Design of Lorentz Force Type Integrated Motor-Bearing System Using Permanent Magnets and Concentrated Windings

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Abstract: The integrated motor-bearing system integrates both functions of active magnetic bearings and electric motor into a unit. In this paper, a generalized approach to generation of torque and radial force in integrated motor-bearing systems using Lorentz forces is presented. The constraints to arrangement of permanent magnets and distribution of concentrated coil windings are derived for the disk and cylindrical type configurations to be realized. It is found that the difference between the number of permanent magnets and the number of magnetic poles for motoring should be an integer multiple of the number of windings. The phase difference of position control current is determined by the rule that the number of poles for position control should be more or less by 2 than the number of the motoring poles. Typical feasible combinations of design parameters satisfying the constraints are also listed.

Keywords: Bearingless motor, Magnetic bearing, Lorentz force, AC Synchronous motor

Nomenclature

\[ A \] Magnitude of motoring current
\[ B \] Peak of air gap flux density generated by rotor magnets
\[ C \] Magnitude of radial force current
\[ k \] Number of turns per concentrated winding
\[ l \] Effective length of winding part crossing the flux
\[ M \] Number of pole pairs for motoring current
\[ N \] Number of pole pairs for radial force current
\[ \mathbb{N} \] Set of natural numbers
\[ Q \] Number of pole pairs for rotor permanent magnets
\[ r \] Effective radius of stator windings
\[ t \] Time
\[ W \] Number of windings \((W \geq 5)\)
\[ 2\alpha \] Winding pitch angle
\[ \eta \] Winding angular interval
\[ \theta \] Angular coordinate fixed at stator
\[ \phi \] Phase shift of radial force control current
\[ \varphi \] Phase shift of motoring current
\[ \omega \] AC current frequency \([\text{rad/sec}]\)

1 Introduction

Active magnetic bearing (AMB) is an electromagnetic device that supports the rotor using the controlled electromagnetic forces without any contact. There has been a wide range of AMB applications in industry, owing to such advantages as free of mechanical contact and lubrication, high peripheral speed and precision operation, and adjustability of the bearing stiffness and damping within the physical limits [1]. Nevertheless, the conventional AMB system normally requires a relatively long bearing span or shaft to accommodate an electric motor to drive the rotor as well as AMBs, lowering the flexural critical speeds. On the other hand, the integrated motor-bearing system integrates both functions of AMB and electric motor into a unit and thus is capable of simultaneously generating torque for motoring and radial or axial forces for levitating the rotor through the control of rotating magnetic flux distribution. It can therefore possess a simple and compact mechanical structure, yet preserving inherent advantages of AMB. In return, however, the design and analysis of its magnetic field and the controller design become a little complex because the motoring and levitation of the rotor has to be simultaneously controlled.

Since one and half decades ago, extensive works have been carried out in order to develop various kinds of integrated motor-bearing systems [2], such as induction type, permanent magnet type, homopolar type, reluctance type, etc. Most of them have the design principles based on use of the Maxwell forces for the position control of the rotor. In recent years, new integrated motor-bearing systems using the Lorentz forces for the rotor position as well as torque control were proposed [3-5]. Maxwell force is an attractive force between the magnetic poles of opposite polarity produced by the magnetic fluxes going through the air gap. It has a nonlinear relationship between the force magnitude and the length of air gap, and heavy laminated cores are necessary for most of its applications. On the other hand, Lorentz force is bidirectional and holds a good linear property, by virtue of which the analysis and design of the system can be made easy. In addition, it can be obtained even if no ferrite core is employed. A coreless configuration serves to reduce the system weight and takes away such troubles as iron loss, magnetic saturation, hysteresis, eddy current effect, etc. However, the system proposed by Han, et al. [3] necessitates a complex frequency demodulation skill with two stator disks to avoid singularity in calculating control inputs, resulting mainly from the fact that the system has the same number of poles in the stator windings as that of the permanent magnets on the rotor disk. Okada, et al. [4] developed two prototypes of integrated motor-bearing system: one is of the disk type and the other is of the cylindrical type, using eight permanent magnets on the
rotor and six concentrated stator windings driven by three phase currents. Though the proposed disk type system, equipped with one rotor disk and one stator disk, accomplished stable operation, the capability of force generation was not satisfactory. The cylindrical type one was designed as a remedy to the defect of the disk type, but they reported that their theory was valid only for eight-pole motors. Kim, et al. [5] developed another Lorentz force type integrated motor-bearing system, which has four permanent magnets inside the ring-shape rotor and six-concentrated windings on the slotted stator core, but the design formula is also limited to the case of four pole motors.

In this paper, a general formula for generation of torque and radial force in integrated motor-bearing systems using Lorentz forces is presented. The formula is based on the use of common windings for both torque and position control, as it is beneficial to the compactness of the system. In spite of the difference in configuration, the principle can be applicable to the cylindrical and disk types in a similar way; this paper deals with both of them.

2 Principles of Torque and Radial Force Generation

Consider a pair of concentrated windings in an orthogonal magnetic flux field. Figure 1 shows the conceptual drawing for a disk type, and Fig. 2 is the equivalent one for a cylindrical type. The dot and cross in the circles shown in Fig. 1 mean the magnetic flux coming out of and going into the page, respectively, the loop in the windings indicates the current flow, and the arrow on the loop shows its direction. In Fig. 2, the dot and cross in the circles represent the current flow direction, and the magnetic flux runs across the windings in the radial direction. Figures 1 (a) and 2 (a) illustrate the basic principle of torque generation. According to the Lorentz law, electromagnetic forces, which are represented by the thick arrows in Figs. 1 and 2, are produced in the upper and lower sides of windings for the counter-clockwise current flows in the right and left windings. The reaction forces to the rotor have counter direction to the forces generated in the windings, resulting in a couple moment. The contour arrow at the center indicates the motoring torque.

For radial force generation, the current flow direction in the left side winding is reversed while the current flow in the right winding and all the magnetic fluxes remain unchanged as shown in Figs. 1 (b) and 2 (b). The direction of the force generated in the left side winding is then reversed. Hence, the resultant of the reaction force vectors generated by both windings will head for the upper direction. Reversing the current flows in both windings, of course, can easily change the radial force direction. As the rotor rotates at a certain speed, the magnetic flux polarity may alternate periodically, leading to oscillation or fluctuation of the forces. Therefore, a proper combination of fluctuating forces is essential in order to obtain stable and reliable forces for the radial position control of the integrated motor-bearing system.

3 Theoretical Approach to Generalized System

Let us consider the $W$ equi-spaced concentrated windings along the periphery of the stator as shown in Fig. 3. The derivation of a new design formula developed here begins on the basis of the so-called ‘$P \pm 2$’ formula presented by Okada, et al. [6]: the difference between the numbers of poles for the motoring and position control current distributions should be plus or minus two in order to obtain the decoupled controllability of torque and rotor position. The positive sign is assigned to the magnetic flux generated by rotor permanent magnets coming out of the page, and to the effective current running from the center of the stator to the outer radial direction in Fig. 3 (a). Under that sign convention, the stator will deliver a positive torque in the counterclockwise direction. To keep consistency of analysis in cylindrical configuration as shown in Fig. 3 (b), the magnetic flux in the outer radial direction and the current going into the page should take the positive value. For simplicity of analysis, assume that the magnetic flux density generated by the permanent magnets on the rotor takes a sinusoidal waveform of
\[ B_R = B \cos(\omega t - Q\theta) \]  
where \( B \) is the amplitude of the flux density, \( \omega \) is the AC circular frequency, \( Q \) is the number of pole pairs of the rotor permanent magnets and \( \theta \) is the angular coordinate. Now consider the currents for motoring torque given in the form of

\[ I_i^M = A\cos(\omega t + \varphi - M\eta i) \quad i = 0, 1, \ldots, W - 1 \]  
where \( M \) denotes the number of pole pairs for motoring current in the stator windings, \( A \) is the magnitude of the current, \( \varphi \) is the phase shift, and the angular spacing of the windings \( \eta \) is defined as

\[ \eta = \frac{2\pi}{W} \]  

Note that the phase difference between the currents in the two adjacent windings is \( M\eta \) and equation (2) forms the spatial current distribution of \( M \) pole pairs in the stator windings. The stator winding distribution can be represented by using Dirac’s delta function as

\[ D_i = k\delta(\theta - \eta i + \alpha) - k\delta(\theta - \eta i - \alpha) \quad i = 0, 1, \ldots, W - 1 \]  
where \( k \) is the number of turns in a concentrated winding and \( 2\alpha \) is the winding pitch angle. Then the motoring torque produced by these currents can be obtained, using Lorentz law, as

\[ T = \sum_{i=0}^{W-1} r l \int_{0}^{2\pi} B_R I_i^M D_i d\theta = ABkr l \sin(Q\alpha) \times \left( \sum_{i=0}^{W-1} [\sin(\varphi + (Q-M)\eta i) - \sin(2\omega t + \varphi - (Q+M)\eta i)] \right) \]  

Thus, the conditions for the torque to be a nonzero constant, regardless of the rotating speed of the rotor become

\[ (Q+M)\eta = 2m\pi \\
(Q-M)\eta = 2n\pi \]  

where \( m \) and \( n \) are arbitrary integers. In other words, the number of pole pairs \( Q \) for the rotor permanent magnets and the number of pole pairs \( M \) for the motoring current should be determined so that they satisfy the relation, for the number of windings \( W \), given by

\[ Q = M + nW \]

Equation (7) implies that the phase difference, \( M\eta \) in equation (2), should be restricted to a proper fraction of \( \pi \) so that all motoring currents are not given in phase or out of phase of the reference state corresponding to, say, \( i = 0 \). It also means that \( (Q-M) \) should be an integer multiple of \( W \). Note, however, that \( Q \), physically a natural number, can take a negative integer, since equation (1) still holds for \( -Q \), which is equivalent to reversal of the sign convention for \( \theta \). For a negative integer value of \( Q \), the torque direction will also be reversed so that the rotor will rotate clockwise. Substituting equation (7) into equation (5), we obtain

\[ T = ABkr l \sin(Q\alpha) \sin\varphi \]  

implying that the motoring torque is proportional to the number of windings, the magnitudes of the motoring current and the magnetic flux, the number of turns, and the effective length of the winding. It suggests that the torque attains its maximum value when the winding pitch angle \( 2\alpha \) is tuned to the pitch angle of the rotor permanent magnets. In addition, we can control the system like a standard synchronous AC motor by adjusting the phase shift \( \varphi \) from 0° to 90°, depending on the increase of load torque. When \( \varphi = 90^\circ \), we can operate the system as a servomotor controlled by the current magnitude \( A \). The relation between the rotor speed \( N_S \) in rpm and the motoring current frequency \( f \) in Hz can be written as

\[ N_S = \frac{60f}{Q} \text{ [RPM]} \]
where $\phi = 2\pi f \tau$.

Similarly to the previous procedure for generation of motoring torque, consider the currents for the radial control force in the form of

$$I_C = C \cos(\alpha t + \phi - N\eta) \quad i = 0, 1, \ldots, W-1 \quad (10)$$

where $N$ denotes the number of pole pairs for radial force control current in the stator windings, $C$ is the magnitude of the current, and $\phi$ is the phase shift. Note that the phase difference between the currents in the two adjacent windings is $N\eta$. Applying these currents to the windings represented by equation (4), we obtain the expressions for the radial control forces in the $x$ and $y$ directions as

$$F_x = -\sum_{i=0}^{W-1} \int_0^{2\pi} B_R I_C^i D_i \sin \theta d\theta = \frac{1}{2} BClk \times \left\{ \begin{array}{l} \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \cos(\phi + (Q - N + 1)\eta) \\
+ \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \cos(2\alpha t + \phi - (Q + N + 1)\eta) \\
- \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \cos(\phi + (Q - N - 1)\eta) \\
- \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \cos(2\alpha t + \phi - (Q + N - 1)\eta) \end{array} \right. \quad (11)$$

$$F_y = \sum_{i=0}^{W-1} \int_0^{2\pi} B_R I_C^i D_i \cos \theta d\theta = \frac{1}{2} BClk \times \left\{ \begin{array}{l} \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \sin(\phi + (Q - N + 1)\eta) \\
- \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \sin(2\alpha t + \phi - (Q + N + 1)\eta) \\
+ \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \sin(\phi + (Q - N - 1)\eta) \\
- \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \sin(2\alpha t + \phi - (Q + N - 1)\eta) \end{array} \right. \quad (12)$$

Eliminating the terms including $\alpha t$, we derive the conditions for generation of a stable and controllable radial force given by

$$N = M \pm 1 \quad \frac{2N}{W} \notin N \quad (13)$$

Note here that the condition for a stable radial force coincides with the ‘$P \pm 2$’ formula and the control currents also should not be given in phase or out of phase, in the same way as for the motoring currents. Substituting equations (7) and (13) into equation (12), we obtain

$$F_x = \pm \frac{1}{2} WBCl \sin((Q \pm 1)\alpha) \cos \phi \quad (14)$$

where the double signs are aligned. Equation (14) proves that the strength and the two dimensional direction of the radial force can be linearly controlled by the magnitude $C$ and the phase shift $\phi$ of the current, respectively. The sign of the radial force in equation (14) depends completely upon how to locate two orthogonal coordinates of the system. It also reveals that the winding pitch angle to get the maximum radial force is slightly different from that for the maximum torque, i.e., the angle becomes narrower or wider according to the pole pairs of the control current and the sign of $Q$. The radial control force is also independent of the rotational speed of the rotor, which makes it simple to design the radial position controller of the system, not requiring a complex frequency demodulation step. Finally, the torque generated by the radial force currents and the radial force generated by the motoring current are calculated as

$$T_c = \sum_{i=0}^{W-1} \int_0^{2\pi} B_R I_C^i D_i \sin \theta d\theta = BCkrl \sin(\phi t) \times \left\{ \begin{array}{l} \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \cos(\phi + (Q - M + 1)\eta) \\
+ \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \cos(2\alpha t + \phi - (Q + M + 1)\eta) \\
- \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \cos(\phi + (Q - M - 1)\eta) \\
- \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \cos(2\alpha t + \phi - (Q + M - 1)\eta) \end{array} \right. \quad (15)$$

$$F_x^M = -\sum_{i=0}^{W-1} \int_0^{2\pi} B_R I^M_M D_i \sin \theta d\theta = \frac{1}{2} ABkl \times \left\{ \begin{array}{l} \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \sin(\phi + (Q - M + 1)\eta) \\
- \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \sin(2\alpha t + \phi - (Q + M + 1)\eta) \\
+ \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \sin(\phi + (Q - M - 1)\eta) \\
- \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \sin(2\alpha t + \phi - (Q + M - 1)\eta) \end{array} \right. \quad (16)$$

$$F_y^M = \sum_{i=0}^{W-1} \int_0^{2\pi} B_R I^M_M D_i \cos \theta d\theta = \frac{1}{2} ABkl \times \left\{ \begin{array}{l} \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \sin(\phi + (Q - M + 1)\eta) \\
- \sin((Q+1)\alpha) \sum_{i=0}^{W-1} \sin(2\alpha t + \phi - (Q + M + 1)\eta) \\
+ \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \sin(\phi + (Q - M - 1)\eta) \\
- \sin((Q-1)\alpha) \sum_{i=0}^{W-1} \sin(2\alpha t + \phi - (Q + M - 1)\eta) \end{array} \right. \quad (17)$$

From equations (15), (16), and (17), it is possible to decouple the torque and control force generation, when they hold

$$\frac{2M+1}{W} \notin N, \quad \frac{2M-1}{W} \notin N \quad (18)$$

Equations (7), (13), and (18) suggest that the minimum number of windings required for generation of the decoupled stable torque and control force is five.
4 Conclusions

In the previous section, the mechanisms of generating the torque and the radial force have been described for a generalized integrated motor-bearing system using Lorentz forces. The constraints to arrangement of permanent magnets and distribution of concentrated coil windings are derived for the disk and cylindrical type configurations to be realized. It is found that the number of windings should not be less than five if the motoring and radial force currents share common windings. The difference between the number of permanent magnets on the rotor and the magnetic poles for motoring has to be an integer multiple of the number of windings. The phase difference of position control current is determined by the rule that the number of poles for position control should be more or less by 2 than that of the motoring poles. The radial control force is independent of the motoring torque and the rotational speed of the rotor, not requiring a complex frequency demodulation scheme. It is also found that the winding pitch angle required for the maximum radial force is slightly different from that for the maximum torque. In recent years, a few works with some special arrangements of permanent magnets and windings have been performed in an attempt to achieve the decoupled stable torque and control forces in integrated motor-bearing systems. For example, the formula proposed by Okada, et al. [4] corresponds to the case of $W = 6$, $M = 2$, $N = 1$, $Q = -4$, $\alpha = 22.5^\circ$, and the system developed by Kim, et al. [5] to $W = 6$, $M = 2$, $N = 1$, $Q = 2$, $\alpha = 45^\circ$. Typical feasible combinations of design parameters satisfying the constraints discussed in the previous section are listed in Table 1.

Table 1 Feasible combinations of design parameters

<table>
<thead>
<tr>
<th>$W$</th>
<th>$M$</th>
<th>$N$</th>
<th>$Q$</th>
<th>Phase difference</th>
<th>Winding pitch angle</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$M \eta$ [rad]</td>
<td>$N \eta$ [rad]</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>2\pi/5</td>
<td>4\pi/5</td>
</tr>
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<td>1\pi/3</td>
<td>2\pi/3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>7</td>
<td>8\pi/7</td>
<td>4\pi/7</td>
</tr>
<tr>
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<td>1</td>
<td>8</td>
<td>1\pi/3</td>
<td>2\pi/3</td>
<td>8\pi/7</td>
</tr>
<tr>
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<td>9</td>
<td>1\pi/4</td>
<td>$\pi/2$</td>
<td>$\pi/10$</td>
<td>$\pi/11$</td>
</tr>
<tr>
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<td>1</td>
<td>10</td>
<td>2\pi/2</td>
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<td></td>
<td></td>
<td>1\pi/4</td>
<td>$\pi/2$</td>
<td>$\pi/11$</td>
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<tr>
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<td>10</td>
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<td>$\pi/12$</td>
<td>$\pi/11$</td>
</tr>
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</table>

* The optimum winding pitch angles were calculated from equations (8) and (14). Practicality of design, however, is another issue for discussion, considering the concept and performance of the design. In fact, the winding pitch angle, which may differ from the optimal value, should be determined considering practical restrictions. For example, in case when the neighboring windings are allowed to overlap, the air-gap length tends to become large and the magnetic saturation problem may occur. The handling of winding ends should be also considered, especially in the disk type configuration.

References