Sound Image Constructed by Line Array Sweeping Method

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ABSTRACT

There are various methods which can be used to visualize acoustic field of interest. Acoustic holograph is one of them. Comparing with other methods; for example, acoustic intensity mapping, acoustic holograph requires complicated, laborious, and costly measurement set-up. Therefore, there have been many researches which mainly convey the idea of which reduces or simplifies the measurement configuration into practically achievable one. In fact, STSF(Spatial Transformation of Sound Fields) essentially achieved this objective. It uses a reference microphone and line array system. The array scans the sound field of interest step by step. This method is very powerful for measuring sound field which comes from stationary sound sources. Other than stationary sound field, such as non-stationary sound field but stationary in space, this method cannot be used. This paper introduces the way in which one can construct necessary hologram by using single line array. This method does not have limitation on the type of sound field. The theoretical formulation starts with introducing appropriate coordinate system which essentially uses three coordinate system; inertia, hologram, and microphone coordinate. The relations between these coordinates have been theoretically found. Various examples which include noise from running car, motorcycle, Zing; one of Korean gong, show the usefulness of the method.

1. INTRODUCTION

Planar acoustic holograph is one of the most valuable sound visualization techniques for expressing sounds with respect to time and space. The beauty of holographic methods is that one can reconstruct entire sound field based on the hologram measurement; hologram is the pressure distribution with respect to each frequency on the hologram plane. This enables one to see every details of sound field of interest, which include not only pressure but also particle velocity, acoustic power, and intensity. The practical limitation however also exists in the
realization of the method. The predicted results are directly affected by the number of measurement points in practical circumstances. The more one uses, the better quality of predicted sound pictures one expects to have [1]. There have been many researches which mainly convey the idea of which reduces or simplifies the measurement configuration into practically achievable one [2-3].

For stationary sound field, this heavy investment can be reduced by employing the technique of scanning over hologram plane. STSF(Spatial Transformation of Sound Fields) [2] and BAHIM(Broadband Acoustical Holography from Intensity Measurements) [3] essentially achieved this objective. Although these methods make the hologram measurement easier than the conventional simultaneous measurement method [1], these works only for non-moving source which emitting stationary sound field. It is because these methods scan the hologram step by step. This is a significant limitation of the available methods.

The problems associated with the conventional methods motivated us to develop a new scanning method which brings a useful alternative. This method generalizes the holographic method so that one can use it for the visualization of the sound field generated by moving sources, such as moving vehicles [4].

2. THEORETICAL BACKGROUNDS OF SWEEPING ARRAY METHOD

Pressure expression in relative coordinate

A general configuration of hologram measurement by using a sweeping array is depicted in Fig. 1. Three coordinates are employed in this measurement system. One is “inertia coordinate” which is fixed in space, the second one is “microphone coordinate” which is allowed to move, and the last one is “hologram coordinate” which can either move or be stationary depending on whether source is moving or not. The hologram coordinate is the coordinate system where one obtains the hologram which will be used to predict all acoustic variables on unmeasured planes. Sound pressure on each coordinate system can be written as \( p(x, y, z; t) \), \( p_{mic}(x_m, y_m, z_m; t) \) and \( p_{hol}(x_h, y_h, z_h; t) \). The subscript \( \text{mic} \) and \( \text{hol} \) express microphone and hologram coordinate system respectively. When one assumes that all coordinates are at the same location at \( t = 0 \), and they move in \( x \) direction afterward, then

\[
y = y_m = y_h, \quad z = z_m = z_h, \quad x_m = x - u_m t, \quad x_h = x - u_h t.
\]

Let’s denote the relative speed between microphone coordinate and hologram coordinate as \( u \), and assume that the microphone array is fixed at \( x_m = 0 \) in microphone coordinate as shown in Fig. 1. This will lead to

\[
p_{mic}(0, y, z_H; t) = p_{hol}(ut, y, z_H; t).
\]

This equation essentially allows us to construct the hologram in hologram coordinate whether acoustic source moves or not. Other interpretation of Eq. (2) is also possible, that is one can scan the hologram plane, which means that source does not move but microphone array moves instead, and from this information, construction of the hologram can be achieved.
Construction of acoustic hologram

Hologram is the expression of pressure distribution on the hologram plane with respect to frequency. The pressure in frequency domain is expressed by using temporal Fourier transform, that is

$$P_{\text{hol}}(x, y, z_H; f) = F_T \{ p_{\text{hol}}(x, y, z_H; t) \} = \int_{-\infty}^{\infty} p_{\text{hol}}(x, y, z_H; t)e^{i2\pi ft} dt,$$  \hspace{1cm} (3)

Here, $F_T \{ \}$ presents temporal Fourier transform. Also $P_{\text{hol}}(x, y, z_H; f)$ is the hologram expressed in hologram coordinate at frequency; $f$. This is true when both the hologram and the microphone array are not moving. However, one cannot simply use the above equation for the case of sweeping microphone array or moving source situation. Instead, the following expression holds;

$$F_T \{ p_{\text{hol}}(ut, y, z_H; t) \} = \int_{-\infty}^{\infty} p_{\text{hol}}(ut, y, z_H; t)e^{i2\pi ft} dt = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_{\text{hol}}(ut, y, z_H; f_h)e^{-i2\pi f_h df_h} df_h e^{i2\pi ft} dt.$$  \hspace{1cm} (4)

Our main objective is, then, to obtain hologram which is in the integrand in the expression of Eq. (4) \cite{4}. To achieve this goal, one has to utilize inverse spatial Fourier transform with respect to $x$, that is

$$P_{\text{hol}}(x, y, z_H; f) = F_{\text{X}}^{-1} \{ \tilde{p}_{\text{hol}}(k_x, y, z_H; f) \} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{p}_{\text{hol}}(k_x, y, z_H; f)e^{i2\pi k_x x} dk_x.$$  \hspace{1cm} (5)

Using the above transform, one can rewrite Eq. (4) as

$$F_T \{ p_{\text{hol}}(ut, y, z_H; t) \} = \frac{1}{u} \int_{-\infty}^{\infty} \tilde{p}_{\text{hol}}(\frac{2\pi(f_h - f)}{u}, y, z_H; f_h) df_h.$$  \hspace{1cm} (6)

Here, Eq. (6) essentially states that the Fourier transform of measured sound pressure with respect to time can be obtained from wave number spectrum of $x$ variable centered at $f_h$ which is the frequency of interest of sound field. The integrand has the argument of frequency; $f$, which is shifted to $f_h$. And the distribution of frequency is dependent on the type of acoustic field. For example, for a planar wave front, this turns out to be shifted frequency of $f_h - f$, corresponding to the wave number; $k_x$, which depends on the direction of the wave. But, for a monopole source, such wave numbers are cosines of free space wave number; $k$, therefore they are continuously distributed around $f_h$. This essential feature in estimating hologram imposes fundamental limitation. This is due to the effect of spreading frequencies around $f_h$, which are inherent when there is relative motion between the source (or hologram) and the microphone array. When one has single frequency of interest, it is not a problem at all. However, when one has more than one frequency of interest; say, $f_{h1}$ and $f_{h2}$, then one has an overlap problem and one would not be able to decompose the spread components of $f_{h1}$ from those of $f_{h2}$. The criteria to avoid this rather undesirable effect is addressed in the following section.

When we have multiple frequency components in the sound field of interest, which is
usually the case we have interest, then we have multiple \( f_\text{hi} \)'s and associated side bands (Fig. 2). The following equations expresses this situation mathematically;

\[
p_{\text{hol}}(ut, y, z_\text{hi}; t) = \frac{1}{2} \left( \sum_{i=1}^{l} P_{\text{hol}}(ut, y, z_\text{hi}; f_\text{hi}) e^{-j2\pi u t} + \sum_{i=1}^{l} P_{\text{hol}}^*(ut, y, z_\text{hi}; f_\text{hi}) e^{j2\pi u t} \right), \tag{7}
\]

and

\[
F_T \{ p_{\text{hol}}(ut, y, z_\text{hi}; t) \} = \frac{1}{2u} \left( \sum_{i=1}^{l} \tilde{P}_{\text{hol}} \left( \frac{2\pi(f_\text{hi} - f)}{u}, y, z_\text{hi}; f_\text{hi} \right) e^{-j2\pi u t} \right.
\]
\[
\left. + \sum_{i=1}^{l} \tilde{P}_{\text{hol}}^* \left( \frac{2\pi(f_\text{hi} + f)}{u}, y, z_\text{hi}; f_\text{hi} \right) e^{j2\pi u t} \right\}. \tag{8}
\]

**Condition to avoid side band overlapping**

Let’s assume that the wave number spectral band in \( x \) direction is limited by the two times the free space wave number. It is a general requirement to avoid spatial aliasing in sampling the hologram. Then, by using the \( x \) directional wave number - frequency relation in the proposed method as

\[
\frac{2\pi(f_\text{hi} - f)}{u} = k_x,
\]

we obtain the following condition

\[
(1 - 2M)f_\text{hi} \leq f \leq (1 + 2M)f_\text{hi}, \quad M = \frac{u}{c}. \tag{10}
\]

where \( M \) is Mach number. Without loss of generality, minimum frequency has to be positive, therefore the Mach number must be less than 0.5. The requirement on allowable frequency separation between adjacent frequency components can be obtained from Eq. (10), that is

\[
f_{\text{hi}+1} > \frac{1 + 2M}{1 - 2M} f_{\text{hi}}. \tag{11}
\]

This is a quite pleasant requirement in practice.

### 3. EXPERIMENTAL RESULTS

Experiments were attempted to demonstrate how well the proposed method works. Fig. 3 shows fundamental features of the method in comparison with other existing methods. Fig. 4 is the case of moving vehicle. We used one loudspeaker, locating at right front window as depicted in Fig. 5. The result shows that the method very accurately estimate the radiated sound field, especially the location of the source. Last figure, Fig. 6 actually demonstrates how well the method mimics the actual sound field generated from moving source. In this specific case, we used motorcycle and had attention to the frequency of noise generated at the
engine; other cases of frequencies are also available in reference [4]. This figure shows the noise picture in space and time.

4. CONCLUDING REMARKS

A new hologram measurement method by using a sweeping line array was introduced. This method is very useful to measure the hologram of moving source, therefore extends the possible applications of acoustic holographic method. This paper only shows the case of which source moves with constant speed. We now extends to the case when one have variable speed of moving sources.

REFERENCES


Fig. 1. Coordinate systems (inertia coordinate, microphone coordinate, and hologram coordinate) and their relative motions

Fig. 2. Procedure to calculate the hologram from the measured pressure signal
Fig. 3. Comparison of the effects of hologram measurement methods by the reconstructed pressure images on the source plane; two identical speakers radiate the sound of 500 Hz in this experiment, and the holograms were in an anechoic room by each method.
Fig. 4. Experimental set-up to measure the hologram of moving source by using a fixed vertical line array of 16 microphones; two photo-electric sensors were used to check the position with respect to time and the speed of moving source.

Fig. 5. Predicted results of moving source experiment by using a speaker which was attached on the front side window of a moving automobile, and which radiates the pure sound of 700 Hz; the automobile moved at the speed of 8.204m/s on the paved road, 62cm away from the array.
Fig. 6. Predicted results of moving source experiment by using a moving motorcycle (pressure images on the source plane with respect to time; motion pictures), about which radiates the tonal sound of 807 Hz; the motorcycle moved at the speed of 8.901 m/s on the paved road, 43 cm away from the array.