Multi-Criteria Group Decision Making under Imprecise Preference Judgments : Using Fuzzy Logic with Linguistic Quantifier

Duke Hyun Choi
Graduate School of Management, KAIST
(dhchoi@kgsm.kaist.ac.kr)

Byeong Seok Ahn
College of Business Administration, ChungAng University
(bsahn@cau.ac.kr)

Soung Hie Kim
Graduate School of Management, KAIST
(seekim@kgsm.kaist.ac.kr)

The increasing complexity of the socio-economic environments makes it less and less possible for single decision-maker to consider all relevant aspects of problem. Therefore, many organizations employ groups in decision making. In this paper, we present a multiperson decision making method using fuzzy logic with linguistic quantifier when each of group members specifies imprecise judgments possibly both on performance evaluations of alternatives with respect to the multiple criteria and on the criteria. Inexact or vague preferences have appeared in the decision making literatures with a view to relaxing the burdens of preference specifications imposed to the decision-makers and thus taking into account the vagueness of human judgments. Allowing for the types of imprecise judgments in the model, however, makes more difficult a clear selection of alternative(s) that a group wants to make. So, further interactions with the decision-makers may proceed to the extent to compensate for the initial comforts of preference specifications. These interactions may not however guarantee the selection of the best alternative to implement. To circumvent this deadlock situation, we present a procedure for obtaining a satisfying solution by the use of linguistic quantifier guided aggregation which implies fuzzy majority. This is an approach to combine a prescriptive decision method via a mathematical programming and a well-established approximate solution method to aggregate multiple objects.

Keywords : Group Decision Making, Imprecise Preference Judgments, Fuzzy Linguistic Quantifier

Received: November 2005 Accepted: June 2006 Corresponding Author: Duke Hyun Choi

1. Introduction

For the increasing complexity of the socio-economic environments, it becomes less and less possible for single decision-maker (DM) to consider all relevant aspects of a problem. Therefore, many organizations employ group members in decision making [17]. In general, a group decision making process can be defined as a decision situation in which (i) there are two or more individuals, each of them characterized by his or her own perceptions, attitudes, motivations, and personalities, (ii) who recognize the existence of a common problem, and (iii) who attempt to reach a collective decision [2, 7, 9]. According to the type or degree of information assumed to be elicited from DMs, different solution techniques exist to find a single collective preference or group preference that is best acceptable by the DMs as a whole with their different preferences.

With regard to the preference judgments, we can frequently find that most of researchers have focused on dealing with the group decision problems whose decision parameters are characterized by exact numerical values. Sometimes, however, an individual cannot provide his opinions with exact numerical values [1, 7, 8, 12, 17]. The possible reasons are time pressure, lack of data, unquantifiable information due to its nature(e.g., intangible or non-monetary), and/or DM's limited attention and information processing capabilities [14, 28]. As a result, many attempts have been made with a view to relaxing the burdens of preference specifications imposed to the DMs and thus taking into account the inexactness or vagueness of human judgments.

With regard to the situations in which only incomplete or vague information is available, there exist two categories of research efforts resolving (group) decision problems. One group of studies is a series of research efforts on multi-attribute decision making (MADM) with *partial information* such as [1], [3], [4], [5], [16], [17], [19], [20], [21], [22], [25], [26], [28], and [29]. Though no definite terminology has been developed for the general case, some authors describe 'partial information', 'imprecise informaion', 'incomplete knowledge', or 'set inclusion'. All of these are, however, similar to each other and commonly imply linear inequality of preferences such as rankings and bounded descriptions. The preference relations assessed by the set inclusions with respect to multiple conflicting criteria, on one hand, may provide the DMs with opportunities that are enhanced freedom of choice and comforts of specification. On the other hand, however, it is an extremely rare case to find the best alternative(s) dominating all of the remaining others and/or being accepted by all DMs.

The other group of studies is focused on the preference judgments that are closer to human knowledge representation with *fuzzification* of (group) decision making model, e.g., preference relations based on fuzzy sets [12, 13] and linguistic preference relations [6, 7, 8, 9]. In this approach, each of DM's preference judgments are supposed to be assessed by means of fuzzy preference relation as it incorporates the uncertainty in the decision by the particular DM. And to find a solution best acceptable by the group of DMs as a whole, the concept of *fuzzy majority* manifested by the so-called *linguistic quantifier* is used. In those applications, anyhow, we find some common situations in which each of group members states pairwise comparison judgments between alternatives in a *holistic* manner. It is, however, widely recognized that obtaining preference relations between alternatives with respect to the decomposed multiple criteria is more preferable in view of *divide and conquer* principle than judging them on just one holistic criterion, since alternatives can be evaluated by multiple aspects of characteristics [15, 23, 24]. In addition, the preference relations between alternatives with respect to each of criteria, in some cases, can be better assessed by set inclusions that imply relational expressions on considered alternatives (e.g., a certain alternative is at least twice as preferred as to the other alternative with respect to a certain criterion).

In this paper, we are considering the model in which two parameters (attribute weight and preference judgment) are specified imprecisely by each DMs. This imprecisely identified information constructs region of linear constraints and therefore, pairwise dominance relationship between alternatives reduces to incomplete form, i.e., bounds, which is not sufficient to find the best alternative(s) dominating all the remaining others and acceptable by all DMs. To deal with these difficulties, fuzzy logic with linguistic quantifier is applied for approximation.

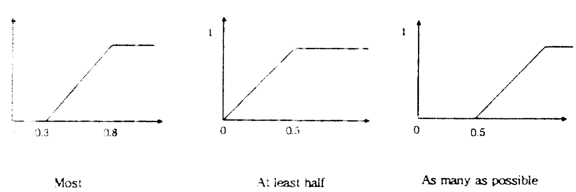
The paper is organized as follows : in Section 2, we introduce the background of our methodology, i.e., the aggregation method guided by fuzzy linguistic quantifier. In Section 3, the whole group decision making process is outlined step by step. A numerical example is presented in Section 4 followed by concluding remarks in Section 5.

2. Background of Methodology : Fuzzy Linguistic Quantifier Guided Aggregation

In a group decision making, a solution is to be an option(s) best acceptable by the group as a whole, that is by most of its members since in no real situation it would be accepted by all [12]. Applying fuzzy-logic-based calculi of linguistically quantified propositions proposed by [30, 32, 33, 34] and [35] which can make it possible to handle fuzzy linguistic quantifiers, Kacprzyk [11] suggested to use the concept of ***fuzzy majority*** represented by ***linguistic quantifier*** to make aggregation phase in group decision models more human-consistent. A general form of a linguistically quantified statement is *Qy*'s are *F*, where *Q* is a linguistic quantifier (e.g., about 8, most, at least half, as many as possible), *y* is a class of objects (e.g., DMs) and *F* is some property [12]. Zadeh [35] distinguished between two types of quantifiers : absolute and proportional. Absolute quantifiers are used to represent amounts that are absolute in nature. These quantifiers are closely related to the concepts of the counting of the number of elements. Zadeh [35] suggested that these absolute quantifiers values can be represented as fuzzy subsets of the nonnegative real numbers, R^+. In particular, he suggested that an absolute quantifier can be represented by a fuzzy subset *Q*, where for any $r \in^+$, $Q(r)$ indicates the degree to which the value *r* satisfies the concept represented by *Q*. Relative quantifiers represent proportion type statements. Thus, if *Q* is relative quantifier, then *Q* can be represented as fuzzy subset of [0, 1], such that, for each $r \in 0, 1]$, $Q(r)$ indicates the degree to which proportion of objects satisfies the concept devoted by Q. The membership function of a relative quantifier can be represented as

$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ \dfrac{r-a}{b-a} & \text{if } a \le r \le b \\ 1 & \text{if } r > b \end{cases}$$

with a, b, r∈[0, 1]. Some examples of relative quantifier in our consideration may be depicted by following [Fig. 1].



[Fig. 1] Linguistic Quantifiers

Yager (1988) computes the weights w_i of the aggregation from the function Q describing the quantifier. In the case of an absolute proportional quantifier

$$w_i = Q(i) - Q(i\text{-}1),\ i = 1, \cdots, m,$$

and in the case of a relative proportion quantifier,

$$w_i = Q(i/m) - Q((i\text{-}1)/m),\ i = 1, \cdots, m.$$

After determining the weights that quantify linguistic quantifiers in order to signify fuzzy majority, a collective opinion can be obtained by means of the aggregation of all individual opinion, and indicates the group's global opinion according to the majority of DM's opinions. The aggregation operation is carried out by the use of OWA operator [32].

Definition 1. An OWA operator of dimension M is a mapping $F : R^M \rightarrow R$ that has an associated weighting M vector $W = [w_1, w_2, \cdots, w_M]^T$, such that $w_i \in [0, 1]$ and $\sum_{i=1}^{M} w_i = 1$, and where the function value $F(a_1, a_2, \cdots, a_M)$ determines the aggregated value of arguments the $a_1, a_2, \cdots, a_M$ in such a manner that

$$F(a_1, a_2, \ldots, a_M) = W \cdot B^T = \sum_{i=1}^{M} w_i b_i ,$$

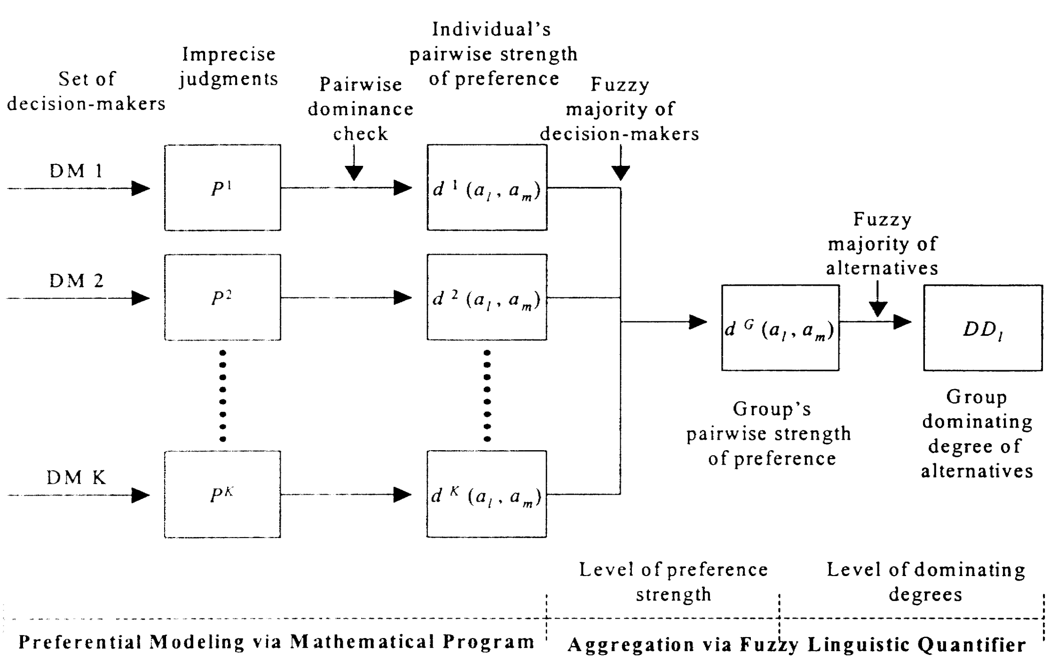
where b_i is the ith largest element of the collection of the M aggregated objects $a_1, a_2, \cdots, a_M$.

The fundamental aspect of the OWA operator is the re-ordering step, in particular, an argument a_i is not associated with a particular weight w_i, but rather a weight w_i is associated with a particular ordered position, i of the argument a_1, a_2, $\cdots$, a_M, thus yielding a nonlinear aggregation. Its generality lies in the fact that by selecting the weights, we can implement different aggregation operators. Specifically, by appropriately selecting the weights in W we can emphasize different arguments based upon their position in the ordering. Thus, if we place most of the weights near the top of W, we can emphasize the higher scores, while placing the weights near the bottom of W emphasizes the lower scores in the aggregation [30, 31].

This aggregation technique have been applied into group decision making and consensus formation models in which each DM's preference between alternatives are specified with fuzzy linguistic label associated with semantics of linear trapezoidal membership function [6, 7, 8, 9]. Sometimes, however, it is difficult for all individuals to agree on the same associations between membership functions and linguistic labels. In some situation, it can be more preferable to obtain preference relations between alternatives with respect to the decomposed multiple criteria than to judge them on just one holistic criterion. Moreover, the vagueness of evaluation can be varied across the criteria according to their own nature or the degree of expertise each DM has. For example, the value of a car with respect to *horse power* (quantitative) criterion can be specified in exact numerical number while its evaluation with respect to *design* (subjective) criterion should be specified in imprecise way. Thus, this paper considers a situation in which each DM's are allowed to specify their preference judgments in partial information mentioned above.

3. Methodology : A Fuzzy Multi-criteria Group Decision Making

Two-stage group decision process, as can be seen in [Fig. 2], is considered in this paper. In the first stage, a preferential modeling via mathematical program, the dominance checks for identifying individual DM's preferences toward alternatives are performed by solving mathematical programs since the preferences judgments are expressed in terms of relational set inclusions which constitute feasible regions.



[Fig. 2] The framework of group decision making process

As can be easily seen, however, it is an extremely rare case to find the alternative(s) dominating all the remaining others. Due to the wide use of imprecise information about performance evaluations and criteria weights, the induced dominance relations are not sufficient to resolve a decision problem. Kirkwood and Corner [18] show in their simulation study that imprecise information solely on the ordinal preference information of weights is often not sufficient to determine the most preferred alternative for realistic decision problems. In addition to the imprecise information on criteria weights, it can be conceived that the consideration of imprecise judgments on the evaluations of alternatives makes the clear selection of best alternative more difficult. An interactive decision aid has features to make the DMs specify additional statements or modify existent preference judgments for the purpose of restricting the feasible decision regions. Further interactions with the DM may proceed to the extent to compensate for the initial comforts of preference specification. In addition, these interactions may not, from the perspective of group decision making, guarantee to result in the best alternative. Rather than ending up a group decision making process with nothing in hand in an attempt to trying to satisfy all the members, it would be more fruitful and productive to compromise to a certain level of accommodation by allowing for an appropriate degree of aggregation. It seems that "all or nothing" is inappropriate especially in group decision support environments since in almost all group decision making problems there exist contradictory opinions, though some of them are adjusted, among the group members who are participating to resolve the problem.

These considerations lead to the adoption of fuzzy majority concept in the second stage, i.e., aggregation via fuzzy linguistic quantifier. In this study, OWA operator guided by linguistic quantifier is applied to aggregate individual DM's imprecise preference judgments, according to two types of fuzzy majority : (1) *fuzzy majority of decision-makers* in which DMs are considered as objects, used to quantify group's aggregated pairwise preferences relation between alternatives by means of aggregating each of individuals' pairwise preference relation and (2) *fuzzy majority of alternatives* in which alternatives are considered as objects, used to quantify the dominating degree that one alternative has over all the remaining ones, according to the decision group's opinions, considered as a whole.

More detail process of group decision making with fuzzy linguistic quantifier can be outlined in the following five steps.

3.1 Step 1: Eliciting individual DM's preferences in imprecise forms

Individual DM of a group specifies his or her preferences in natural language styles with respect to performance evaluations of alternatives under multiple criteria considered (i.e., $P_i^k(\cdot)$) and importance of criteria weights among the criteria (i.e., C_i^k). For example, the *k*-th DM states his or her preference judgments between criteria as following :

- Criterion 3 is more important than criterion 2 but is not more important than sum of criterion 1 and criterion 2.

- The importance of criterion 1 lies between 0.2 and 0.3.

These statements, then, can be summarized as

$$C^k = \{c_3^k \geq c_2^k,\ c_1^k + c_2^k \geq c_3^k,\ 0.2 \leq c_1^k \leq 0.3,\ c_1^k + c_2^k + c_3^k = 1,\ c_1^k, c_2^k, c_3^k \geq 0\}$$

3.2 Step 2: Identifying individual DM's pairwise dominance relation via a mathematical programming method

Individual DM's pairwise dominance values between alternatives are identified with the preference statements obtained in Step 1, via a mathematical programming method.

$$\varphi_{\min(\max)} : A \times A \rightarrow [-1, 1].$$

$$\varphi^k_{\min}(a_l, a_m) = \min \sum_{i \in I} c_i^k [p_i^k(a_l) - p_i^k(a_m)]$$

$$s.t \quad C^k, P_i^k \ \ for\ i \in I, k \in K.$$

$$\varphi^k_{\max}(a_m, a_l) = \max \sum_{i \in I} c_i^k [p_i^k(a_m) - p_i^k(a_l)]$$

$$s.t \quad C^k, P_i^k \ \ for\ i \in I, k \in K.$$

We obtain a *pairwise dominance relation* in the form of value interval $[\varphi_{\min}(a_l, a_m), \varphi_{\max}(a_l, a_m)]$, which represents a range of all possible difference of evaluation values between alternatives a_l and a_m. Notice that a pairwise dominance value, $\varphi_{\min}(a_l, a_m) \geq 0$ (or $\varphi_{\max}(a_m, a_l) \leq 0$) represents an individual DM's preference of al over a_m. If one can find an alternative that has $\varphi_{\min}(a_l, a_m) \geq 0$ (or $\varphi_{\max}(a_m, a_l) \leq 0$) for $\forall\ m \neq l$, then alternative a_l is the most preferred alternative in his or her perspective.

3.3 Step 3: Calculating the individual DM's strength of preference measure

To make each individual DM's pairwise dominance relation, $[\varphi_{\min}(a_l, a_m), \varphi_{\max}(a_l, a_m)]$ obtained in Step 2, equivalent to the ones in fuzzy relations, we define a function d^k, $d^k : X \times X \rightarrow [0, 1]$, which denotes the *strength of preference* [27] of alternative a_l over a_m with the k-th DM's opinion.

Let $\Psi(a_l, a_m) = V(a_l) - V(a_m)$, $a_l, a_m \in X$ and define $d^k(a_l, a_m) = P(\Psi(a_l, a_m) \geq 0)$ where $P(\bullet)$ denotes the probability of $(\bullet)$ being greater or equal to zero. For a given pair $a_l, a_m \in X$, $\varphi_{\min}(a_l, a_m)$ is the minimum value of $\Psi(\bullet)$ and $\varphi_{\max}(a_l, a_m)$ is the maximum value of $\Psi(\bullet)$. Thus a measure, $P(\Psi(a_l, a_m) \geq 0)$ denotes the strength of preference indicating the degree of overall evaluation of alternative al exceeding that of alternative am when the measure, $P(\Psi(a_l, a_m) \geq 0)$ ranges in one dimensional interval $[\varphi_{\min}(a_l, a_m), \varphi_{\max}(a_l, a_m)]$. The equations in (1) show a way of determining the strength of preference of alternative a_l over a_m, which depends on the probability distribution function f_{a_l, a_m} of the random variable $\Psi(a_l, a_m)$.

$$P\{\Psi(a_l, a_m) \geq 0\} = \begin{cases} 1 & \text{if } \varphi_{\min}(a_l, a_m) \geq 0, \quad (1a) \\ \int_0^{\varphi_{Max}(a_l, a_m)} f_{a_l, a_m}(x)dx & \text{if } \varphi_{\min}(a_l, a_m) < 0 \text{ and } \varphi_{\max}(a_l, a_m) > 0, \quad (1b) \\ 0 & \text{if } \varphi_{\max}(a_l, a_m) \leq 0. \quad (1c) \end{cases}$$

Considering the value interval, the fact of a_l over a_m is definitely supported if the lower bound of the interval is nonnegative, i.e., $\varphi_{\min}(a_l, a_m) \geq 0$.

Therefore, $\Psi(a_l, a_m)$, the strength of preference of al over am is perfectly identified (see 1a). To the contrary, if the fact of a_l over a_m is definitely supported, which implies $\varphi_{\min}(a_l, a_m) \ge 0$ and in turn $\varphi_{\max}(a_l, a_m) \le 0$, then the strength of preference of a_l over a_m does not exist at all, thus resulting in value zero (see 1c). In case of (1b), none of two alternatives strictly dominates each other. For deriving a specific strength of preference, suppose that the distribution, $f_{a_l, a_m}(\Psi(a_l, a_m))$ on $[\varphi_{\min}(a_l, a_m), \varphi_{\max}(a_l, a_m)]$ can be approximated by a symmetric triangular distribution with a vertical axis through Ψ_m in which $\Psi_m = (\varphi_{\max}(a_l, a_m) + \varphi_{\min}(a_l, a_m))/2$. Then, the $\Pr\{\Psi(a_l, a_m) \ge 0\}$ becomes

$$P\{\Psi(a_l, a_m) \ge 0\} = \begin{cases} 1 - \dfrac{[\varphi_{\min}(a_l, a_m)]^2}{2[\Psi_m - \varphi_{\min}(a_l, a_m)]^2} & \text{if } \Psi_m \ge 0, \\ \dfrac{[\varphi_{\max}(a_l, a_m)]^2}{2[\varphi_{\max}(a_l, a_m) - \Psi_m]^2} & \text{if } \Psi_m < 0. \end{cases} \quad (2)$$

For example, let us suppose that the value intervals among alternatives a_1, a_2, and a_3 be given as follows:

$$[\varphi_{\min}(a_1, a_2), \varphi_{\max}(a_1, a_2)] = [-0.209, \quad 0.652],$$
$$[\varphi_{\min}(a_1, a_3), \varphi_{\max}(a_1, a_3)] = [-0.465, \quad 0.501],$$
$$[\varphi_{\min}(a_2, a_3), \varphi_{\max}(a_2, a_3)] = [-0.484, \quad 0.376].$$

From the value intervals, let us further compute the strength of preference of each alternative based on the formulae (2), then we can obtain

$$P\{\Psi(a_1, a_2) \ge 0\} = 0.88,$$
$$P\{\Psi(a_1, a_3) \ge 0\} = 0.54,$$
$$P\{\Psi(a_2, a_3) \ge 0\} = 0.38.$$

From those values, we can conclude that the DM recognizes that the confidence of alternative a_1 over a_2 is relatively as high as 88%, but confidence of a_2 over a_3 is as low as 38%.

To obtain group's pairwise strength of preference and select the best alternative(s) or rank alternatives from these individual DM's strength of preference, we consider the adoption of fuzzy majority concept on two levels : (1) *level of preference strength*, on which the group's pairwise strength of preferences of an alternative over another one are calculated according to the opinions of each DMs, and (2) *level of dominating degrees*, on which the aggregated dominating degree of an alternative over all the remaining ones are calculated according to the opinions of the group, considered as a whole. We can use different fuzzy linguistic quantifiers on each level, since the concept of fuzzy majority is different between those two levels, i.e., majority of decision-makers and majority of alternatives, respectively as followings.

3.4 Step 4: Calculating the group's pairwise strength of preference measure

With the individual DM's strength of preferences between alternatives, the *group's strength of preference* of an alternative over another one, signifying the relative strength to which the alternative is preferred to each of the other alternatives by the fuzzy majority of the decision-makers, is calculated according to Definition 2.

Definition 2. Given two alternative a_l, $a_m \in X$, the group's pairwise strength of preference guided by linguistic quantifier, $d^G(a_l, a_m)$, is defined as

$$d^G(a_l, a_m)=F_{Q_1}[d^k(a_l, a_m)],\ k=1,\cdots,K,$$

in which F_{Q_1} is the OWA operator guided by *fuzzy majority of decision-makers* and $d^k(a_l, a_m)$ represents the pairwise strength of preference of a_l over a_m, according to the k-th DM's opinions.

We can use the group's pairwise strength of preference in a manner that the confidence of alternative a_l over a_m is recognized relatively as high as $d^G(a_l, a_m)$ by the whole DMs in a group, since the aggregation is performed with OWA operator guided by the linguistic quantifier, Q_1, in which the individual DMs in a group are considered as objectives. Aggregating the results of group's pairwise strength of preference, the dominating degree of each alternative can be obtained for final decision making.

3.5 Step 5: Calculating dominating degree of each alternative

With the group's strength of preference of each alternative obtained in Step 4, the *dominating degree*of each alternative, signifying the degree to which an alternative dominates the fuzzy majority of the remaining ones, is calculated according to-Definition 3.

Definition 3. Given an alternative $a_l \in X$, the dominating degree of a_l guided by linguistic quantifier, DD_l, is defined as

$$DD_l = F_{Q_2}[d^G(a_l, a_m)],\quad m=1,\cdots,K,\ l\neq m$$

in which F_{Q_2} is the OWA operator guided by *fuzzy majority of alternatives* and DD_l represents the dominating degree of al over all the remaining others according to the whole decision group's opinions.

We can use the dominating degree in a manner that an alternative to have larger is better from the viewpoint of the whole DMs in a group. Thus, the alternative which is to be selected from the magnitude of quantifier guided dominating degree is the one that dominates the fuzzy majority of remaining alternatives since the aggregation is performed by the use of OWA operator guided by linguistic quantifier, Q_2, in which the remaining other alternatives are considered as objects.

4. Numerical Example

Considering an artificial example with five alternatives ($M=5$) that are characterized by three criteria ($N=3$), we shall show how the proposed approach can be applied in imprecisely specified group decision making problem with six group members ($K=6$). Let us suppose that the DMs' imprecise preference judgments are given as follows:

$$P^1 = \begin{pmatrix} 1 & 0.1 \le p_2^1(a_1) - p_2^1(a_2) \le 0.2 & 0.4 \\ 0.5 & 0.1 \le p_2^1(a_2) \le 0.2 & 0.9 \\ 0.1 & p_2^1(a_3) = 0 & 0.95 \\ 0 & 2p_2^1(a_2) \le p_2^1(a_4) \le 3p_2^1(a_2) & 1 \\ 0.3 & p_2^1(a_5) = 1 & 0 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.5 & p_2^2(a_1) = 1 & 0.1 \\ 0 & p_2^2(a_2) + p_2^2(a_4) \geq 0.6 & 0.6 \\ 0.9 & 0.1 \leq p_2^1(a_3) \leq 0.2 & 0.2 \\ 1 & p_2^2(a_3) \leq p_2^2(a_4) \leq 3p_2^2(a_3) & 0.1 \\ 0.3 & p_2^1(a_5) = 0 & 1 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 1 & p_2^3(a_1) = 1 & 0.5 \\ 0.6 & 0.2 \leq p_2^3(a_2) \leq 0.4 & 0.9 \\ 0 & p_2^3(a_3) = 0 & 1 \\ 0.1 & p_2^3(a_2) \leq p_2^3(a_4) \leq 2p_2^3(a_2) & 0.8 \\ 0.3 & p_2^3(a_5) - p_2^3(a_2) \leq 0.3 & 0 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.7 & p_2^4(a_1) = 0 & 0 \\ 1 & 0.1 \leq p_2^4(a_2) \leq 0.2 & 0.8 \\ 0.1 & 0.1 \leq p_2^4(a_3) - p_2^4(a_2) \leq 0.2 & 0.4 \\ 0.5 & p_2^4(a_4) = 1 & 1 \\ 0 & 2p_2^4(a_2) \leq p_2^4(a_5) \leq 3p_2^4(a_2) & 0.3 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.9 & 0.5 \leq p_2^5(a_1) + p_2^5(a_2) \leq 0.8 & 1 \\ 1 & 0.2 \leq p_2^5(a_2) \leq 0.4 & 0.9 \\ 0.3 & p_2^5(a_2) \leq p_2^5(a_3) \leq 2p_2^5(a_2) & 0.7 \\ 0.6 & p_2^5(a_4) = 0 & 0 \\ 0 & p_2^5(a_5) = 1 & 0.3 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.3 & p_2^6(a_1) = 1 & 1 \\ 0 & 0.2 \leq p_2^6(a_2) \leq 0.4 & 0.3 \\ 0.8 & p_2^6(a_2) + p_2^6(a3) \geq 0.6 & 0.2 \\ 1 & p_2^6(a_3) \leq p_2^6(a_4) \leq 2p_2^6(a_3) & 0.8 \\ 0.5 & p_2^6(a_5) = 0 & 0 \end{pmatrix}$$

The performance scores with respect to first and third criteria are normalized between zero and one in which zero is attached to the lowest unnormalized performance and one to the highest (see the normalization methods in [10] and [15]). We assign the imprecise judgments to the performance scores with respect to the second criterion in which the types of imprecision include strict preferences, preferences with ratio comparisons, and interval preference. For example, let us suppose that the first DM makes the following preference judgments with respect to second criterion:

- the difference in performance scores between alternative 1 and 2 lies between 0.1 and 0.2,
- the performance score of alternative 2 lies between 0.1 and 0.2,
- the performance score of alternative 3 is 0 (the lowest),
- the performance score of alternative 4 is 2 times greater than or equal that of alternatives 2,
- the performance score of alternative 5 is 1 (the highest).

These statements can be summarized as P^1.

Let us suppose that the first DM makes the following preference statements among the criteria :

- criterion1 is more important than criterion 2, which is in turn more important than criterion 3,
- the importance of criterion 3 lies between 0.1 and 0.3.

These statements can be summarized as

$$C^1 = \{c_1^1 \geq c_2^1 \geq c_3^1,\ 0.1 \leq c_3^1 \leq 0.3,\ c_1^1 + c_2^1 + c_3^1 = 1, c_1^2, c_2^2, c_3^2 \geq 0\}.$$

With the same manners, the judgments between criteria of other decision-makers can be expressed as

$$C^2 = \{c_3^2 \geq c_2^2,\ c_1^2 + c_2^2 \geq c_3^2,\ 0.2 \leq c_1^2 \leq 0.3,\ c_1^2 + c_2^2 + c_3^2 = 1, c_1^2, c_2^2, c_3^2 \geq 0\}.$$

$$C^3 = \{c_2^3 \geq c_1^3 \geq c_3^3,\ 0.3 \leq c_1^3 \leq 0.6,\ c_1^3 + c_2^3 + c_3^3 = 1, c_1^3, c_2^3, c_3^3 \geq 0\}.$$

$$C^4 = \{c_3^4 \geq c_1^4,\ c_1^4 + c_3^4 \geq c_2^4,\ 0.1 \leq c_1^4 \leq 0.2,\ c_1^4 + c_2^4 + c_3^4 = 1, c_1^4, c_2^4, c_3^4 \geq 0\}.$$

$$C^5 = \{c_2^5 \geq c_1^5 \geq c_3^5,\ 0.3 \leq c_3^5 \leq 0.5,\ c_1^5 + c_2^5 + c_3^5 = 1\}.$$

$$C^6 = \{c_1^6 \geq c_2^6,\ c_1^6 + c_2^6 \geq c_3^6,\ 0.4 \leq c_1^6 \leq 0.5,\ c_1^6 + c_2^6 + c_3^6 = 1, c_1^6, c_2^6, c_3^6 \geq 0\}.$$

With precise or imprecise preference judgments specified in P^k and $C^k(k = 1, 2, \cdots, 6)$, we want to identify each individual DM's preferences of alternatives. For example, a mathematical program for identifying the first individual DM's preferences between a_1 and a_2 can be formulated as follows(a maximization program can also be formulated by using max instead of min and hereafter we deal with only minimization program) :

$$Min \quad c_1^1(p_1^1(a_1) - p_1^1(a_2)) + c_2^1(p_2^1(a_1) - p_2^1(a_2)) + c_3^1(p_3^1(a_1) - p_3^1(a_2))$$
$$\text{subject to } P^1,\ C^1.$$

When we encounter seemingly complicated imprecise decision problem, the solution process is so simple that we only have to solve a series of small linear programs backwards from performance to weight. In the first phase, the performance differences between alternative a_1 and a_2 are minimized subject to the constraints comprised of imprecise value scores, i.e., P^1, with respect to each of criteria. The solutions indicate

$$Min(p_1^1(a_1) - p_1^1(a_2),\ p_2^1(a_1) - p_2^1(a_2),\ p_3^1(a_1) - p_3^1(a_2)) = (0.50, 0.10, -0.50)$$

The minimization program of performance difference with respect to the second criterion for the first DM is formulated below and the result can be obtained by the use of linear program package.

$$Min(p_2^1(a_1) - p_2^1(a_2))$$
$$\text{subject to}$$
$$p_2^1(\cdot) \in P^1$$

In the second phase, each of minimized performance differences is used for the corresponding coefficient of weight variables and then individual weighted performance is minimized subject to the constraints comprised of imprecise weights judgments, i.e., C^1. For the first DM's case, the problem can be formulated as follow :

$$Min(0.5c_1^1 + 0.1c_2^1 - 0.5c_3^1)$$
$$\text{subject to}$$
$$C^1 = \{c_1^1 \geq c_2^1 \geq c_3^1,\ 0.1 \leq c_3 \leq 0.3,\ c_1^1 + c_2^1 + c_3^1 = 1\}.$$

The result indicates $\varphi_{\min}(a_1, a_2) = 0.060$ for the first DM. Note that we can get maximized performance differences by formulating maximization program as follow :

$$Max(p_1^1(a_1) - p_1^1(a_2),\ p_2^1(a_1) - p_2^1(a_2),\ p_3^1(a_1) - p_3^1(a_2)) = (0.50, 0.20, -0.50).$$

Then, individual weighted performance is maximized as follow :

$$Max(0.5c_1^1 + 0.2c_2^1 - 0.5c_3^1)$$
$$\text{subject to } C^1.$$

The result indicates $\varphi_{\max}(a_1, a_2) = 0.370$ for the first DM. Thus, the first DM's pairwise dominance relation between alternative 1 and 2 is obtained as $[\varphi_{\min}(a_1, a_2), \varphi_{\max}(a_1, a_2)] = [0.060, 0.370]$. With the same manners, one can obtain all the pairwise dominance relations as shown in [Fig. 3]. The dominance relation $[\varphi_{\min}(a_l, a_m), \varphi_{\max}(a_l, a_m)]$ of DM$k(k = 1, 2, \cdots, 6)$ represents a range of all possible differences of performance scores between

DM1	Alternatives				
Alt.	1	2	3	4	5
1	-	[0.060, 0.370]	[0.185, 0.735]	[-0.030, 0.760]	[-0.050, 0.520]
2		-	[0.125, 0.400]	[-0.090, 0.390]	[-0.270, 0.200]
3			-	[-0.410, 0.075]	[-0.445, -0.095]
4				-	[-0.485, -0.195]
5					-

DM2	Alternatives				
Alt.	1	2	3	4	5
1	-	[-0.150, 0.325]	[-0.010, 0.240]	[-0.070, 0.260]	[-0.190, 0.095]
2		-	[-0.200, 0.340]	[-0.200, 0.360]	[-0.335, 0.180]
3			-	[-0.290, 0.030]	[-0.140, -0.030]
4				-	[-0.140, 0.105]
5					-

DM3	Alternatives				
Alt.	1	2	3	4	5
1	-	[0.200, 0.680]	[0.50, 1.000]	[0.260, 0.850]	[0.420, 0.910]
2		-	[0.230, 0.500]	[0.230, 0.250]	[-0.130, 0.333]
3			-	[-0.470, -0.033]	[-0.590, 0.233]
4				-	[-0.590, 0.500]
5					-

DM4	Alternatives				
Alt.	1	2	3	4	5
1	-	[-0.750, -0.350]	[-0.300, -0.100]	[-0.880, -0.760]	[-0.350, -0.050]
2		-	[0.150, 0.500]	[0.150, -0.060]	[-0.480, 0.600]
3			-	[0.100, -0.560]	[-0.680, 0.100]
4				-	[-0.680, 0.730]
5					-

DM5	Alternatives				
Alt.	1	2	3	4	5
1	-	[-0.180, 0.310]	[0.160, 0.510]	[-0.190, 0.700]	[0.350, 0.667]
2		-	[0.200, 0.445]	[0.200, 0.450]	[-0.010, 0.670]
3			-	[-0.030, 0.250]	[-0.350, 0.233]
4				-	[-0.350, 0.470]
5					-

DM6	Alternatives				
Alt.	1	2	3	4	5
1	-	[0.450, 0.580]	[-0.250, 0.280]	[-0.350, 0.080]	[0.400, 0.520]
2		-	[-0.800, -0.220]	[-0.800, -0.400]	[-0.900, 0.020]
3			-	[-0.150, -0.100]	[-0.430, 0.650]
4				-	[-0.430, 0.760]
5					-

[Fig. 3] Individual DM's pairwise dominance relation

DM1 Alt	Alternatives				
	1	2	3	4	5
1	-	1.000	1.000	0.997	0.985
2	0.000	-	1.000	0.930	0.362
3	0.000	0.000	-	0.048	0.000
4	0.003	0.070	1.000	-	0.000
5	0.015	0.638	1.000	1.000	-

DM2 Alt	Alternatives				
	1	2	3	4	5
1	-	0.801	0.997	0.910	0.222
2	0.199	-	0.726	0.745	0.244
3	0.003	0.274	-	0.018	0.000
4	0.090	0.255	1.000	-	0.367
5	0.778	0.756	1.000	0.633	-

DM3 Alt	Alternatives				
	1	2	3	4	5
1	-	1.000	1.000	1.000	1.000
2	0.000	-	1.000	1.000	0.842
3	0.000	0.000	-	0.000	0.160
4	0.000	0.000	0.840	-	0.421
5	0.000	0.158	0.840	0.579	-

DM4 Alt	Alternatives				
	1	2	3	4	5
1	-	0.000	0.000	0.000	0.000
2	1.000	-	1.000	1.000	0.605
3	1.000	0.000	-	1.000	0.033
4	1.000	0.000	0.967	-	0.535
5	1.000	0.395	0.967	0.465	-

DM5 Alt	Alternatives				
	1	2	3	4	5
1	-	0.730	1.000	0.909	1.000
2	0.270	-	1.000	1.000	1.000
3	0.000	0.000	-	0.977	0.319
4	0.091	0.000	0.681	-	0.636
5	0.000	0.000	0.681	0.364	-

DM6 Alt	Alternatives				
	1	2	3	4	5
1	-	1.000	0.555	0.069	1.000
2	0.000	-	0.000	0.000	0.001
3	0.445	1.000	-	0.000	0.683
4	0.931	1.000	0.317	-	0.739
5	0.000	0.999	0.317	0.261	-

[Fig. 4] The individuals'pairwise strength of preferences alternatives l and m under the k-th DM's preference judgments. Note that the entries in lower diagonal are omitted, since it holds $\varphi_{\min}(a_1, a_2) = -\varphi_{\max}(a_2, a_1)$ or equivalently $\varphi_{\max}(a_1, a_2) = -\varphi_{\min}(a_2, a_1)$. From the results with the third individual DM's judgments, it is evident that alternative a_l dominates all the other alternatives, implying $\varphi_{\min}(a_1, a_m) > 0$ for $\forall m \neq 1$. With the other DM's judgments, however, none of alternatives dominate all the remaining ones.

Instead of relying on the interactive questions and responses for the purpose of restricting the imprecise judgments, we turn our attention to the well-established aggregation method by the use of fuzzy linguistic quantifier. This is mainly because there are a number of DMs who are participating in the decision problem. If we are to support a single decision making problem with imprecise judgments, the final decision might be made after several rounds of interactions. In most of cases, however, ensuing questions and responses might be required to obtain sufficient results from insufficient information. The problem is that the final decisions obtained from many interactions are so sensitive to the methods adopted to elicit more specific preference judgments. Thus, we intend to support group decision making problem using the results as they are up to the point. Based on the formulae (2) in subsection 3.3, each of interval values (i.e., pairwise dominance relation) is transformed into the individual's strength of preferences, that is $d^k(a_l, a_m)$. In [Fig. 4], the degree of an alternative l over alternative m under k-th DM's preference judgment (i.e., $d^k(a_l, a_m)$) were presented. For example, from the first DM's dominance relation between alternative 1 and 2 (i.e., [0.060, 0.370]) is transformed to 1, that means he or she recognized that the confidence of alternatives 1 over alternative 2 is 100%.

On the level of preference strength, we calculate the group's strength of preferences signifying the degree of alternative a_l over other alternative a_m, $d^G(a_l, a_m)$ by means of OWA operator, $F_{Q_1}[d^k(a_l, a_m)]$, $k=1, 2, \cdots, 6$, according to the fuzzy majority of individuals specified by the linguistic quantifier. With the weighting vector W=[0, 0, 0, 0.333, 0.333, 0.333] implying the linguistic quantifier, *as many as possible*, the pairwise strength of preference based on group's opinion is calculated as shown in <Table 1>. For example, the group's pairwise strength of preference of alternative 1 over alternative 2 is calculated as follows.

$$d^G(a_1, a_2)=1.000\times0.000+1.000\times0.000+1.000\times0.000 +0.801\times0.333+0.730\times0.333+0.000\times0.333=0.510$$

<Table 1> The group's pairwise strength of preferences

Group Alt.	Alternatives				
	1	*2*	*3*	*4*	*5*
1	–	0.510	0.517	0.326	0.202
2	0.066	–	0.575	0.558	0.202
3	0.000	0.000	–	0.006	0.011
4	0.031	0.000	0.612	–	0.263
5	0.000	0.184	0.612	0.364	–

Finally on the level of dominating degrees, we calculate dominating degree, DD_l, by means of OWA operator, $F_{Q_2}[d^G(a_l, a_m)]$, $m=1, \ldots, 5, m \neq l$, according to the fuzzy majority of alternatives specified by the linguistic quantifier. With the weighting vector W=[0, 0.4, 0.5, 0.1], implying the linguistic quantifier, *most*, the dominating degree of an alternative is calculated by the OWA aggregation of elements in the row. Therefore, we can make a final decision by the dominating degree in a manner that an alternative to have larger is better.

DD_1 =0.517×0.000+0.510×0.400+0.326×0.500+0.202×0.100 =0.387

DD_2 =0.575×0.000+0.558×0.400+0.202×0.500+0.066×0.100 =0.331

DD_3 =0.011×0.000+0.006×0.400+0.000×0.500+0.000×0.100 =0.002

DD_4 =0.612×0.000+0.263×0.400+0.031×0.500+0.000×0.100 =0.121

DD_5 =0.612×0.000+0.364×0.400+0.184×0.500+0.000×0.100 =0.238

Based on the magnitudes of dominating degrees, we can conclude alternative 1 is the best alternative in group's perspective.

5. Concluding Remarks

In this paper, we present a group decision making method using fuzzy logic with linguistic quantifier when each of group members specifies their own imprecise judgments possibly both on performance evaluations with respect to the multiple criteria and on the importance of each criteria. This is an approach to combine a prescriptive preference modeling via a mathematical programming and an approximate solution method to aggregate multiple objects. The former makes it possible to identify the strength of preferences between alternatives, which are then utilized for aggregation according to the fuzzy majority specified by linguist quantifier. In our approach, approximations on two levels (i.e., the level of pairwise preference strength and the level of dominating degrees) are required since the solution process is as follows : individual DMs' problems are solved, then their pairwise strength of preferences are aggregated to produce a group's aggregated strength of preferences (i.e., on the level of pairwise strength of preference), and then a group's pairwise strength of preferences are further aggregated to identify which alternative is better than others (i.e., on the level of dominating degrees). In our case, linguistic quantifiers such as "most" or "as many as possible" among others can be used for the purpose of aggregation.

As further researches, it will be interesting to develop a prototype system in which our methodology is embedded. By adopting it to support real group decision making situations (e.g., project selection), it will be possible to investigate the users' response to the prototype system.

References

[1] Ahn, B. S., "Extending Malakooti's Model for Ranking Multi-Criteria Alternatives with Preference Strength and Partial Information", *IEEE Trans. Systems Man Cybernet (Part A)*, Vol. 33(2003), 281-287.

[2] Bui, T. X., Co-op. *A Group Decision Support System for Cooperative Multiple Criteria Group Decision Making*, Springer-Verlag, Berlin, 1987.

[3] Cook, W. and M. Kress, "A Multiple Criteria Decision Model with Ordinal Preference Data", *European Journal of Operational Research*, Vol.54(1991), 191-198.

[4] Hannan, E. L., "Obtaining Nondominated Priority Vectors for Multiple Objective Decision Making Problems with Different Combinations of Cardinal and Ordinal Information", *IEEE Transactions on Systems, Man, and Cybernetics*, Vol.11(1981), 538-543.

[5] Hazen, G. B., "Partial Information, Dominance and Potential Optimality In Multiattribute Utility theory", *Operations Research*, Vol. 34(1986), 296-310.

[6] Herrera-Viedma, E., F. Herrera, and F. Chiclana, "A Consensus Model for Multiperson Decision Making with Different Preference Structures", *IEEE Trans. Systems Man Cybernet*, Vol.32 (2002), 394-402.

[7] Herrera, F., E. Herrera-Viedma, and J. L. Verdegay, "A Sequential Selection Process in Group Decision Making with a Linguistic Assessment Approach", *Infor. Sci.*, Vol.85 (1995), 223-239.

[8] Herrera, F., E. Herrera-Viedma, and J. L. Verdegay, "A Model of Consensus in Group Decision Making Under Linguistic Assessments", *Fuzzy Sets and Systems*, Vol.78(1996a), 73-87.

[9] Herrera, F., E. Herrera-Viedma, and J. L. Verdegay, "Direct Approach Processes in Group Decision Making Under Linguistic OWA Operators", *Fuzzy Sets and Systems*, Vol.79 (1996b), 175-190.

[10] Hwang, C. L. and M. J. Lin, *Group Decision Making under Multiple Criteria : Lecture Notes in Economics and Mathematical Systems, Springer-Verlag*, New York, 1986.

[11] Kacprzyk, J., "Group Decision Making with a Fuzzy Linguistic Majority", *Fuzzy Sets and Systems*, Vol.18(1986), 105-118.

[12] Kacprzyk, J., M. Fedrizzi, and H. Nurmi, "Group Decision Making and Consensus Under Fuzzy Preferences and Fuzzy Majority", *Fuzzy Sets and Systems*, Vol.49(1992), 21-31.

[13] Kacprzyk, J., H. Nurmi, and M. Fedrizzi, *Consensus under fuzziness*, KAP, Boston, 1997.

[14] Kahneman, D., P. Slovic, and A. Tversky, "Judgment Under Uncertainty : Heuristics and Biases", Cambridge Univ. Press, 1982.

[15] Keeney, R. L. and H. Raiffa, H. *Decisions with Multiple Objectives : Preferences and Value Tradeoffs*, Wiley, New York, 1993.

[16] Kim, S. H. and B. S. Ahn, "Group Decision Making Procedure Considering Preference Strength Under Incomplete Information", *Computers and Operations Research*, Vol.24 (1997), 1101-1112.

[17] Kim, S. H. and B. S. Ahn, "Interactive Group Decision Making Procedure Under Incomplete Information", *European J. Oper. Res.*, Vol.116 (1999), 498-507.

[18] Kirkwood, C. W. and J. L. Corner, "The Effectiveness of Partial Information About Attribute Weights for Ranking Alternatives In Multiattribute Decision Making", *Organ. Behave. Hum.*, Vol.54(Dec. 1993), 456-476.

[19] Kirkwood, C. W. and R. K. Sarin, "Ranking with Partial Information : A Method and an Application", *Operations Research*, Vol.33 (1985) 38-48.

[20] Kmietowcz, Z. W. and A. D. Pearman, "Decision Theory, Linear Partial Information and Statistical Dominance", *Omega*, Vol.12 (1984), 391-399.

[21] Kofler, E., Z. W. Kmietowcz, and A. D.

Pearman, "Decision Making with Linear Partial Information (LPI)", *Journal of Operational Research*, Vol.35, No.12(1984), 1079-1090.

[22] Park, K. S. and S. H. Kim, "Tools for Interactive Multiattribute Decision Making with Incompletely Identified Informations", *European Journal of Operational Research*, Vol.98 (1997), 111-123.

[23] Saaty, T. L., *Group Decision Making and the AHP. In the Analytic Hierarchy Process : Application and Studies*, Edited by B. Golden, E. Wasil, and P. Harker, Springer-Verlag, Berlin, 1989.

[24] Saaty, T. L., *The Analytic Hierarchy Process*, McGraw-Hill, New York, 1980.

[25] Sage, A. P. and C. C. White, "ARIADNE : A Knowledge-Based Interactive System for Planning and Decision Support", *IEEE Transactions on Systems, Man, and Cybernetics*, Vol.14, No.1(1984), 35-47.

[26] Sarin, R. K., "Screening of Multiattribute Alternatives", *Omega*, Vol.5, No.4(1977), 481-489.

[27] Weber, M., "A Method of Multiattribute Decision Making with Incomplete Information", *Mgmt. Sci.*, Vol.31(1985), 1365-1371.

[28] Weber, M., "Decision Making with Incomplete Information", *European Journal of Operation Research*, Vol.28(1987), 44-57.

[29] White, C. C., A. P. Sage, and S. Dozono, "A Model of Multiattribute Decision Making and Trade-Off Weight Determination Under Uncertainty", *IEEE Transaction on Systems. Man and Cybernetics*, Vol.14, No.2(1984), 223-229.

[30] Yager, R. R., "Analytic Properties of Maximum Entropy OWA Operators", *Inform. Sci.*, Vol.85 (1995), 11-27.

[31] Yager, R. R., "On Ordered Weighted Averaging Aggregation Operators in Multi-Criteria Decision Making", *IEEE Trans. Systems Man Cybernet*, Vol.18(1988), 83-90.

[32] Yager, R. R., "On the Issue of Obtaining OWA Operator Weights", *Fuzzy Sets and Systems*, Vol.94(1998), 157-169.

[33] Yager, R. R., "Quantifiers in the Formulation of Multiple Objective Decision Functions", *Infor. Sci.*, Vol.31(1983a), 107-139.

[34] Yager, R. R., "Quantified Propositions in a Linguistic Logic", *Internat. J. Man-Machine Stud*, Vol.19(1983b), 195-227.

[35] Zadeh, L. A., "A Computational Approach to Fuzzy Quantifier in Natural Languages", *Comput. Math.*, Appl. Vol.9(1983), 149-184.

요약

불명료한 선호정보 하의 다기준 그룹의사결정 : Linguistic Quantifier를 통한 퍼지논리 활용

최덕현* · 안병석** · 김성희*

본 논문에서는 각 대안의 속성 평가와 속성 자체의 중요도에 대한 평가에 있어 불명료한 선호정보 형태로 주어진 경우, linguistic quantifier를 통한 퍼지논리를 활용하여 그룹의사결정을 지원하는 방법을 제시하였다. 불명료한 선호정보는 의사결정 관련 문헌에서 의사결정자에게 요구되는 선호정보 명시의 부담을 줄여주고, 판단의 모호성을 받아들이고자 하는 시각으로서 다뤄져 왔다. 그러나 불명료한 유형의 선호정보를 허용할 경우 의사결정그룹이 원하는 대안의 명확한 선택이 보다 어려워진다. 따라서 추가적인 정보획득을 위한 의사결정자들과의 상호작용이 요구되지만, 이는 불명료한 선호정보를 허용하였던 초기의 취지를 반감시킬 뿐더러, 반드시 최적의 대안을 보장하는 것도 아니다. 이러한 상황을 타계하기 위하여, fuzzy majority의 의미를 반영하고 있는 linguistic quantifier를 활용함으로써 satisfying solution을 구하는 절차를 제시하였다. 이는 mathematical programming을 활용한 의사결정 기법과 다수의 객체를 집성하기 위한 개략적 해법을 결합한 접근방식이다.

* 한국과학기술원 테크노경영대학원
** 중앙대학교 경영학부