Spectrum-Efficient Call Control in a Dual-Mode TDMA Cellular System

Dong-wan Tcha, Member, IEEE, Chun-hyun Paik, and Yong-jo K. Chung

Abstract—This study deals with a call control strategy in a dual-mode time-division multiple access (TDMA) cellular system, which provides services both to analog and digital calls. Since analog calls consume the frequency resource several times as much as digital calls, we consider a call control strategy of the threshold type that the number of active analog calls is restricted within a prespecified level. Given the arrival rates and the GOS's for both types of calls in the cells, two nonlinear integer optimization problems are considered for a multicell system as well as for a single cell system. The one is to find the threshold parameters optimizing the relevant objective measures. The other is to obtain the minimum number of required channels in the cells satisfying the GOS's of both types of calls. The solution methods for the two kinds of optimization problems are devised based on the interesting properties of the objective function and the blocking probabilities of both call types. And the efficiency of the proposed algorithms is verified by extensive computational experiments with realistic input data.

I. INTRODUCTION

THE shortage of bandwidth, particularly severe in some metropolitan areas, from the rapid growth in demand for cellular services has accelerated the transfer of the cellular technology from the existing analog to the digital, specifically, time-division multiple access (TDMA) system. The TDMA cellular system is known to be capable of providing not only higher capacity but also better voice quality [11].

For the graceful transition from an existing analog system to a digital TDMA system, the cellular operator has to support both analog and digital calls during the interim period. The dual-mode system has thus emerged as an indispensable transitional one from all-analog to all-digital systems [3], [7], [13]. According to the current establishments, one analog channel accommodates either three or six digital calls, or some combination of the two [7]. This underscores the need to employ an effective channel control strategy, coordinating analog and digital calls which have different bandwidth requirements and different service qualities as well [3], [7], [14].

There are several strategies in the literature to control a variety of call types with different characteristics [11], [7], [10], [12]. Focusing only on dual-mode cellular systems, Kakae's [7] proposes two strategies. The one is the complete partitioning strategy where a certain fraction of the channel pool assigned for a cell is allocated permanently to each call type. The other is the strategy of the threshold type (TT), which accepts analog call requests only when the number of free channels is not less than a value prespecified for the cell. A modified complete partitioning strategy, which allows the dual mode (digital) calls that cannot find a free digital channel in the digital channel pool to overflow into the analog channel pool, is suggested by Lara-Rodriguez [3]. By Tawfik [13] this strategy is extended to the one in which both types of calls are allowed to overflow into the channel pool of the other type.

In this paper, we adopt another TT strategy, slightly different from Kakae's, such that the number of active analog calls in a cell is instead limited by a given value. Note that our TT strategy is at least as effective as Kakae's version in giving priority to a certain traffic stream (see (12)). An important side benefit gained from its employment is its mathematical tractability, as evidenced in the forthcoming development. Note from the literature that Kakae's version is more often referred to as the cutoff priority strategy [10].

Under our TT call control strategy, the objective of the study is to determine the number of required channels and to find the threshold value for each cell which optimizes the performance criterion while satisfying the prespecified grade of service (GOS) for each call type. The performance criterion used is quite general to include as special cases the three most meaningful measures: the weighted average blocking probability, the blocking probability of digital calls, and the average bandwidth utilization. We derive some interesting relations between blocking probabilities, based on which efficient algorithms are developed for the associated optimization models. This analytic method is indeed in good contrast to the existing simulation-based approach [7].

The organization of this paper is as follows. In Section II, we describe our traffic model and define explicitly the TT strategy. Then some fundamental properties of the blocking probabilities of analog and digital calls are provided. In Section III, focusing on a single cell system where the arrival rates and the GOS's for both call types in the cell are given, two basic problems are formulated as nonlinear integer models. The one is to find the optimal threshold value. The other is to obtain the minimum number of required channels in the cell. Exact algorithms yielding optimal solutions are then provided. Section IV extends the problems defined for a single-cell system to the multicell case and presents an efficient solution method. After summarizing extensive computational experiments performed with the proposed solution methods, the paper concludes in Section V.
II. TRAFFIC MODEL AND CALL-CONTROL STRATEGY

Consider a cellular mobile network supporting both analog and digital calls, which consists of M cells such that N_i (analog) channels are assigned to cell i. Suppose that analog and digital call attempts are generated in cell i according to independent Poisson processes with rates λi and λi, respectively. The service times, irrespective of call types, are exponentially distributed with mean 1/μ, based on the assumption that the network is mostly for voice traffic [3], [6].

To make a clear distinction from here on, either a channel or an a-channel will be used to represent a bandwidth required for transmitting a single analog call, while a d-channel to denote a subchannel of an a-channel required for carrying a single digital call. We assume that a single channel corresponds to d-channels, i.e., the bandwidth required for a single analog call can carry d digital calls. Therefore cell i having N_i channels can support N_i analog calls or d N_i digital calls when fully utilized by either type.

From among several control strategies in the literature prioritizing call (job) classes [7], [10], [12], we shall employ the strategy called the threshold type.

Threshold Type (TT) Strategy: The TT strategy limits the number of analog calls in cell i to l_i (≤ N_i) but always accepts a digital call while a free d-channel is available.

Note that the TT strategy includes the so-called complete sharing (CS) strategy [1], which does not give priority to either type.

Let the threshold value of cell i be l_i, i = 1, · · · , M. Then, the behavior of cell i can be modeled as a 2-D Markov process, characterized by n_i(t) = (n_1(t), n_2(t)): t ≥ 0, where n_1(t) and n_2(t) are the numbers of analog and digital calls active at time t in the cell, respectively. The state space for cell i is represented by the following:

\[ \Omega_i = \{ (n_1, n_2) : n_1 + n_2 ≤ N_i, 0 ≤ n_1 ≤ l_i \}. \]

A. Product-Form Solution and Blocking Probabilities

The TT strategy mentioned above belongs to the so-called coordinate convex strategies [8], which leads the steady-state probability for the Markov process n_i(t) to the product-form solution. Let \( P_i(n_1, n_2) \) be the steady-state probability that the state of cell i is (n_1, n_2). Then we obtain the steady-state probabilities as follows [8]:

\[ P_i(n_1, n_2) = \frac{\lambda_i^{n_1} \lambda_i^{n_2}}{\mu n_1 n_2} \left[ G(\Omega_i) \right]^{-1} \]

where

\[ a_i = \frac{\lambda_i}{\mu}, \quad b_i = \frac{\lambda_i}{\mu} \quad \text{and} \quad G(\Omega_i) = \sum_{(n_1, n_2) \in \Omega_i} \frac{\lambda_i^{n_1} \lambda_i^{n_2}}{n_1 n_2}. \]

Further denote

\[ \Omega_i(A) = \{ (n_1, n_2) \in \Omega_i : n_1 + n_2 ≤ N_i - 1, n_1 ≤ l_i - 1 \}. \]

1 A more general type of threshold strategy would simultaneously limit both types of calls to prespecified values [12]. But it is meaningless to take this strategy in the situation where definite priority is given to digital calls over analog calls.

Note that \( \Omega_i(k) (k = A, D) \) is the set of states in which a call attempt of type k (A and D represent analog and digital calls, respectively) will be accepted at cell i.

Let \( B_A(l_i, N_i) \) and \( B_D(l_i, N_i) \) be the blocking probabilities of analog and digital calls in cell i when the threshold value is \( l_i \) and there are \( N_i \) available channels. Then we obtain the following:

\[ B_A(l_i, N_i) = 1 - \frac{G(\Omega_i(A))}{G(\Omega_i)}. \]
\[ B_D(l_i, N_i) = 1 - \frac{G(\Omega_i(D))}{G(\Omega_i)}. \]

B. Properties of Blocking Probabilities

We observe some interesting properties of blocking probabilities of analog and digital calls, which are not only meaningful by themselves but also act as a basis for further development.

Proposition 1: Under the TT strategy, the following relations hold for 0 ≤ l_i ≤ N_i

\[ B_A(l_i, N_i) > B_A(l_i, N_i + 1) > B_A(l_i + 1, N_i + 1) \quad (1) \]
\[ B_D(l_i, N_i) > B_D(l_i + 1, N_i + 1) > B_D(l_i, N_i + 1) \quad (2) \]

Remark 1: The above relations are only for the marginal channel increase. Note the reality that the allocation of frequency spectrum is done on a channel basis, implying that the increase in d-channels is made only by d units. It is thus meaningless to consider the marginal effect of a single d-channel increase.

Remark 2: Note that \( B_A(l_i, N_i) > B_D(l_i, N_i) \), 0 ≤ l_i ≤ N_i. Under the CS strategy, the blocking probability of analog calls is known to be approximately \( \nu \) times larger than that of digital calls [8]. The above relation is thus obvious, considering that the blocking probability of digital calls with higher priority becomes further smaller under the TT strategy.

The first inequality of (1) for \( B_A(l_i, N_i) \) shows that the addition of a channel strictly decreases the blocking probability of analog calls, while the second inequality indicates the strict decrease from increasing the threshold value and thus loosening the restriction on analog calls. The inequalities of (2) for \( B_D(l_i, N_i) \) may be understood similarly: the first showing the overall positive effect on digital calls from adding a channel, while the second, the probability decrease from accepting less analog calls and thus allowing more digital calls. The proofs for these inequalities are given in the Appendix.

III. SINGLE CELL SYSTEM

Focusing on a single cell system where the arrival rates and the prespecified QOS levels for both types of calls are given, we consider two optimization problems: the call control problem (CCP) and the cell dimensioning problem (CDP).

For expediency, we shall drop subscript i from the associated notations used to designate cell i in this section.
A. Call Control Problem

Given \( N \) channels in the cell, the objective of the CCP is to find a threshold value optimizing the specified performance objective while ensuring the GOS's for both types of calls. The CCP for a given \( N \) is formally stated as follows:

\[
\begin{align*}
\text{(CCP)} \quad & \min_{l} Z(l) = \alpha_1 BA(l, N) + \alpha_2 BD(l, N) \\
\text{s.t.} \quad & BA(l, N) \leq \text{GOS}_A \\
& BD(l, N) \leq \text{GOS}_D \\
& 0 \leq l \leq N, \quad l: \text{integer}
\end{align*}
\]

where \( \text{GOS}_A \) and \( \text{GOS}_D \) are the GOS levels required for analog and digital calls.

Note that the objective function of \( Z(l) \) takes the most general form with the weight parameters \( \{\alpha_i\} \) whose values depend on the specific performance objective to be employed. Three prominent alternatives, which will be referred to as Type I, II, and III objectives, are respectively, of our particular interest: the minimization of the weighted average blocking probability given by \( (\lambda_1 BA(l, N) + \lambda_2 BD(l, N))/\lambda_3 \), the minimization of the digital blocking probability, \( BD(l, N) \), and the maximization of the average bandwidth utilization, \( \mu_0 (1 - BA(N, N)) + \mu_1 (1 - BD(N, N)) \). Therefore it is clear that \( \alpha_j = \lambda_j/\lambda_3, \quad j = 1, 2 \) for Type I objective, \( \alpha_3 = 0, \quad \alpha_2 = 1 \) for Type II, and \( \alpha_3 = \mu_0, \quad \alpha_2 = \mu_1 \) for Type III.

The CCP is an integer programming problem with a nonconvex objective function for which no effective solution method is available in the literature. One may thus consider the use of the enumeration method, but the associated computational burden, aggravated by the difficulty of calculating blocking probabilities [15], is prohibitively large even for moderately-sized systems. However, we have the following interesting property of the objective function as well as those of the blocking probabilities as shown in Proposition 1, which will be exploited for developing an efficient solution algorithm. Before proceeding further, we list the definition of the unimodality of a function on integer domain for self-containment [2].

**Definition:** A function \( f \) is said to be unimodal on an interval \( L \subset Z \) if there is an \( x^* \in L \) minimizing \( f \) on \( L \) and, for any two points \( x^1 \in L, x^2 \in L \) such that \( x^1 < x^2 \), we have

\[
\begin{align*}
x^2 \leq x^* \implies f(x^1) \geq f(x^2) \\
x^1 \leq x^* \implies f(x^2) \geq f(x^1).
\end{align*}
\]

**Theorem 1:** The objective function of the CCP, \( Z(l) \), is unimodal on \( [0, N] \).

**Proof:** See the Appendix.

Fig. 1 illustrates that the objective functions, \( Z(l) \)'s, of three different types of objectives, with the parameters set at \( \lambda_1 = 10, \lambda_2 = 90, \mu = 1, \mu = 3, N = 30 \), are all unimodal.

The unimodality of the objective function naturally suggests the approach of incrementally improving the objective. Start with an appropriate threshold value and incrementally increase the value insofar as both GOS constraints are satisfied until reaching the point from which the objective value starts to increase. The efficiency of this incremental method depends greatly on that of the initializing process. Now recall that the Erlang-B formula is convex and strictly decreases in the number of channels (servers) [9]. From this property, it is straightforward to calculate the number of channels required to meet the GOS level for analog calls when only analog traffic is present in the cell. Taking this value as the initial threshold point, we now summarize the algorithm.

**Algorithm CC:**

Step 1) Initialization: Find the smallest integer \( l \) satisfying \( E(\lambda_j/\mu, l) \leq \text{GOS}_A \), where \( E(\rho, \kappa) \) is the Erlang-B formula with offered load \( \rho \) and \( \kappa \) servers. If \( l > N \), the CCP for the given \( N \) is infeasible. Otherwise go to Step 2.

Step 2) Optimality check: Compute \( BA(l, N) \) and \( BD(l, N) \). If \( BD(l, N) > \text{GOS}_D \), terminate; the CCP is infeasible.

If \( BA(l, N) > \text{GOS}_A \), go to Step 3.2.

If \( BA(l, N) < \text{GOS}_A \) and \( BD(l, +1, N) \leq \text{GOS}_D \), compute \( BA(l, +1, N) \) and \( BD(l, +1, N) \). If \( Z(l) < Z(l + 1) \) or \( BD(l, +1, N) > \text{GOS}_D \), \( l \) is optimal; terminate. Otherwise go to Step 3.1.

Step 3) Update of \( l \):

Step 3.1) If \( l = N \), \( l \) is optimal. Otherwise, set \( l = l + 1 \); go to Step 2.

Step 3.2) If \( l = N \), the CCP is infeasible. Otherwise, set \( l = l + 1 \); go to Step 2.

**Theorem 2:** Algorithm CC finds an optimal solution of the CCP.

**Proof:** It is straightforward to see from Proposition 1 and Theorem 1 that Steps 2 and 3 of the algorithm CC maintain the feasibility and minimize the objective function.

**Experiment 1:** As shown in Table 1(a), a variety of input data instances are generated by differentiating arrival rates and GOS requirements, such that class \( (i, j) \), \( i, j = 1, 2, 3 \), therein denotes the combination of the arrival rate of type \( i \) and the GOS requirement of type \( j \). Three types of objectives, I, II, and III, are also considered by discriminating weight parameters. As expected, Table 1(b) shows that the Type II objective of minimizing \( BD(l, N) \) forces the optimal threshold values to be as small as possible, whereas the Type III of maximizing

![Fig. 1. Unimodality of Z(l).](image-url)


TABLE I

Computational Experiments for Algorithm CC: (a) Sample Data for Traffic Parameters, (b) Optimal Solutions of the CCP's

<table>
<thead>
<tr>
<th>Type ( \lambda_j )</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
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<tr>
<td>( \lambda_1 )</td>
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<td>25</td>
<td>40</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>105</td>
<td>60</td>
<td>15</td>
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(Arrival rates)

<table>
<thead>
<tr>
<th>Type ( GOS )</th>
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<th>2</th>
<th>3</th>
</tr>
</thead>
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<tr>
<td>( GOS_A )</td>
<td>0.08</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>( GOS_D )</td>
<td>0.015</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

(GOS's)

(a)

<table>
<thead>
<tr>
<th>Class ((i,j))</th>
<th>((1,1))</th>
<th>((1,2))</th>
<th>((1,3))</th>
<th>((2,1))</th>
<th>((2,2))</th>
<th>((2,3))</th>
<th>((3,1))</th>
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<tr>
<td>Obj.</td>
<td>15</td>
<td>17</td>
<td>30</td>
<td>31</td>
<td>33</td>
<td>45</td>
<td>48</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>( BA )</td>
<td>6.9813</td>
<td>5.9507</td>
<td>6.8630</td>
<td>6.3319</td>
<td>5.9052</td>
<td>6.1027</td>
<td>5.7551</td>
<td>5.7551</td>
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</tr>
<tr>
<td>( BD )</td>
<td>1.6215</td>
<td>1.7424</td>
<td>1.4157</td>
<td>1.6122</td>
<td>1.8038</td>
<td>1.3135</td>
<td>1.8429</td>
<td>1.8429</td>
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</tr>
<tr>
<td>( WAB^* )</td>
<td>2.0876</td>
<td>2.1083</td>
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<td>3.0094</td>
<td>3.0101</td>
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<td>50</td>
<td>50</td>
<td>45</td>
<td>50</td>
<td>50</td>
<td></td>
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<tr>
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<td>6.8630</td>
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<td>5.8300</td>
<td>6.1027</td>
<td>5.7549</td>
<td>5.7549</td>
<td></td>
</tr>
<tr>
<td>( BD )</td>
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<td>1.8455</td>
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<td></td>
</tr>
<tr>
<td>( ABU^* )</td>
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<td>146.42</td>
<td>144.00</td>
<td>144.52</td>
<td>142.48</td>
<td>142.48</td>
<td>142.82</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. The service rate and the total number of available channels are set to 1 and 50, respectively.
2. Class \((i,j)\) means that the arrival rate is of type \(i\) and the GOS is of type \(j\).
3. \(^*\) indicates the optimal value of the corresponding objective function.
4. \(I\) : The optimal threshold value.
5. \(BA\) : The blocking probability of analog calls \(\times 10^9\).
6. \(BD\) : The blocking probability of digital calls \(\times 10^9\).
7. \(WAB\) : The weighted average blocking probability \(\times 10^9\).
8. \(ABU\) : The average bandwidth utilization.

(b)

the bandwidth utilization lifts up the value as high as possible, which is observed in [12]. Under the Type I objective of minimizing the weighted average blocking probability, the optimal threshold values are in between but closer to those under Type II.

B. Cell-Dimensioning Problem

The objective of the cell-dimensioning problem (CDP) is to determine the number of channels at least required in the cell for meeting the GOS requirements of both analog and digital calls. Noting the difference with the CCP where the number of channels is given, the CDP is now stated as follows:

\[
(CDP) \quad \min_{N \geq 0, N: \text{integer}} N \quad \text{s.t.} \quad BA(I, N) \leq GOS_A, \quad BD(I, N) \leq GOS_D, \quad N \geq 0, N: \text{integer.}
\]

Algorithm CD:

Step 1) Initialization: Set \((I, N) = (N, N)\), where \(N\) is the smallest integer satisfying \(BA(N, N) \leq GOS_A\).

Step 2) Update of \(I\):

Step 2.1) If \(BD(I, N) \leq GOS_D\), \(N\) is optimal. Otherwise, go to Step 2.2.

Step 2.2) If \(I > 1\) and \(BA(I - 1, N) \leq GOS_A\), set \(I = I - 1\). Otherwise, go to Step 3.

Step 3) Update of \(N\): Set \(N = N + 1\) and go to Step 2.

Note that the initial point is taken as the number of channels at least required for meeting the GOS of analog calls under the CS strategy. For the blocking probability of analog calls is always lower under the CS strategy than under the TT strategy. Besides its computation is easy and simple by the algorithm in [8].

Theorem 3: Algorithm CD finds the minimum number of channels that guarantees the prespecified GOS's for analog and digital calls.
Proof: For given \( N \), the search process is exhaustive over all possible threshold values within the range satisfying both constraints. By varying \( N \), the algorithm then finds out the optimal solution by Proposition 1.

Experiment 2: Table II shows the minimum number of channels required for both analog and digital cells under different arrival rates and GOS levels. Since a channel usually corresponds to three or six \( d \)-channels [3], [7], we consider two cases: \( \nu = 3 \) and \( \nu = 6 \), which are treated within Table II. As digital traffic increases, the optimal values for \( \nu = 6 \) become significantly smaller compared with those for \( \nu = 3 \), which conforms with our expectation.

IV. MULTICELL SYSTEM

The two problems for a single-cell system are here extended to a multicell system consisting of a number of cell clusters. Note that a cluster implies the set of cells that may interfere with each other, and hence no two cells in a cluster can use the same frequency channel [5]. Hence for our purpose of assigning channels, we can let the system be a single cluster of cells without loss of generality. Assume that the cluster consists of \( M \) cells and that the arrival and service rates of both analog and digital calls in each cell are represented the same as in Section II.

Of the two problems for a single-cell system, the CDP can be straightforwardly extended to a multicell system. Thus, the minimum number of channels required for the cluster that satisfies the GOS's of both analog and digital calls can be obtained simply by summing those numbers of channels, one for each cell generated by an algorithm CD run. But the extension of the CCP to a multicell system is not so obvious, since not only the number of required channels but also the threshold value for each constituent cell has to be determined.

Suppose that a total of \( K \) interference-free frequency channels are available for the cluster. Let \( N = (N_1, \ldots, N_M) \) and \( 1 = (I_1, \ldots, I_M) \). Then the problem (MCP) is formulated as follows:

\[
\begin{align*}
\text{min} \quad & \sum_{i=1}^{M} \left[ \omega_1 A_i(l_i, N_i) + \omega_2 D_i(l_i, N_i) \right] \\
\text{s.t.} \quad & B_i(l_i, N_i) \leq \text{GOS}_A, \quad i = 1, \ldots, M \\
& D_i(l_i, N_i) \leq \text{GOS}_D, \quad i = 1, \ldots, M \\
& \sum_{i=1}^{M} N_i = K \\
& 0 \leq N_i, \quad N_i \geq 0, \quad i = 1, \ldots, M \\
& I_i, N_i: \text{integer}, \quad i = 1, \ldots, M.
\end{align*}
\]

In this multicell system, the weight parameters for three types of objectives are given as

\[
(\omega_1, \omega_2) = \begin{cases} (\lambda_1, \lambda_2), & \text{for Type I} \\ (0, \lambda_2), & \text{for Type II} \\ (\lambda_1, \beta_i), & \text{for Type III} \end{cases}
\]

where \( \lambda_j = \sum_{i=1}^{M} \lambda_{ij}, \quad j = 1, 2 \).

\[
\text{Table II:}
\]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \text{GOS}_A = 0.08 )</th>
<th>( \text{GOS}_D = 0.02 )</th>
<th>( \text{GOS}_A = 0.08 )</th>
<th>( \text{GOS}_D = 0.02 )</th>
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<td>29</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

The service rate is set to 1.

Two figures in each cell indicate the minimum numbers of channels; the upper one for \( \nu = 3 \), and the lower for \( \nu = 6 \).

Since the unimodality is not defined for multidimensional functions like \( Z(I, N) \), the strategy taken for algorithm CC cannot be directly applied to the MCP. On the other hand, full enumeration alternative demands additionally as many as \( (M+K-1) \) operations only for distributing \( K \) channels to \( M \) cells. The aim is thus set at developing a heuristic that generates a good approximate solution.

The proposed heuristic, called heuristic MC, is outlined as follows: A round of algorithm CD runs are first made to generate the minimum number of required channels for each cell, guaranteeing both GOS's. The remaining channels are then assigned one by one to the cells so that each assignment maximizes the marginal improvement of the objective value. For that, algorithm CC is repetitively executed to evaluate the marginal effect of every potential assignment. Finally, the phase of transferring channels from one cell to another follows for further improvement.

Let \( \theta_i(N_i) \) be the optimal objective value of the CCP for cell \( i \) with \( N_i \) channels and the parameter \((\alpha_1, \alpha_2) = (\omega_1, \omega_2)\). Assume that \( \theta_i(N_i) = \infty \) if the CCP is infeasible. \( C \) denotes the set of cells in the cluster. The precise statement of heuristic MC is given below.

**Heuristic MC:**

1. **Step 1:** Initialization: Set \( r = 1 \). Apply algorithm CD to obtain \( N_i \) for each cell \( i \in C \). Set \( T = K - \sum_{i=1}^{M} N_i \).

   If \( T > 0 \), apply algorithm CC to obtain \( \theta_i(N_i) \) for each cell \( i \in C \). Otherwise, the MCP is infeasible; terminate.

2. **Step 2:** Branch: If \( T > 0 \), go to Step 3. Otherwise, go to Step 4.

3. **Step 3:** Marginal channel assignment: Apply algorithm CC to obtain \( \theta_i(N_i + 1) \) for each cell \( i \in C \). Let \( j \) be the index at which the maximum of \( \{ \theta_i(N_i) - \theta_i(N_i + 1), k \in C \} \) is attained. Set \( T = T - 1 \) and \( N_j = N_j + 1 \); go to Step 2.

4. **Step 4:** Channel transfer: Let \( \Delta_j^r(\tau) = \theta_i(N_i + 1) - \theta_i(N_i) \) and \( \Delta_j^r(\tau) = \theta_i(N_i) - \theta_i(N_i + 1) \). Set \( p, q \) be the \((i, j)\) at which \( \xi(r) = \max_{(i,j)}(\Delta_i^r(\tau) - \Delta_j^r(\tau), i, j \in C, i \neq j) \) is attained. If \( \xi(r) = -\infty \),

2. The idea of this step is taken from the marginal capacity allocation procedure by Fox [4].
The results with heuristic MC, summarized in Table III, show that the optimal threshold values have the same tendency as in single-cell systems with respect to the types of objective functions. It is also worth noting that this trend also holds for additionally allocating the surplus \((K - L)\) channels when \(L\) is the number of channels at least required for satisfying both GOS's. Namely, to the cells with more digital traffic, i.e., the cells with smaller indices, more surplus channels are assigned under the Type II objective and fewer channels under Type III. For the case of Type I, the channels are evenly distributed among the cells, as compared with two other cases.

Our computational experience, strengthened by some additional test runs not reported here, indicates that the quality of the solutions generated by heuristic MC is so satisfactory that the optimal solutions were found for most of the test problem instances, as well evidenced by those in the above experiment 3. Also confirmed from the experience is the high speed of heuristic MC, mainly attributed to the effectiveness of Step 3 of the heuristic.

### Table III

**Computational Experiments for Algorithm MC**

(A) The results by heuristic MC for cluster sizes \((M)\) 4 and 7

<table>
<thead>
<tr>
<th>Obj.</th>
<th>(K)</th>
<th>(M = 4 (L = 34))</th>
<th>(M = 7 (L = 55))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>L</td>
<td>110</td>
<td>109</td>
<td>108</td>
</tr>
<tr>
<td>L + 5</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>L + 10</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>L + 15</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>L + 20</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>II</td>
<td>110</td>
<td>109</td>
<td>108</td>
</tr>
<tr>
<td>L + 5</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>L + 10</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>L + 15</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>L + 20</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>III</td>
<td>110</td>
<td>109</td>
<td>108</td>
</tr>
<tr>
<td>L + 5</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>L + 10</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>L + 15</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
<tr>
<td>L + 20</td>
<td>113</td>
<td>111</td>
<td>108</td>
</tr>
</tbody>
</table>

1. \(\mu = 1.0, GOS_{A1} = 0.08, GOS_{A2} = 0.02, v = 3, M = 4, L = 20 + 20, \lambda_2 = 260 - 60i, i = 1, ..., 4,\)
2. \(\lambda_2 = 55 + 5i, \lambda_2 = 115 - 15i, i = 1, ..., 7,\)
3. \(L : \text{the minimum number of channels for the cluster required to satisfy the preassigned GOS's.}\)
4. \(\rho : \text{The solution obtained after the channel transfer step.}\)
5. \(\rho : \text{The solution is not optimal.}\)

---

terminate. If \(\xi(r) > 0\), set \(N_p = N_q + r\) and \(N_q = N_p - r\). Set \(r = r + 1; \text{go to Step 2}\).

Step 4 explores the possibility of further improving the objective value by transferring channels from one cell to another insofar as both GOS's are ensured. The search is complete by gradually increasing the value of \(r\) starting from one.

**Experiment 2:** For a comprehensive test, we consider three different clusters with sizes \(M = 4, 7, 12\). Associating three types of objective functions for each cluster, we have a total of nine problems. For each problem, five input instances are tested by differentiating \(K\). The value of \(\nu\) is set at three.

Arbitrarily numbering the cells in each cluster from 1 to \(M\), we let \(\lambda_1\) and \(\lambda_2\) be chosen for cell \(i\) such that the relative weight of the former increases as \(i\) gets larger. This parameterization is to assimilate the traffic environment to the reality. For the cluster of size 4, \(\lambda_1 = 20 + 20i, \lambda_2 = 260 - 60i\), for size 7, \(\lambda_1 = 55 + 5i, \lambda_2 = 115 - 15i\), and for size 12, \(\lambda_1 = 5 + 5i, \lambda_2 = 185 - 15i\).
### TABLE III (Cont.)
(B) The Results by Heuristic MC for Cluster Size (M) 12

<table>
<thead>
<tr>
<th>Obj</th>
<th>$K$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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<tbody>
<tr>
<td>$L$</td>
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<td>71</td>
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<td>71</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>70</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>(15)</td>
<td>(20)</td>
<td>(26)</td>
<td>(36)</td>
<td>(35)</td>
<td>(40)</td>
<td>(40)</td>
<td>(49)</td>
<td>(54)</td>
<td>(59)</td>
<td>(64)</td>
<td>(67)</td>
<td></td>
</tr>
<tr>
<td>$L + 5$</td>
<td>73</td>
<td>73</td>
<td>72</td>
<td>72</td>
<td>72</td>
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<td>(23)</td>
<td>(30)</td>
<td>(35)</td>
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<td>(40)</td>
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<td>(49)</td>
<td>(54)</td>
<td>(59)</td>
<td>(64)</td>
<td>(67)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L + 10$</td>
<td>74</td>
<td>73</td>
<td>73</td>
<td>73</td>
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<td>72</td>
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<td>71</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(16)</td>
<td>(20)</td>
<td>(26)</td>
<td>(32)</td>
<td>(36)</td>
<td>(41)</td>
<td>(46)</td>
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<td>(64)</td>
<td>(67)</td>
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<tr>
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<td>(36)</td>
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<td>$L + 30$</td>
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</tr>
<tr>
<td></td>
<td>(18)</td>
<td>(23)</td>
<td>(27)</td>
<td>(33)</td>
<td>(39)</td>
<td>(44)</td>
<td>(48)</td>
<td>(54)</td>
<td>(58)</td>
<td>(63)</td>
<td>(68)</td>
<td>(71)</td>
<td></td>
</tr>
</tbody>
</table>

| $L$ | 72  | 72 | 71 | 71 | 71 | 72 | 71 | 71 | 70 | 70 | 70 | 69 |
|     | (15) | (20) | (26) | (30) | (35) | (39) | (44) | (49) | (54) | (59) | (64) | (67) |
| $L + 5$ | 73  | 73 | 73 | 73 | 72 | 72 | 71 | 71 | 70 | 70 | 70 | 69 |
|     | (14) | (19) | (24) | (30) | (34) | (39) | (44) | (49) | (54) | (59) | (64) | (67) |
| $L + 10$ | 74  | 74 | 73 | 73 | 72 | 72 | 71 | 71 | 70 | 70 | 70 | 69 |
|     | (14) | (19) | (24) | (29) | (34) | (39) | (44) | (49) | (53) | (58) | (62) | (67) |
| $L + 20$ | 75* | 75 | 74 | 74 | 73 | 73 | 73 | 72 | 72 | 71 | 70 | 69 |
| $L + 30$ | 77  | 76 | 76 | 75 | 75 | 74 | 73 | 73 | 72 | 71 | 70 | 69 |

1. $\mu = 1.0, \text{GOS}_L = 0.09, \text{GOS}_D = 0.09, \nu = 3, \lambda_1 = 5 + 5i, \lambda_2 = 105 - 15i, i = 1, \ldots, 12.$
2. $L$ is the minimum number of channels for the cluster required to satisfy the specified GOS's ($L = 848$).
3. $\star$: The solution obtained after the channel transfer step, b: The solution is not optimal.

### V. CONCLUSION

For the graceful transition from all-analog to all-digital systems, the dual-mode cellular system supporting both types of calls appears to be the only solution for the interim period. Focusing on the TDMA code, this paper has discussed the problem of managing the frequency resource by incorporating the so-called threshold-type call-control strategy of giving priority to digital calls consuming less bandwidth, which limits the number of active analog calls in each cell.

Under the TT strategy, two nonlinear integer optimization problems, namely the call-control problem and the cell-dimensioning problem, were suggested for a multicell as well as a single-cell system. We let the objective function take the most general form so as to include as special cases the three most popular performance measures: the weighted average blocking probability, the blocking probability of digital calls, and the average bandwidth utilization. Exploiting the unidimensionality of the objective function and the interesting relations between the blocking probabilities of both types of calls, exact algorithms for CCP and CDP in single-cell systems were first developed. Incorporating these algorithms as subroutines, an efficient heuristic solution method for a multicell system has also been developed. The extensive computational experiments indicate that the proposed solution methods perform so satisfactorily in both the quality of solutions generated and the speed as to even fit for dynamically assigning frequencies in real-world dual-mode systems.

### APPENDIX

To simplify the notation, the subscript $i$ indicating cell $i$ is suppressed. In the following, we shall use the measure of carried traffic instead of blocking probability for the convenience of exposition. Defining $CA(i, N)$ and $CD(i, N)$ as the carried analog and digital call traffic, respectively, note the relations that $CA(i, N) = \lambda_1 (1 - BA(i, N))$ and $CD(i, N) = \lambda_2 (1 - BD(i, N)).$ Before proceeding further, we first list the next lemma without proof, which will be utilized for the forthcoming main proofs.

**Lemma 1:** For $A_k > 0$ and $B_k > 0,$ $k = 0, 1, \ldots$

- if $B_k < \frac{B_{k+1}}{A_k},$
then \( \sum_{k=0}^{n-1} B_k < \sum_{k=0}^{n-1} A_k \) for \( n = 1, 2, \ldots \).

Proof of Proposition 1: Let

\( q(n_1, n_2) = \frac{a^{n_1} b^{n_2}}{n_1! n_2!} \) and \( q^N(n_1) = \frac{a^{n_1} b^{(N-n_1)}}{n_1! [N-k-n]}! \)

1) Proof of \( CA(l, N) < CA(l+1, N) \).

With the notation

\[ A_k = \sum_{n_2=0}^{N-k} q(n_1, n_2) \quad \text{and} \quad B_k = \sum_{n_2=0}^{N-k} kq(n_1, n_2) \]

\( CA(l, N) \) can be expressed as \( \sum_{n_2=0}^{N-l} B_k \). The result follows from \( \frac{B_k}{A_k} < \frac{B_{k+1}}{A_{k+1}} \).

2) Proof of \( CD(l, N) < CD(l+1, N+1) \).

If we let

\( S_k = \{ (n_1, n_2) \in Z^2 \mid \nu n_1 + n_2 = \nu(N-l-k) \text{ or } n_1 = k \} \)

\[ A_k = \sum_{(n_1, n_2) \in S_k} q(n_1, n_2) \]

and

\[ B_k = \sum_{(n_1, n_2) \in S_k} n_2 q(n_1, n_2) \]

\( CD(l, N) \) and \( CD(l+1, N+1) \) are given as follows:

\[ CD(l, N) = \sum_{n_2=0}^{N-l} B_k \sum_{l=0}^{N} A_k \]

\[ CD(l+1, N+1) = \sum_{k=0}^{N-l} B_k \sum_{k=1}^{N} A_k \]

It then suffices to show that \( \frac{B_k}{A_k} < \frac{B_{k+1}}{A_{k+1}} \), i.e., \( A_l B_{l+1} - B_l A_{l+1} > 0 \) for completing the proof.

Let

\[ X_1 = \sum_{n_2=0}^{N-l} q(l, n_2) \]

\[ Y_1 = \sum_{n_2=0}^{N-l} n_2 q(l, n_2) \]

\[ X_2 = \sum_{n_1=0}^{l-1} q^N(n_1) \]

\[ Y_2 = \sum_{n_1=0}^{l-1} n_1 q^N(n_1) \]

\[ X_3 = \sum_{n_1=1}^{l} q^{N+1}(n_1) \]

\[ Y_3 = \sum_{n_1=1}^{l} n_1 q^{N+1}(n_1) \]

Then

\[ A_l = X_1 + X_2 \]

\[ A_{l+1} = \frac{a}{l+1} X_1 + X_3 + q^{N+1}(0) \]

\[ B_l = Y_1 + Y_2 \]

\[ B_{l+1} = \frac{a}{l+1} Y_1 + Y_3 + \nu(N+1) q^{N+1}(0) \]

Hence we have

\[ A_l B_{l+1} - B_l A_{l+1} \]

\[ = \left[ X_1 Y_3 + \frac{a}{l+1} X_2 Y_1 - \left( Y_1 X_3 + \frac{a}{l+1} Y_2 X_1 \right) \right] \]

\[ + [X_2 Y_3 - Y_2 X_3] + q^{N+1}(0) \]

\[ \times [\nu(N+1)(X_1 + X_2) - (Y_1 + Y_2)] \]

Equation (3) consists of three parts of square brackets.

We complete the proof by showing that the value of each of the three brackets is greater than zero. Noting that \( X_3 \) and \( Y_3 \) can be expressed as

\[ X_3 = \sum_{n_1=0}^{l-1} q^{N+1}(n_1) \]

\[ Y_3 = \sum_{n_1=0}^{l-1} \nu(N-n_1) q^{N+1}(n_1) \]

we have for the for part

\[ X_1 Y_3 + \frac{a}{l+1} X_2 Y_1 - \left( \frac{a}{l+1} Y_1 X_3 + Y_2 X_1 \right) \]

\[ = \sum_{n_2=0}^{N-l} \sum_{n_1=0}^{l-1} n_2 q(l, n_2) q^N(n_1) \]

\[ \times a \left( \frac{\nu(N-n_1) + n_2}{n_1 + 1} + \frac{n_2}{n_1 + 1} - \frac{\nu(N-n_1)}{l+1} \right) \]

\[ = \sum_{n_2=0}^{N-l} \sum_{n_1=0}^{l-1} a q(l, n_2) q^N(n_1) \]

\[ \times a [\nu(N-n_1) - n_2] \left( \frac{1}{n_1 + 1} - \frac{1}{l+1} \right) > 0. \]

For the second part, let

\[ Q_{ij} = q^N(i) \nu(N-j) q^{N+1}(j+1) \]

\[ - \nu(N-i) q^N(i) q^{N+1}(j+1) \]

then the following is satisfied

\[ X_3 Y_3 - Y_3 X_3 \]

\[ = \sum_{j=0}^{l-1} \sum_{i=j+1}^{l-1} [Q_{ij} + Q_{ji}] \]

\[ = \sum_{j=0}^{l-1} \sum_{i=j+1}^{l-1} q^N(i) q^N(j) \]

\[ \times a \left( \frac{\nu(N-j) + \nu(N-i)}{j+1} + \frac{n_2}{j+1} - \frac{\nu(N-j)}{l+1} \right) \]

\[ - \frac{\nu(N-i)}{l+1} - \frac{\nu(N-j)}{l+1} \]
Finally, from the definition of \( X_i \) and \( Y_i \), it can be easily verified that the last part is also greater than zero.

3) Proofs of the remaining two strict inequalities.

The proofs are similar to the above cases and thus omitted.

**Proof of Theorem 1:**

Let

\[
A_k = \sum_{n=0}^{\nu(N-k)} \frac{a_k b_{N-n}}{k! n!}
\]

and

\[
B_k = \sum_{n=0}^{\nu(N-k)} \frac{(a_1 k + a_2 n) a_k b_{N-n}}{k! n!}
\]

Then

\[
CT(l) = \alpha_1 CA(l, N) + \alpha_2 CD(l, N) = \sum_{k=1}^{l} A_k B_k.
\]

We complete the proof by showing that \( CT(l) \) is unimodal over \([0, N]\).

First note that

\[
\frac{B_k}{A_k} = \alpha_1 k + \alpha_2 \frac{b_k}{a_k} (N - k),
\]

where \( T(p, m) \) is the carried traffic of an \( M/M/m \) queueing system with offered load \( p \). Also note that \( T(b, \nu(N - k)) \) is concave for \( k \) [9], rendering the concavity of \( \frac{B_k}{A_k} \). So, with \( l = \arg \max_{k \in [0, N]} \frac{B_k}{A_k} \), \( \frac{B_k}{A_k} \) increases on \([0, l]\) and decreases on \([l, N]\). Furthermore, note that \( CT(l) \leq \frac{B_k}{A_k} \) if and only if \( CT(l) \leq CT(l + 1) \).

If we let

\[
l = \min \left\{ \frac{1}{\nu(N-k)} \mid \sum_{k=1}^{l} A_k B_k \right\}
\]

then \( l \geq l \) and hence

\[
CT(0) \leq \cdots \leq CT(l) \leq \cdots \leq CT(N).
\]

That is, \( CT(l) \) is unimodal on \([0, N]\) with its maximum at \( l \).

**REFERENCES**


