user subroutine.

\[
\Phi\left(\sigma_y, \xi \right) = \Phi^e \left(\sigma_y^{p}, \xi^{p} \right) + \frac{\partial \Phi^e}{\partial \sigma_y} \Delta \sigma_y^{p+1} + \frac{\partial \Phi^e}{\partial \xi} \Delta \xi^{p+1} \tag{13}\]

The state variables $\xi^{p+1}$ and $\xi_y^{p+1}$ can then be updated as follows:

$$\xi^{p+1} = \xi^{p} + \Delta \xi^{p+1}, \quad \xi_y^{p+1} = \xi_y^{p} + \Delta \xi_y^{p+1} \tag{15}$$

and the stress can then be obtained by using the constitutive equation

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\Phi\left(\sigma_y, \xi \right) = \Phi^e \left(\sigma_y^{p}, \xi^{p} \right) + \frac{\partial \Phi^e}{\partial \sigma_y} \Delta \sigma_y^{p+1} + \frac{\partial \Phi^e}{\partial \xi} \Delta \xi^{p+1} \tag{13}\]

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\xi^{p+1} = \xi^{p} + \Delta \xi^{p+1}, \quad \xi_y^{p+1} = \xi_y^{p} + \Delta \xi_y^{p+1} \tag{15}\]
The iterative procedure ends if $\Delta \xi^{n+1}$ is less than a specified tolerance ($10^{-6}$). If the convergence criterion is not satisfied, calculations given by Equations (12) to (16) are repeated until convergence is achieved. The numerical algorithm of SMA constitutive equation for the ABAQUS user subroutine is illustrated in Figure 1. In the numerical algorithm, Newton-Raphson iteration method is used to solve the increment of martensite fraction in Equation (14).

3. RESULTS AND DISCUSSIONS

3.1. Analysis of static bending of pressured beam

In the case of membrane structures, their lack of bending rigidity caused by extreme thinness and/or low elastic modulus, leads to an essentially constrained structure that can have equilibrium configurations only for certain loading fields such as prestress or internal pressure. Under other loading conditions, large rigid-body deformations can take place and these characteristics lead to an inability to sustain the deformed shape. So there are many limitations and difficulties to predict the behaviors of membrane structures with numerical programs. The finite element method (FEM) is almost universally applicable and is capable of getting accurate solution for membrane structures. But, there are many limitations and difficulties to observe the behaviors of membrane structures with FEM. These difficulties are typically related to a common problem in the analysis of the behaviors of membrane structures, namely, the inability of most regular numerical solution procedures to handle singular structural states, such as a flat membrane state with no lateral stiffness before pressurization or prestressing. But one of the difficulties to consider nonlinear analysis is the extreme sensitivity of membrane structures to numerical modeling. Generally, using the shell and membrane elements, we can get accurate solutions of the inflatable structures. But before its prediction can be deemed acceptable, numerical models must be benchmarked by investigating both convergence and parameter sensitivity studies and simplified experimental test results for membranes with a quality comparable to numerically theoretical predictions.

In this section, static behavior of inflatable beam structure is numerically investigated using shell and membrane elements of ABAQUS program. These numerical results are compared with experimental result of previous studies to verify the accuracy of present numerical model using ABAQUS program. The finite element modeling technique for the static bending analysis utilizes two different modeling approaches. The first one is to create shell element models using S4R5. The other is to create membrane elements model using M3D4. All of these elements make the pressured inflatable beam models by applying the appropriate internal pressure as well as the bending loading. Also, the method for analyzing the finite element models should be carefully considered. All nonlinear finite element solvers, including the algorithms used in ABAQUS, rely on an incremental loading approach in order to reach a convergent solution.

Figure 2: Numerical model of pressured beam for static bending analysis. Figure 3: Load-deflection curve.
Table 1: Material parameters of shape memory alloys

<table>
<thead>
<tr>
<th>Material constants</th>
<th>Values</th>
<th>Model variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^a$</td>
<td>$7.0 \times 10^9 \text{Pa}$</td>
<td>Used to calculate isotropic compliance tensor, $S^a$ and $S^\mu$</td>
</tr>
<tr>
<td>$E^\mu$</td>
<td>$3.0 \times 10^9 \text{Pa}$</td>
<td></td>
</tr>
<tr>
<td>$\nu = \nu^\mu$</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>$\alpha^a$</td>
<td>$2.2 \times 10^6 / \text{K}$</td>
<td>Used to calculate isotropic thermal expansion</td>
</tr>
<tr>
<td>$\alpha^\mu$</td>
<td>$1.0 \times 10^6 / \text{K}$</td>
<td>coefficient tensor, $\alpha^a$ and $\alpha^\mu$</td>
</tr>
<tr>
<td>$\rho \Delta c = c^\mu - c^a$</td>
<td>$0.0 J / (m^3 \text{K})$</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>0.05</td>
<td>$\rho \Delta s_e = \left( \frac{d \sigma}{dT} \right)^a$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\left( \frac{d \sigma}{dT} \right)^\mu = 7.0 \times 10^9 \text{Pa} / (m^3 \text{K})$</td>
</tr>
<tr>
<td>$A^\gamma$</td>
<td>315.0 K</td>
<td>$\gamma = \rho \Delta s_e + \mu_1 = \frac{1}{2} \rho \Delta s_e \left( M^\mu + A^\gamma \right)$</td>
</tr>
<tr>
<td>$A^\nu$</td>
<td>295.0 K</td>
<td>$\rho b^a = -\rho \Delta s_e \left( A^\gamma - A^\nu \right)$</td>
</tr>
<tr>
<td>$M^\nu$</td>
<td>291.0 K</td>
<td>$\mu_1 = \frac{1}{4} \left( \rho b^a - \rho b^\mu \right)$</td>
</tr>
<tr>
<td>$M^\gamma$</td>
<td>271.0 K</td>
<td>$Y^* = -\frac{1}{2} \rho \Delta s_e \left( A^\nu - M^\nu \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+ \frac{1}{4} \rho \Delta s_e \left( M^\mu - M^\nu - A^\gamma + A^\nu \right)$</td>
</tr>
</tbody>
</table>

Due to rapid stiffness changes, displacement and instability of the static solutions during the early stage of analysis, the users should use the geometrically nonlinear, updated Lagrangian equilibrium formulation implemented in ABAQUS. Although computationally demanding, this method keeps the analysis stable in the early stages until the model stiffened due to the internal pressure. Figure 2 shows the numerical model of pressured beam structure which is modeled using shell and membrane elements. Actually, in order to apply bending loading to this very long column, the ends of the column should be capped and bonded to rigid plugs. So, the rigid body constrain condition is applied to the end of the structure where load is applied. This constrain condition should prevent local deformations due to a concentrate loading. Also, cantilevered boundary condition is applied at the other end. The material properties of Kapton-HN polyimide film used in this analysis are as follows:

$$E = 2492 \text{MPa}, \ \nu = 0.34, \ \text{and} \ \rho = 1420 \text{kg/m}^3$$

and the geometric dimension of the pressured beam is 0.152 m in the diameter, 2.44 m in the length, and a thickness, 50.8 \mu m. In case of static bending analysis, internal pressure, 3447 Pa, has been applied in this inflatable beam structure and after that lateral load, 2.7 N, is applied to the end of beam. Figure 3 shows the numerical results of static bending analysis with using shell and membrane elements. The geometric nonlinear static analysis in ABAUQS program is performed to account for large displacement and pressure stiffening effects of the internal pressure on the thin film material. The present results are plotted along with the experimental results [14]. The results for all models are similar until some loading point and after that the curves are separated at the end of loading. In the practical experimental test [14], a distinct ‘popping’ sound could be heard and it was correspond to the deviation in linearity and to the onset of wrinkling near the cantilevered end of the structure. It can be known that numerical result with shell element is more appropriate than membrane elements to describe the nonlinear characteristic of inflatable beam structure.
3.2. Thermomechanical responses of SMA strip actuator

For the numerical results presented in this paper, the ABAQUS finite element program has been utilized with an appropriate user supplied subroutine (UMAT) for the modeling SMA. The material properties of SMA [12] used in the finite element analysis are given in the Table 1. SMA strip is numerically modeled to investigate the thermomechanical responses and capabilities of SMAs as actuators. A cantilevered SMA strip model subjected to uniaxial force is illustrated in Figure 4. This analysis uses 16×10×1 mesh with 3-D eight-node elements (C3D8). Figure 5 shows the unique characteristics of SMA strip at the different temperatures such as shape memory effect and pseudoelastic behavior observed at $\alpha/a = 1$ and $\gamma/b = 0.5$. The hysteresis loop of recovery stress with respect to temperature variation is illustrated in Figure 6. In this analysis, initial state of this strip is completely martensite state with residual strain, $\varepsilon_{str} = 5\%$. SMA strip is restrained during the phase transformation induced by heating process, so that the large recovery stress is generated by shape memory effect. As can be seen from this hysteresis loop, recovery stress is not decreased to zero at the end of complete cycle. The distribution of residual recovery stress of SMA strip is illustrated in Figure 7. The distributions of martensite fraction at various temperatures are also investigated. Figure 8 illustrates the martensite fraction of SMA strip at $T = 200\,^\circ C$ and at the end of temperature cycle, $T = 0\,^\circ C$. The numerical results show that the SMA strip does not recover the initial martensite fraction and generates the residual recovery stress at the end of temperature cycle. These phenomena should cause SMA strip to be irreversible actuator, even if SMA is coupled to an elastic structure which compels SMA to recover the initial condition. Therefore, the research about the interactions between host structure and SMA strip is necessary to design reversible shape adaptive structures.
3.3. Applications of SMA strip actuator to inflatable beam structure

In this section, the interactions between SMA strip actuators and the inflatable beam structure are investigated. The SMA strip actuators coupled with inflatable beam structure can be used to generate bending force and to adjust the shape with the appropriate heating and cooling cycle. Figure 9 shows the numerical model of adaptive inflatable structure with SMA strip actuators. For the numerical analysis, following assumptions are considered: i) SMA strips are perfectly bonded with membrane surface. ii) SMA strips are activated by electrically heating, and iii) SMA strips are thermally insulated from the rest of inflatable beam structure. Inflatable beam and SMA strips are respectively modeled by shell elements (S4R5) and 3-D eight-node elements (C3D8), respectively. SMA strips are subjected to initial strain in Y-direction. When SMA strip is activated by raising its temperature above the austenite start temperature, strain recovered in the activated SMA strips cause bending deformation due to the off-center placement of the SMA strips. The inflatable beam structure is made of Kapton-HN polyimide film of which properties are given in Equation (17) and inflatable beam has the geometric dimensions such as a diameter (D), 0.152 m, a length (L), 1.0 m, and a thickness, 126 µm. Also, internal pressure, 3447 Pa, has been applied in this inflatable beam structure. Due to the extreme flexibility of an inflatable structure’s skin compared to SMA strip, achieving global shape adjustment of inflatable beam structure is very complicated.
So, the seam is introduced numerically in this model. This seam concept mimics the bat analogy that can achieve both deployment and shape precision just as the bones and tendons of a bat together deploy the wing and keep the skin aerodynamically configured [4]. A system of seam with actuator can aid the shape refinement and achieve the global shape adjustment of inflatable structure. Before utilizing the model of such concept, one should consider a number of practical concerns, such as packageability, deployability material and configuration compatibility, integration, weight/cost, and degree of surface precision achievable by the system. But the seam in this model is numerically modeled just to have such material properties,

$$E = 70\text{GPa}, \quad \nu = 0.3$$

(18)

and a thickness, $5 \times 10^{-3} m$. Each SMA strip actuator has dimensions, $a = 0.16 m$, $b = 0.016 m$ and thickness, $t = 0.002 m$. The center of SMA strip (A), (B) and (C) are located at $Y_{A} = 0.08 m$, $Y_{B} = 0.40 m$, and $Y_{C} = 0.72 m$, respectively. The deformed shape of inflatable beam structure at the incremental temperature, $\Delta T = 30^\circ C$ is illustrated in Figure 10. The deformed shape is magnified to see more detailed deformed pattern of inflatable beam structure. As can be seen, local wrinkling and stress concentration phenomena are observed near the location of each SMA strip (A), (B) and (C). Figure 11 shows the hysteresis of vertical tip deflection versus temperature cycle on SMA strip with 3% initial strain, where the deformed shape cannot be fully recovered at the end of the first temperature cycle.

![Figure 9: Numerical model of adaptive inflatable beam.](image)

![Figure 10: Deformed shape of inflatable beam structure at incremental temperature, $\Delta T = 30^\circ C$.](image)

![Figure 11: Hysteresis of tip deflection of inflatable beam structure.](image)
To investigate internal condition of SMA strip actuators at the end of the first temperature cycle, the residual recovery stress and distribution of martensite fraction of SMA strip with 3% initial strains is illustrated in Figure 12. The recovery stress of SMA strip actuators does not decrease to zero. So, the residual recovery stress of SMA strips causes inflatable beam structure to remain deformed shape and martensite fraction of SMA strip not to be recovered its initial value, 0.6. As the SMA strips are heated again from here, one obtains the closed hysteresis cycle. After the first temperature cycle, this adaptive inflatable structure with SMA strips has the range of tip deflection ($U_z$) from 7mm to 10mm with heating and cooling cycle. Therefore, it is difficult to design a perfect reversible shape adaptive structure using this one-way SMA strip. Although the closed hysteresis cycle should be obtained, the initial structural shape shifts to deformed state after the first temperature cycle.

### 4. CONCLUSIONS AND FUTURE WORK

The interactions between the inflatable structure and shape memory alloy (SMA) strip actuators are investigated using finite element simulation. The numerical algorithm of the 3-D SMA thermomechanical constitutive equations based on Lagoudas model is implemented to analyze the unique characteristics of SMA strip. For the numerical results presented in this paper, the ABAQUS finite element program has been utilized with an appropriate user supplied subroutine (UMAT) for the modeling SMA strip. The geometrically nonlinear, updated Lagrangian equilibrium formulation implemented in ABAQUS is used for the numerical model of inflated membrane structures. In this model of SMA strip, the shape memory effect is restricted to one-way applications. The shape change of inflatable structure is caused by initially strained SMA strip bonded on the surface of the structure when thermally activated. The SMA strip starts transformation from the martensite to the austenite state upon actuation through heating, simultaneously recovering the initial strain, thus making inflatable structure to adjust the shape. In the numerical results, SMA strip actuator can generate enough recovery force to deform the inflatable structure. Therefore, the deformed inflatable structure cannot fully recover its initial shape due to residual recovery stress of SMA. Although this adaptive inflatable structure with SMA strips can obtain the closed hysteresis cycle, the initial structural shape shifts to deformed state after the first temperature cycle. So, it is difficult to make a reversible shape adaptive structure using one-way shape memory alloys and necessary to investigate the interactions between host structure and SMA actuator. The two-way shape memory effect could be a solution to make the actuation reversible. However, if high precision is needed in terms of activation magnitude versus the number of cycles, the issues of thermal fatigue and drift in the response are still not completely solved. Therefore, the accurate prediction of the thermomechanical behavior of the SMA should be further studied to design the actuator and shape adaptive structure, taking into account the nonlinear and hysteretic behavior of the SMAs. Due to the extreme flexibility of an inflatable structure’s skin comparing that of SMA strip, it is very complicated to use SMA strip for the global shape adjustment of inflatable structure. So, there remain some researches to apply thin film shape memory alloy for the shape adjustment of inflatable structure. Thin film SMA which should be compatible with
membrane material, has only a small amount of thermal mass to heat or cool, thus the cycle (response) time can be reduced substantially and the speed of operation may be significantly increased. The distribution of seam should be also investigated for the shape adjustment of inflatable structure.

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