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Authors: Cheol-Ung Kim  
Dong-Hoon Kang  
Chang-Sun Hong  
Chun-Gon Kim

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SEND PAPER TO: Professor Lin YE  
Chair of ACCM-4  
Centre for Advanced Materials Technology  
School of Aerospace, Mechanical & Mechatronic Engineering  
Building No. J07  
University of Sydney  
NSW 2006, Australia

Tel: ++61-2-9351-4798  
Fax: ++61-2-9351-3760  
E-mail: ye@aeromech.usyd.edu.au

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Optimal Design of Filament Wound Structures
Based on The Semi-geodesic Path Algorithm

Cheol-Ung Kim, Dong-Hoon Kang, Chang-Sun Hong and Chun-Gon Kim*
Division of Aerospace Engineering, KAIST

ABSTRACT

This research aims to establish an optimal design method of filament wound structures.
Filament winding is one of the most reliable and affordable production techniques for
high performance composite structures such as pressure tanks, pipes and motor cases of
rockets which are widely used in the aerospace application. However, the problem with
filament winding is that the trajectory of the fiber path and the corresponding fiber angles
cannot be chosen freely because of the stability requirement of fiber path. Most design and
manufacturing of filament wound structures are based on manufacturing experiences, and
there is no established design rule. In this research, possible winding patterns considering
the windability and the slippage between fiber and mandrel surface were calculated using
the semi-geodesic path algorithm. In addition, finite element analyses using commercial
code, ABAQUS, were performed to predict the behavior of filament wound structures. On
the basis of the semi-geodesic path algorithm and the verified finite element analysis
method, an optimal design algorithm for filament wound structures was suggested using
the genetic algorithm.

1. INTRODUCTION

Finite element analyses, which can predict the deformation of filament wound
structures, have been performed by many researchers. However, the results have been
utilized only to understand structural characteristics of filament wound structures because
some limited path equations were applied to the analyses. Even though such finite element
analyses are helpful to design filament wound structures, most design and manufacturing
of filament wound structures have been based on manufacturing experience and
experiment. Thus, most designs are not optimized to account for filament wound
structures.

Previous research about filament wound structures are categorized into fiber path
predictions, structural analyses and designs. However, there is no established design
method for general filament wound structures under internal pressure satisfying given
design requirements.

In this study, an optimal design method of filament wound structures under internal
pressure was established. Possible winding patterns considering the windability and the
slippage between fiber and mandrel surface were calculated using the semi-geodesic path
algorithm. In addition, finite element analyses using commercial code, ABAQUS, were
performed to predict the behavior of filament wound structures. On the basis of the

*Correspondence Author : Division of Aerospace Engineering, KAIST, 373-1 Guseong-dong, Yuseong-gu,
Daejeon, 305-701, Republic of Korea.
tel : (82-42) 869-3719, fax : (82-42) 869-3710, email : cgkim@kaist.ac.kr
semi-geodesic path algorithm and the finite element analysis method, an optimal design algorithm was suggested using the genetic algorithm and applied to a symmetric composite pressure vessel.

2. SEMI-GEODESIC PATH ALGORITHM

The design of a filament wound structure consists of the design of the mandrel shape and the calculation of the fiber path. In general, the mandrel shape can be determined by imposed design requirements such as internal pressure, volume and manufacturing convenience. When the liner or mandrel surface is given without considering winding patterns, various slip conditions must be taken into account.

In this study, the semi-geodesic path equation was utilized in order to describe the realistic winding pattern on general filament wound structures [1].

\[
\frac{d\alpha}{dx} = \lambda \left( \frac{r A^2 \sin^2 \alpha - r r^* \cos^2 \alpha - r' A^2 \sin \alpha}{r A^2 \cos \alpha} \right) \quad (|\lambda| = \left| \frac{f_\lambda}{f_\mu} \right| \leq \mu, \quad A = \sqrt{1 + r'^2})
\]

Equation (1) is defined on an arbitrary surface where \( \alpha, x, \theta, r, \lambda \) are the winding angle, the axial coordinate parameter, the circumferential coordinate parameter, the radial coordinate parameter and the slippage tendency between the fiber and the mandrel. And more detailed derivation is shown in Reference [1].

By integrating Equation (1) with a known value, the winding angle can be calculated for the entire mandrel surface. There are two assumptions in the calculation of the thickness: the fiber volume fraction is maintained consistently and the number of fibers in a cross section is always constant. With these assumptions, the thickness along the longitudinal direction can be derived as follows:

\[
t = \frac{n_p w t_c}{2\pi r \cos \alpha} = \frac{2\pi r_c \cos \alpha_c w t_c}{2 \mu \pi \cos \alpha} = \frac{r_c \cos \alpha_c}{r \cos \alpha} \times t_c
\]

where \( r_c, \alpha_c, t_c, n_p, w \) are radius, winding angle, thickness, number of fiber bands in a layer, and band width, respectively.

![FIGURE 1 Semi-geodesic path algorithm](image-url)
Windability is very important because some of the patterns calculated by Equation (1) may be useless for the manufacture of filament wound structures unless uniform coverage is considered. The concept of windability was presented concurrently by Lossie [2] and Lowery [3]. Though notations used in the two studies are different, fundamental concepts are identical.

The semi-geodesic path algorithm suggested in this study considers the fiber slippage and the windability. Figure 1 is the flow chart of the semi-geodesic path algorithm. In this research, a GUI (graphic user interface) based program was developed using Microsoft Visual C++ to perform the semi-geodesic path algorithm.

3. PROGRESSIVE FAILURE ANALYSIS

In this research, the 3-D layered solid element was utilized for finite element modeling. Progressive failure analysis was performed for the forward part of the ASTEB by the commercial code, ABAQUS. The finite element model are shown in Figure 2. In the modeling, C3D20 type elements with 20 nodes per element were used. The model has 100 elements and 1113 nodes. The modeling was performed for a 1.5° strip of a full tank using a cyclic symmetry boundary condition. The element size is larger at the center of the dome than that near the interface of the dome/cylinder and dome/polar opening. In the cylindrical part, 20 elements to the x-axial direction were enough to gain a converged result, but 60 elements to the meridian direction were used in the dome area. The winding path was calculated using the established semi-geodesic path algorithm. And, a preprocessor PREAFT which imposes the calculated path information on each element was developed and used. The inner pressure is 13.79 MPa (2000 psi).

![FIGURE 2 Finite element model and deformed shape](image)
In order to perform the progressive failure analysis, the modified Hashin’s failure criterion, which is commonly used in recent studies and has many failure modes, was selected and applied to the analysis and optimal design in this research. For the purpose of failure analysis, a subroutine, USDFLD of ABAQUS ver 6.3 was coded to define the change of mechanical properties due to failure. An element failure is first identified and later the failed element is replaced with a degraded element. The degradation method should be carefully chosen because the results of progressive failure analyses often depend on the mesh size, degree of reduction, the increment size, etc. The equivalent properties of the damaged element might exist between zero values and solid ones. In this study, the stiffness reduction coefficient (SRC) varies from 0.1 to 0.9 by using Reddy’s method [4]. When SRC is 0.1, the failed elements almost lose their load-carrying capacity. SRC of 0.9 means that the mechanical properties of the failed element do not decrease much compared to those of unfailed ones. The case of SRC<0.1 was excluded because the analysis becomes a conservative one.

The deformation of each case is magnified 5 times in Figure 2. Figure 3 shows the comparison of results between the finite element analysis using the modified Hashin’s failure criterion and the water-pressurizing test of our previous research [5]. It shows good agreement between them both in trend and quantity. Therefore, this analysis method is suitable for applying to the optimal design.

4. OPTIMAL DESIGN USING GENETIC ALGORITHM

4.1 Design algorithm

In this study, the optimal design algorithm was established, which includes the semi-geodesic path algorithm, progressive failure analysis and genetic algorithms [6]. Figure 4 shows the flow chart of the suggested optimal design algorithm. The genetic algorithm controls the overall procedure. The windows based program for this algorithm was developed using C++ language, and it was named ‘IDOTCOM_FW’.
4.2 Application and verification

The established optimal design algorithm was applied to a symmetric pressure tank of type 3 with a load sharing metallic liner for verification. The half shape of the tank is the same as the forward part of the ASTEB. The material of the composites is T800/Epoxy, and the liner is aluminum alloy 7075-T6. Basic design requirements are as follows:

1. Maximum operating inner pressure is 13.79 MPa (2000 psi).
2. The yield of the liner is not permitted.
3. The safety factor of the composite is 3.0.
4. The weight reduction is the most important goal of this design.

The objective function is defined as follows:

\[
f = \begin{cases} 
\frac{W_{\text{max}}}{W} + 0.1 \times \frac{\sigma_{f,\text{design}}}{\sigma_{\text{liner}}} + 0.1 \times \frac{\sigma_{\text{yield}}}{\sigma_{\text{liner}}} \leq \sigma_{\text{liner}} \leq \sigma_{\text{yield}} \leq \sigma_{f,\text{design}} & \sigma_{\text{liner}} > \sigma_{\text{yield}} \\
\frac{\sigma_{f,\text{design}}}{\sigma_{\text{fiber}}} + 0.1 \times \frac{\sigma_{\text{yield}}}{\sigma_{\text{liner}}} \leq \sigma_{\text{liner}} \leq \sigma_{\text{yield}} > \sigma_{f,\text{design}} \\
\frac{\sigma_{\text{yield}}}{\sigma_{\text{liner}}} > \sigma_{\text{yield}} \end{cases}
\]  

\hspace*{2cm}(3)

where \(W_{\text{max}}\), \(W\), \(\sigma_{f,\text{design}}\), \(\sigma_{\text{fiber}}\), \(\sigma_{\text{yield}}\), \(\sigma_{\text{liner}}\) are the possible maximum weight, the weight of the design point, the fiber directional strength considering the safety factor, the maximum fiber directional stress of the design point, the yield strength of the liner, and the maximum von Mises stress of the liner of the design point, respectively.

Four design variables (the number of helical layers, the number of hoop layers, the winding angle of the cylinder part and the thickness of the liner) were set up and used in this optimal design because the goal of this application is the verification of the suggested algorithm and program. Each of them was separated to discrete values as many as 2 bits, 4
bits, 5 bits and 4 bits in order to be applied to the genetic algorithm. From our previous researches, we determined the required parameters for genetic algorithm as follows; the population size was set as 100, the maximum number of generations as 100, the probability of crossover as 0.7, the probability of mutation as 0.1 and the tournament size for the genetic algorithm as 10. The initial seed value was generated randomly, and a total of ten optimal designs were performed.

Table 1 shows the design results. Results of seven kinds were drawn. Among them, the best case, which satisfies the given design requirements, is a tank with a weight of 4.44 kg.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Helical layer</th>
<th>Hoop layer</th>
<th>Winding angle</th>
<th>Liner</th>
<th>Weight</th>
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<td>1</td>
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<td>9</td>
<td>33.5°</td>
<td>1.9 mm</td>
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<td>3</td>
<td>10</td>
<td>34.5°</td>
<td>1.7 mm</td>
<td>4.47 kg</td>
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</tr>
<tr>
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<td>10</td>
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<td>4.45 kg</td>
</tr>
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</table>

5. CONCLUSION

In this research, possible winding patterns considering windability and slippage were calculated using the semi-geodesic path algorithm. In addition, progressive failure analyses were performed to predict the behavior of filament wound structures. In particular, suitable element types and failure criteria for filament wound structures were studied. In addition, on the basis of the semi-geodesic path algorithm and the finite element analysis method, an optimal design algorithm was suggested using the genetic algorithm. Finally, the developed design code was applied to a symmetric composite pressure vessel for verification.

REFERENCES