On the aerodynamic noise source in circular saws

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The character of the dominant aerodynamic noise source for rotating rigid circular saws is concluded to be represented by a point dipole model. The source strength is directly dependent upon Reynolds's number and saw design. A theoretical model is presented for prediction of the farfield noise. Experimental measurement of the fluctuating lift force on particular tooth models was used to identify the dipole source and a hot wire anemometer, rotating with the saw, measured the tooth wake. The theoretical predictions of dipole noise dependence upon parameter variation are generally consistent with literature noise data.

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INTRODUCTION

The noise radiated from an idling circular saw is a significant problem in the wood and metal working industries. The discussion of idling saw noise sources is commonly separated into vibration noise sources and aerodynamic noise sources. Vibration noise is generated by transverse oscillations of the plate, and aerodynamic noise results from interaction of the rigid saw plate, including teeth, with the surrounding air. A severe example of vibration noise is a self-excited instability generating a screaming or whistling sound exceeding 100 dBA. Two explanations currently exist for identification of this self-excitation source. One proposes the problem is a plate instability excited by flow in the vicinity of the teeth, and the second considers the self-excitation to be mechanically induced by machine imbalance, rotor eccentricity or some other imperfection in the machinery. Aerodynamic noise sources are caused by stresses in the turbulent air surrounding the rotating rigid saw plate (quadrupole sources), by interaction of the rigid plate surface and teeth with the air (dipole sources), and by displacement of the air by the moving plate and teeth (monopole sources). The focus of this paper is on the aerodynamic noise source and its dependence upon tooth design and saw rotation speed. A paper on vibration noise is currently in preparation.

Chanaud investigated the aerodynamic noise produced by a collarless rotating disk without teeth, and he concluded that the sound source is equivalent to a classical acoustic dipole. Aerodynamic noise sources on a rigid circular saw are more complex than the plane disk because of the central collars, the teeth, and possibly geometric discontinuities such as radial edge slots. As a result of this complexity, the dominant aerodynamic noise sources in circular saws remain only partially identified and somewhat of a controversial issue. The dominant aerodynamic noise source has been identified by some researchers as having a dipole character from dimensional analysis, some experimentation, and association of the problem with related problems such as fan and propeller noise. More recently the dominant source has been identified by authors, who previously were attracted to the dipole model, as a combination of monopole and dipole, and as an amplified quadrupole source. The apparent reason for the shift in source-character prediction is that the measured sound power from idling circular saws was not found to be proportional to \( U^6 \) as might be expected of a dominantly dipole source. \( U \) is the velocity of the saw periphery. Throughout the literature the velocity exponent for proportionality to the measured sound power ranges between 4.9-6 depending upon the tooth and saw design. In 1977 Segal et al. argued that the dominant noise source is an amplified quadrupole model induced by turbulent flow near the blade edge. The sound power level of this theoretical source is proportional to \( U^3 \) which is in the middle of the noise data. The authors did not propose a specific acoustic model or seek a direct source evaluation. Experimental studies of turbulent flow interaction with a small flat plate by Sharland and Siddon support a dipole source model. Reiter and Kelte discarded an earlier dipole model for a combined dipole-monopole source. The combined model has sound power proportional to \( U^4 \) for the part due to the monopole sources and proportional to \( U^6 \) for the dipole sources. This combination can make the model fit the farfield noise data to some extent but the monopole source has previously been shown to be insignificant in saw noise.

Saw noise research to date has relied on farfield noise measurements from saws and slotted circular plates to predict the source character by inference. Based upon the current controversy over the dominant source character, one might conclude that these indirect identification methods should be supported by fundamental source-identification studies. The need for a direct approach to aerodynamic source identification motivated the study reported in this paper.

This paper presents an analysis of aerodynamic saw noise that is based in Lighthill's aerodynamic theory and utilizes an experimental determination of the dipole source strength for particular cutting-tooth profiles. The aerodynamic lift force on a single cutting tooth was measured for a number of tooth profiles, tooth spacings, and rotation speeds on a full scale test stand. These data, when interpreted within the context of the theoretical acoustic source model, enable one to assess the importance of tooth and cutter design upon the dipole noise source and to indirectly predict the farfield noise variation from saw design changes.

The results of this study support the conclusion that the aerodynamic source is dominantly dipole; the deviation of the sound power from proportionality to \( U^6 \) is ex-
plained through the measured dependence of the source strength upon $U$.

I. NOISE SOURCE

Aerodynamic noise is generated when the rotation induced air flow interacts with the rotating saw surface including the teeth. The resulting sources are distributed over the saw surfaces and around the saw periphery as illustrated in Fig. 1. Sound emanating from the saw surfaces is caused by the induced turbulent boundary layer. The principal sound sources on the surface are pressure dipoles radiating a rather weak broadband random acoustic pressure. The sound sources at the saw periphery are caused by the aerodynamic loading of the teeth, the fluctuating wake around the teeth, and air mass fluctuation arising from the moving teeth.

Aerodynamic loading of the teeth can be resolved into quasisteady and fluctuating components. The quasisteady forces arise from asymmetry of the wake at the rotating teeth. The moving quasisteady lead on each rotating tooth produces a periodic pressure field at all points fixed in space. Noise attributed to these rotating pressure sources is termed rotational noise. The spectral energy of rotational noise is concentrated at harmonics of the saw rotation frequency. The fluctuating forces components on the saw teeth arise from in-flow turbulence, the in-flow turbulent boundary layer, and the irregular turbulent eddies shed from the trailing edges of the teeth. The fluctuating pressure on the tooth surface produces a broadband dipole noise with sound power proportional to $U^6$.

The fluctuating wake in the region defined by radii beyond the rotating teeth is the principal air turbulence effecting sound generation. These wakes produce Reynolds's stresses which transfer momentum to the surrounding air. These stresses can also be reduced to a sum of quasisteady and fluctuating components with the steady components contributing to rotational noise as before. The fluctuating components generate turbulent broadband noise depending upon the wake fluctuations. These sources are classically of a quadrupole type with sound power proportional to $U^4$.

Air mass excitations occur because of the volumetric displacement of air at a point by the rotating teeth. This mass fluctuation produces monopole sound source with spectral energy concentrated at the tooth passage frequency and sound power proportional to $U^4$.

II. THEORETICAL MODEL

Sound radiation from a rotating rigid saw is governed by the inhomogeneous wave equation

$$\frac{1}{c^2} \nabla^2 p(x, t) = g(x, t),$$

where $p(x, t)$ is the acoustic pressure in the radiation field at $(x, t)$, $g(x, t)$ is the source strength at point $(x, t)$, $c$ is the sound velocity and $i$ designates the rectangular coordinate directions 1, 2, 3 in Fig. 2. The source $g$ is assumed to have the general form

$$g(x, t) = Q(x, t) + q(x, t),$$

where the monopole source strength $Q$ has the character of air mass fluctuation rate per unit volume, the dipole source strength $q$ is a force per unit volume on the air surrounding the saw surface, and the quadrupole source $T_{ij}$ is the fluctuating fluid stress tensor. The solution of Eq. (1) is

$$p(x, t) = \frac{1}{4\pi} \int \frac{Q(y, t) - q(y, t)}{r^2} dV(y),$$

where $r = |x - y|/c$ is the distance between the source and the observer and $r$ is the total volume occupied by all sources. Note that $Q$ is evaluated at an earlier time, $t - r/c$, than $p$. Combining Eqs. (2) and (3), one obtains

$$p_s(x, t) = \frac{1}{4\pi} \int \frac{Q(y, t)}{r^2} dV(y) + \frac{1}{4\pi} \int \frac{q(y, t)}{r^2} dV(y),$$

where $r$ is the air volume displaced by the blade, $r_2$ is the wake volume at radii greater than the tooth tip radius, and $D_1$ and $D_2$ are the saw disk and saw teeth total lateral surface areas, respectively. Differentiating inside the integral in Eq. (4), restricting $x$ to lie in the farfield (i.e., $|x| \approx r$) and restricting $|x|$ to be large compared to the typical noise wavelength, $\lambda$, the Eq. (4) becomes
which represents the total farfield sound pressure radiated from a rotating rigid saw. The dipole and quadrupole sources have directivities \( x_i/|x| \) and \( x_i x_j/|x|^2 \), respectively, and the pressure decays as \( 1/|x|^2 \) for both sources.

A. Point dipoles

Chanaud\(^1\) concluded that the turbulent boundary layer noise for a plane disk, represented by the integral over \( D_1 \) above, is insignificant compared to the noise from flow separation at the disk edge. Sharland\(^2\) also found boundary layer noise to be insignificant when compared to fan blade noise. This suggests that sources associated with the saw teeth will often dominate those associated with the saw surfaces. On that assumption the \( D_1 \) integral term above is discarded. Slone and Robertson\(^3\) observed that the peak frequency of the saw noise is unrelated to the tooth passage frequency. This indicates that monopole sources, and rotating dipole and quadrupole sources as well, cannot be very strong. On that basis the monopole term in Eq. (5) is eliminated. Sialdon\(^4\) predicted the nature of the source from a small plate imbedded in turbulent flow by cross correlating the farfield sound pressure and the local surface or source pressure. His conclusion was that the source associated with his plate is dominantly dipole. Sharland\(^5\) concluded that axial flow fans excite dominantly broadband-dipole noise that originates from lift fluctuations on the blades. Large scale turbulence preceding the blades can increase the lift fluctuation and increase the broadband noise. Further support for the dipole model is given by Clark and Ribner\(^6\). They experimentally showed, through cross-correlation of the acoustic pressure and lift force, that the radiated sound from a small airfoil in turbulent flow was of a dipole character and caused by the fluctuating lift force. The same conclusion was reached by Chih-Ming and Kovasznay\(^7\) for an isolated chambered blade in unsteady flow. These studies support the assumption that the radiated noise from a small plate, like a saw tooth, in a turbulent incident flow should be dominantly of a dipole character. If the quadrupole term in Eq. (5) is discarded, the acoustic pressure assumes the characteristic form

\[
p_s(x, t) = \frac{x_i}{4\pi|x|^2} \int_0^\infty \frac{\partial p_i}{\partial t} \left( y, t - \frac{\tau}{c} \right) dD(y),
\]

where summation on \( i \) gives the normal, radial, and tangential stresses on the tooth surface. The rms pressure fluctuations on a blunt body of rectangular cross section in a turbulent field have been shown by Vickery\(^8\) and Lee\(^9\) to be much larger than the rms shear fluctuations. This supports the assumption that the normal pressure fluctuations on the surface will be large compared with fluctuations of the tangential stresses.\(^9\)

Therefore, based upon the literature and physical arguments, the principal acoustic source for a rotating, rigid saw is assumed representable by a distribution of pressure dipoles on the saw teeth. For a coordinate \( x_i \), normal to the tooth surface, Eq. (6) reduces to

\[
p_s(x, t) = \frac{x_i}{4\pi c|x|^2} \int_0^\infty \frac{\partial p_i}{\partial t} \left( y, t - \frac{\tau}{c} \right) dD(y),
\]

Acoustic wavelengths between 5.6 and 16.7 cm (2–6 kHz), which are typical for saw noise data,\(^2\) are large compared with the largest characteristic tooth dimension. In this case the retarded time variation over each tooth surface can be neglected. Each tooth is considered a single point pressure dipole source. The acoustic pressure \( p_s(\mathbf{x}, t) \) becomes the sum of fluctuating point-pressure dipole sources \( p_{ix} \) for each tooth as illustrated in Fig. 3. Incorporating the farfield assumptions \( |x| > |y| \) and \( x_i/|x|^2 = \sin \delta \), one obtains for an N-tooth saw

\[
p_s(\mathbf{x}, t) = \frac{x_i}{4\pi c|x|^2} \sum_{n=1}^N \left[ \frac{\partial p_{ix}}{\partial \mathbf{t}} \right],
\]

where the square bracket indicates evaluation is at the
retarded time $\tilde{t} = t - \tilde{t}_c$. The total mean square sound pressure generated by the point dipoles is

$$\bar{p}^2_s = \frac{\sin^2 \theta}{16\pi^2 c^2 S |x|^2} \sum_{n \neq 0} \sum_{m \neq 0} \left[ \frac{\partial \phi_n}{\partial t} \right] \left[ \frac{\partial \phi_m}{\partial t} \right],$$

where the super bar indicates the mean operation. Following Lowson and Ollerhead, the fluctuating pressures on the teeth are further assumed to be random variables of identical rms amplitude but random phase so that the individual dipole sources are uncorrelated for a fixed tooth spacing; for $k \neq l$

$$\sum_{n \neq 0} \sum_{m \neq 0} \left[ \frac{\partial \phi_n}{\partial t} \right] \left[ \frac{\partial \phi_m}{\partial t} \right] = 0$$

and for $n = k$

$$\left[ \frac{\partial \phi_n}{\partial t} \right]^2 = \left[ \frac{\partial \phi_n}{\partial t} \right] \left[ \frac{\partial \phi_n}{\partial t} \right]$$

The approximate mean square sound pressure becomes

$$\bar{p}^2_s = \frac{2\pi S_0 U}{h},$$

where the number $S_0$ lies between 0.1 and 0.2. With $\omega$ as a representative fluctuating point dipole frequency, the mean square of the fluctuating pressure rate becomes

$$\left[ \frac{\partial \phi_n}{\partial t} \right] \alpha \frac{U^2}{\pi^2} \int \left[ \bar{p}^2_s \right] d\tilde{t}.$$

The fluctuating transverse force on each tooth is

$$p' \alpha \c''_v \rho u U^2 \bar{p}^2_s,$$

where $\rho$ is the air density, $S$ is the tooth area, and $c''_v$ is the transverse fluctuating force coefficient. The coefficient value depends upon the tooth design and operating parameters; it is a function of the tooth height and spacing, $H$ and $\Delta$, the disk thickness, $h$, the tooth profile, and the incident flow velocity $U$. Substitution of Eqs. (15) and (14) into Eq. (12), leads to the mean square acoustic pressure approximation

$$p^2_s \propto N \frac{S^2 c''_v U^2 \sin^2 \theta}{\pi |x|^2 \bar{p}^2_s}. $$

The total sound power over an enclosing spherical surface becomes

$$W_s = \int_{S} \bar{p}^2_s d\tilde{s},$$

or

$$W_s \propto \frac{N S^2 c''_v dU^6}{h^2}.$$

The rms sound pressure and the total sound power in Eqs. (16) and (18) relate the saw design and operation to a dipole normal pressure noise. The characteristic proportionality of $U^6$ to sound power in dipole noise is demonstrated. However, that conclusion requires $c''_v$, the fluctuating transverse force coefficient, to be independent of $U$, which is not the case, as will be shown. The dependence of the transverse force coefficient on peripheral speed gives a velocity dependent source $\rho'$ in Eq. (15).

### III. AERODYNAMIC FORCE AND FLOW MEASUREMENT

The turbulent flow field incident on each tooth is a complex superposition of turbulent boundary layers separated from the saw surfaces with the turbulent wake of the upstream teeth. This general problem has not been investigated here or previously. The objectives of the experiments reported here were to measure the fluctuating force or $c''_v$ and to examine the wake structure downstream of a rotating tooth. The fluctuating transverse force was measured at different rotation speeds, and with different tooth profiles and tooth spacings; the saw radius and thickness were not varied.

#### A. Apparatus

An instrumented tooth was mounted on an aluminum test disk as shown in Figs. 4 and 5. The disk diameter is 457 mm and thickness is a uniform 4.7 mm with a maximum deviation in flatness of ±0.06 mm. This thickness was necessary for installation of the tooth lift and drag strain gauge dynamometer and a hot wire anemometer. Two recesses were machined in the disk periphery to mount the dynamometer sensing rod and to position the hot wire. A small groove machined in the disk surface from the disk periphery to the hollow drive shaft contained the instrumentation wires. The wires then passed through the shaft to the signal conditioning unit that rotated with the disk, as seen in Fig. 4. The amplified data signals were taken off the shaft through slip rings. The recesses and grooves were flattened with epoxy and small plates prior to testing. The disk was centrally clamped with representative saw collars at a clamping ratio of 0.5. The motor speed was continuously variable in the range 500–5500 rpm.

The tooth models examined are summarized in Table I. These models were bonded to the dynamometer rod which extended radially beyond the disk periphery as shown in Fig. 5. The radial clearance between tooth model and disk rim was less than 1 mm. The adjusted model variables include tooth height $H$, tooth width $l$, clearance angle $\beta$, and back angle $\beta_2$. The rake angle $\beta_2$ was fixed for all tooth models, and the model thickness was always identical to the disk thickness. The models were cut from balsa wood to minimize their mass. This eased a balancing problem where normal acceleration of the tooth during rotation bends the dynamometer sensing rod, and it maximized the dynamometer frequency response at 4 kHz. The tooth profiles were copied after current saw teeth. There are four different profile types listed in Table 1 under models 1–4. The profiles of models 2–1 and 2–2 are similar to model 2 except the height $H$ is modified for investigation of that variable. Models 4, 4–1, and 4–2 represent tooth profiles with brazed car-
bide tips. The carbide tip width is varied in models 4
and 4-1, and model 4-2 is simply model 4 run back-
wards. Dugdale noted that particularly noisy carbide
tipped cutters were quiet when run backwards.

The flow separated from the edge of a plane disk and
the wake behind each tooth model was measured with a
single constant temperature ($400^\circ$F) hot wire anemom-
eter mounted on the disk. The hot wire was constructed
of 0.038-mm-diam. platinum-plated tungsten wire with
length/diam. equal to 508. The frequency response was
raised with compensation to 10 kHz. The rectangular
recess in the disk periphery allowed adjustment of the
hot wire radial and circumferential position and orienta-
tion. The separated flow from the disk alone (no teeth)
was measured at the wake centerline and at four radial
positions $r/R=1.02, 1.03, 1.06, 1.07$. See Fig. 6 for
the geometrical nomenclature. The wake behind each
single tooth model was measured at the above radial
positions and the downstream locations $\delta=0.33, 0.66,$
1.0. These measurements were undertaken at the three
rotation speeds: 900, 1300, and 1700 rpm. The mean
and fluctuating hot wire responses were recorded with a
resolution of 10 mV or an equivalent air velocity of 0.2
m/s.

The dynamometer sensing element was a 2.4-mm-
diam. by 12.7 mm long cylinder with two, semiconduc-
tor, strain gauge half bridges for lift and drag measure-
ment. Aerodynamic force measurements were recorded
on single tooth and two-tooth models. In the two-tooth
model the downstream tooth was attached to the dyna-
mometer and the upstream tooth was positioned at three
stations $\Delta=0.33, 0.66, 1.1$. The dynamometer resolu-
tion was $5.0 \times 10^{-5}$ N.

B. Flow separated from a plane disk edge

The separated flow at the edge of a plane disk without
teeth was measured at the radial positions $r/R=1.02,$
TABLE I. Summary of the examined tooth models.

<table>
<thead>
<tr>
<th>TOOTH PROFILE</th>
<th>MODEL NO.</th>
<th>HEIGHT (H), mm</th>
<th>THICKNESS (d), mm</th>
<th>BACK ANGLE (β₁), deg</th>
<th>CLEARANCE ANGLE (β₂), deg</th>
<th>RAKE ANGLE (β₃), deg</th>
<th>WIDTH (L), mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>14.2</td>
<td>4.7</td>
<td>30</td>
<td>9</td>
<td>19</td>
<td>22</td>
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<tr>
<td></td>
<td>2</td>
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<td>4.7</td>
<td>50</td>
<td>9</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>2-1</td>
<td></td>
<td>13</td>
<td>4.7</td>
<td>50</td>
<td>9</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>2-2</td>
<td>3</td>
<td>17.1</td>
<td>4.7</td>
<td>50</td>
<td>9</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>4*</td>
<td>14.2</td>
<td>8.0</td>
<td>50</td>
<td>9</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>4-1**</td>
<td>14.2</td>
<td>7.0</td>
<td>50</td>
<td>9</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>4-2**</td>
<td>14.2</td>
<td>8.0</td>
<td></td>
<td></td>
<td></td>
<td>20</td>
</tr>
</tbody>
</table>

* MODELS 4 AND 4-1 ARE CARBIDE TIPPED VERSION OF MODEL 2
** MODEL 4-2 IS MODEL 4 ROTATED BACKWARDS

1.03, 1.06, and 1.07. The mean relative tangential velocity \( \bar{V}_t \), normalized with respect to the local peripheral velocity \( r \Omega \), and the turbulence intensity \( \eta_t \), are shown in Fig. 7(a). The tooth heights are marked on the figure to aid the interpretation of the radial scale; no teeth were present. The velocity \( \bar{V}_t \) increases with radial position; the maximum velocity is approximately 70% of the peripheral velocity at the tooth height. The velocity dependence on position may explain the inconsistent velocity predictions in the literature. The tangential turbulence intensity \( \eta_t \) decreases from 9% to 5% in the radial increment equivalent to approximately 4 of the tooth height. The transverse rms-velocity fluctuation \( \bar{V}_w \) is much less than \( \bar{V}_t \), and it is concentrated within the radial zone occupied by the tooth as shown in Fig. 7(b). The data variations seen in Figs. 7(a) and 7(b) illustrate the impracticality of using a single air velocity and a single turbulence-intensity measure to predict noise. Within the radial region of interest, the flow is three dimensional.

C. Fluctuating transverse force

The fluctuating force coefficient \( c'_{aw} \), measured for the tooth models, is presented in Figs. 8-10. The coefficient as defined by

\[
c'_{aw} = \frac{\left( \rho \bar{V}_t^2 \right)^{1/2}}{\frac{1}{2} p S R^3 \Omega^2}
\]

FIG. 7. (a) Flow from a plane disk rim—no teeth, (b) decay of axial turbulence intensity—no teeth.

FIG. 8. Dependence of the fluctuating transverse force coefficients for the tooth models on disk rim velocity.

H. S. Cho and C. D. Mote: Aerodynamic noise source in saws
is determined from the rms fluctuating force $p'$. The load fluctuation dependence on Reynolds's number for the tooth models and for different tooth spacings of model 1 is shown in Fig. 8. Because the incident flow to each single tooth model was identical at a given $Q$, the fluctuating load differences for each model result from the tooth profile design. The load fluctuation is relatively high for models 3 and 4, and low for models 1 and 4-2. The high load fluctuations of models 3 and 4 are caused by a transverse turbulence that is large compared to turbulence of the other models. The increased transverse turbulence is probably caused by the increased tooth height and the presence of a carbide tip. The models are all sharp edged appendages in the turbulent incident flow; the differences in the fluctuating force, which is equal to a dipole source pressure fluctuation, are significant.

The load fluctuation increases with tooth aspect ratio $H/l$ for model 2 as shown in Fig. 9. The incident turbulence and the disk edge flow separation are modified by tooth height and tooth spacing. The dependence of fluctuating force on tooth spacing is examined with two tooth models in Fig. 10. The dash curves project the coefficient $c_w'$ beyond those of the measured spacings to a point where the spacing is sufficiently large that the upstream tooth wake has negligible influence on the downstream tooth incident flow and accordingly the fluctuating load. The tooth spacing for maximum load fluctuation has not been determined here. The implication of these data is that the aerodynamic force fluctuation increases with the number of teeth less than a certain number, say $N_*$, which corresponds to the tooth spacing of maximum load fluctuation. The load fluctuation decreases with tooth numbers larger than $N_*$. Wake measurements downstream of the single tooth models support this conclusion. Briefly, the mean relative tangential velocity $\bar{V}_t$ increases with the downstream position $s/H$, for $0.4 \leq s/H \leq 1.2$ for all tooth models. The tangential turbulence intensity

$$\eta_t = \left( \frac{\bar{V}_t}{V_w} \right)^2$$

is approximately constant in this downstream interval $s/H$, although a relative maximum $\eta_t$ was measured at $s/H = 0.7$. The increase of $\bar{V}_t$ with wake positions causes an increased pressure fluctuation on the downstream tooth surfaces.

IV. THEORETICAL NOISE ANALYSIS AND DISCUSSION

The sound power level in this discussion is predicted from the theoretical point dipole model in Eq. (18), for the tooth models in Table I.

A. Rotation speed

The character of the dominant noise source is an interesting and controversial topic. The expected $U^6$ dependence of the sound power level for a dipole source is not generally observed. Specifically, the velocity exponent $M$ for proportionality of $U^M$ to sound power has been reported as in Table II. The observed differences between the measured $M$ and 6 can be explained in terms of a dipole source that is dependent upon the Reynolds's number and tooth design. This source dependence was measured in the tooth models herein and represented as $c_w'$ in Figs. 8–10. For an assumed power law dependence of $c_w'$ upon velocity

$$c_w' \approx kRU^\beta,$$

where $k_R$ = constant for a particular tooth design, the $\beta$

<table>
<thead>
<tr>
<th>Reference No.</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>4.56 to 5.07</td>
</tr>
<tr>
<td>7</td>
<td>5.05</td>
</tr>
<tr>
<td>2</td>
<td>4.9</td>
</tr>
<tr>
<td>4</td>
<td>5.6</td>
</tr>
<tr>
<td>6</td>
<td>6.0</td>
</tr>
<tr>
<td>3</td>
<td>5.0 to 5.6</td>
</tr>
</tbody>
</table>
for current models are experimentally determined as shown in Table III. The sound power-rim velocity dependence becomes from Eq. (18),

\[ W_s \propto U_0^{\alpha+\beta}. \]  

The theoretical sound power for the models, relative to a reference sound power at \( U_0 = 1 \) m/s, are presented in Fig. 11. The velocity exponents for a dipole source are seen to all fall between 4.9 for model 4-2 and 6.0 for model 3. When cascades of teeth are considered, the velocity exponent is also a function of tooth spacing. This is deduced from model 1 data in Fig. 8. In short both the tooth profile design and the cutter design affect the source dependence upon velocity. Because of this result, tests on slotted circular plates cannot be expected to provide sound power data applicable to circular saws.

### B. Tooth profile

The source data \( c'_w \) in Fig. 8 can be used to theoretically compare the sound power for the tooth profiles shown in Fig. 12; where the single tooth, model 1 at 900 rpm, is the reference. The carbide tip in model 4 theoretically increases the sound power 1.7 dB from the untipped tooth in model 2. Running model 4 backwards, which effectively alters the tooth profile, reduces the theoretical sound power 4 dB at all \( \Omega \) as seen in model 4-2. Dugdale's experiments show similar results.²⁰ The tooth back angle \( \beta_2 \) contribution to sound power is indicated by comparison of models 1 and 2 data. Increasing \( \beta_2 \) from 30° to 50° increases the theoretical sound power at all speeds, the maximum increase is 1.7 dB at 900 rpm. Among the models tested, model 3 is the noisiest.

### C. Tooth height

The dependence of the source \( c'_w \) on tooth height \( H \) in Fig. 9 may be approximated by

\[ c'_w \approx H^2, \]  

#### Table III. The experimental determination of \( \beta \) for current models.

<table>
<thead>
<tr>
<th>TOOTH PROFILE</th>
<th>MODEL NO</th>
<th>( \beta )</th>
<th>( \eta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>SINGLE TOOTH</td>
<td>( \Delta = 0.33 )</td>
<td>( \Delta = 0.66 )</td>
<td>( \Delta = 1.1 )</td>
</tr>
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</tr>
<tr>
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<td>0.15</td>
<td>0.55</td>
</tr>
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<td>0</td>
<td>0.15</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-0.12</td>
<td>0.55</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>4-2</td>
<td>-0.53</td>
<td>0.55</td>
<td>1.17</td>
</tr>
</tbody>
</table>

(The table continues with additional columns for model numbers, etc.)
where \( k_H \) is constant for all parameters fixed except tooth height. The experimental \( \gamma \) are given in Table II for model 2. Because the surface \( S \) is proportional to \( H \), the theoretical sound power becomes

\[
W_s \propto H^{2+\gamma},
\]

(24)

The sound power predictions are referenced to the shortest tooth model, model 2-1, with aspect ratio \( a^* = H/l = 0.65 \), and are presented in Fig. 13. The sound power dependence upon height \( H \) decreases as the tooth spacing increases. The exponent of \( H \) in Eq. (24) is 3.10 for a single tooth and 4.34 for two teeth at \( \Delta = 0.66 \). A 30\% increase in tooth height increases the theoretical sound power 5 dB for a single tooth and 3.4 dB for two teeth at \( \Delta = 0.66 \). These results are in general agreement with published sound power data.\(^3\),\(^4\)

D. Tooth spacing

The dependence of the source strength on the tooth spacing for the limited data in Fig. 10 may be approximated by

\[
c'_w = k_H \Delta^3,
\]

(25)

where \( k_H \) is constant for all parameters fixed except \( \Delta \). The \( \eta \) values in Table II are almost independent of tooth profile. For fixed tooth width \( l \), the empirical relationship between the tooth spacing \( \Delta \) and number \( N \) is approximately

\[
\Delta \propto N^{-1.97}.
\]

(26)

Accordingly, the sound power dependence on tooth number becomes

\[
W_s \propto N^{1.74}.
\]

(27)

The curves in Fig. 14 present the sound power for each tooth model relative to a 40 tooth reference. For \( N > 40 \) doubling the number of teeth increases the theoretical sound power by 1.6 dB for models 2 and 3, and by 1.3 dB for models 1 and 2-2. The result requires the relationship in Eq. (27) to be valid for these tooth numbers \( N \); the number \( N \) corresponding to the maximum source has not been determined.

V. CONCLUSIONS

The saw noise aerodynamic source can be modeled as an acoustic dipole of strength dependent upon flow velocity as well as the tooth profile design and the cutter design. This was concluded by measurement of the transverse lift force on full scale single tooth models and comparing the noise versus tooth velocity relationships available in the literature to these theoretical values. Theoretical prediction of farfield noise from cutter design and operating specifications is possible. No noise measurements were recorded here. However, the source dependence upon cutter design and operation is in general agreement with published noise data.

The dipole source strength was found proportional to exponents of velocity in the interval \( U^{4.9} \) to \( U^{6.9} \) depending upon tooth profile and spacing. Tooth design modifications including changes in tooth profile, height, back angle, and number are observed to produce assumed dipole source power variations of less than 5 dB. The source strength increases with tooth height, back angle, and with the addition of "carbide" tips on the teeth. Sound power may or may not increase with tooth number. Because of the sensitivity of the source strength and of the source dependence upon \( U \) to the cutter design and rotation speed, the use of slotted circular disks and other analog models for circular saws may not produce results that are applicable to any real saw.

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