

# Silencer design by using array resonators for low-frequency band noise reduction

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Helmholtz resonators are often used to reduce noise. They are particularly useful when noise has a narrow frequency band. In this study we aim to broaden its narrow band characteristics by combining many resonators. Serial and parallel arrangements of resonators have been tested to obtain broader impedance mismatch characteristics in the broader band. Theoretical and experimental results explain these characteristics in the absence of mean flow. The serial arrangement mainly increases the peak of TL at the resonance frequency. But the parallel arrangement logarithmically increases the peak of TL and expands the bandwidth. The change of acoustic characteristics is explained by introducing an "equivalent impedance analysis." This shows that the transmission loss has a maximum value when the distance between resonators is  $\lambda/4$  of its wavelength. In this study we propose a novel design method that optimizes the arrangement of resonators while keeping the volume minimized. Various transmission loss characteristics are possible when we select different objective functions under constraints. © 2005 Acoustical Society of America. [DOI: 10.1121/1.2036222]

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## I. INTRODUCTION

The Helmholtz resonator is often used to reduce noise in a narrow frequency band. This type of resonator has a high transmission loss in a narrow band at its resonance frequency. It is easy to design this resonator to have a desired resonance frequency, because this is determined by the geometric ratio of the cavity and its neck.

There have been many studies about the disagreement between analytic and experimental results of the resonator due to the uncertainty of damping around the neck or end correction.<sup>1–3</sup> With regard to characteristics of the silencer using resonators, Anderson<sup>4</sup> studied the effect of flow when a single side branch Helmholtz resonator is attached to a circular duct. Chun<sup>5</sup> studied the acoustic characteristics of concentric pipe resonators to obtain a general solution for the acoustic field in the resonators by using the Green's function. Furthermore, many studies about the applications of a resonator are in progress in various fields.

In this paper, multiple array resonators are used to broaden the narrow band characteristics of a resonator in the low-frequency band. Kim<sup>6</sup> and Kuntz<sup>7</sup> studied the acoustic characteristics of the panel using multiple array resonators to obtain a high absorption coefficient at low frequency. These studies made use of impedance changes in the resonator configuration.

Also, in the case of a silencer using array resonators, in order to obtain broadband transmission loss characteristics, we can make use of an additional effect from many different impedance surfaces.

Therefore, we need to study the acoustic characteristics of array resonators for the design of an effective silencer model. Thus we intend to introduce the equivalent impedance that represents the total acoustic characteristics of the silencer and an optimization for determining resonance frequencies.

## II. THEORY

### A. Transmission loss of a branch resonator

Silencer using a resonator reduces noise by an impedance mismatch that causes reflection of the incident acoustic energy and attenuation in the resonator's neck. When a resonator is attached to a duct by a side branch, as depicted in Fig. 1, the basic assumption is that plane waves propagate in a duct and the reflected waves from downstream of a duct do not exist by using an anechoic termination in the absence of mean flow. Considering effects of grazing flow, if the mean flow's velocity is less than  $M=0.1$  ( $M$ : Mach number), its effect is not serious.<sup>4</sup>

The sound pressure ( $P$ ) and the volume velocity ( $U$ ) can be expressed as follows:

$$P_1 = Ae^{-jkx} + Be^{jkx}, \quad P_2 = Ce^{-jkx}, \quad (1a)$$

$$U_1 = \frac{1}{Z}(Ae^{-jkx} - Be^{jkx}), \quad U_2 = \frac{1}{Z}(Ce^{-jkx}), \quad (1b)$$

where  $A$ ,  $B$ , and  $C$  are the magnitude of the incident wave, reflected wave, and transmitted wave, respectively, and  $Z = \rho c/S$  is the acoustic impedance of the duct. Here  $k$

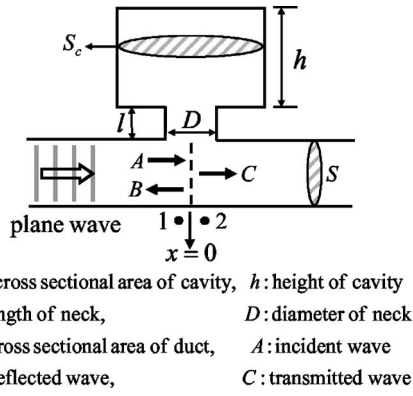


FIG. 1. Helmholtz resonator with a duct.

$=2\pi f/c$  is the wave number,  $\rho$  is the density of air, and  $c$  is the sound speed.

The transfer matrix between point 1 and point 2 can be obtained as follows by using the continuity of the sound pressure and the volume velocity:

$$\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ -jZ_c \cot kh + Z_h & 1 \end{bmatrix} \begin{pmatrix} P_2 \\ U_2 \end{pmatrix} \quad (2)$$

$$= \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{pmatrix} P_2 \\ U_2 \end{pmatrix},$$

where  $Z_c = \rho c / S_c$  is the acoustic impedance of a resonator's cavity.

The impedance of resonator  $Z_r$  can be expressed as

$$Z_r = -jZ_c \cot kh + Z_h, \quad (3)$$

$$Z_h = \frac{\rho c}{S_h} [0.0072 + jk(l + 0.75)], \quad (4)$$

where  $Z_h$  is the hole impedance of a resonator, as suggested by Sullivan.<sup>8,9</sup> Sullivan obtained the hole impedance of perforated elements in a concentric tube resonator by measurement. Here, the resistance of  $Z_h$  was modified in consideration of the experimental results of this study, and  $S_h$  is the cross-sectional area of the hole.

Transmission loss (TL) can be represented as follows by using the transfer matrix:

$$TL = 20 \log_{10} \left| \frac{A}{C} \right|$$

$$= 20 \log_{10} \left| \frac{T_{11} + T_{12}/Z + T_{21} \cdot Z + T_{22}}{2} \right| \quad (\text{dB}), \quad (5)$$

where  $A$  is acoustic pressure of the incident wave and  $C$  is the acoustic pressure of the transmitted wave.

Using Eqs. (2) and (5), the TL of a branch resonator in the duct can be obtained as

$$TL = 20 \log_{10} \left| \frac{2 + Z \left( \frac{1}{-jZ_c \cot kh + Z_h} \right)}{2} \right|. \quad (6)$$

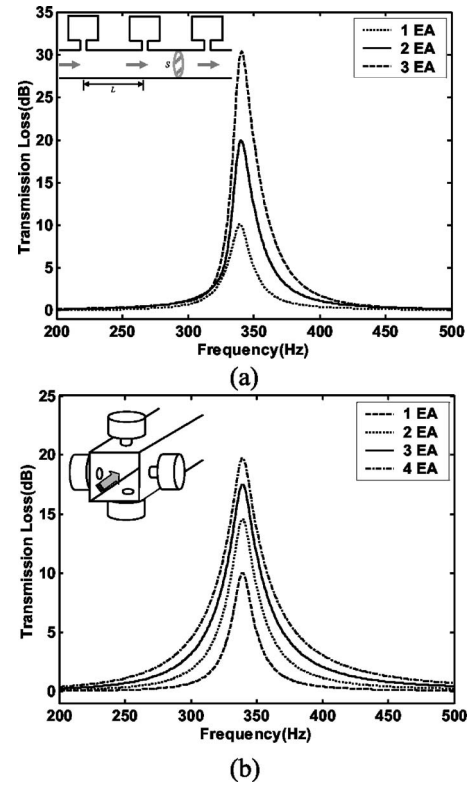


FIG. 2. The TL change varying the number of resonators. (a) The serial arrangement ( $S$ : cross-sectional area of duct,  $L$ : distance of resonators); (b) the parallel arrangement. ( $f_0=340$  Hz,  $L:100$  mm,  $l:25$  mm,  $D:10$  mm,  $S_c:\pi(20)^2$  mm<sup>2</sup>,  $h:48.5$  mm,  $S:50^*50$  mm<sup>2</sup>).

The TL of a branch resonator has a peak at its resonance frequency, as is well known. Thus, in order to obtain broadband characteristics, it is necessary to study the TL change for the serial and parallel array of resonators.

## B. Serial and parallel array

Figure 2 illustrates the serial and parallel arrangement of identical resonators. Considering the TL change varying the resonator's number, the serial arrangement mainly increases the magnitude of TL at the resonance frequency. But the parallel arrangement logarithmically increases the magnitude of TL and the bandwidth.

These characteristics give the resonator array high TL in the broader band. In this sense, we can design the silencer by using multiple array resonators having different resonance frequencies, which are included in the objective frequency band.

Figure 3 illustrates a panel-type silencer using each of 8 resonators ( $i=1$ ; 257 Hz,  $i=2$ ; 297 Hz,  $i=3$ ; 361 Hz,  $i=4$ ; 413 Hz) in the parallel direction. Impedance of each resonator is expressed as follows, as given by Eq. (3):

$$Z_{r(i)} = -jZ_{c(i)} \cot kh_{(i)} + Z_{h(i)} \quad (i = 1, 2, 3, 4). \quad (7)$$

If the transfer matrix that represents a resonator having resonance frequency ( $f_i$ ) is  $T_{f_i \text{ Hz}}$  and the direct tube between the resonators is  $T_{\text{tube}}$ , the total transfer matrix ( $T_{\text{total}}$ ) of the panel-type silencer is obtained as follows:

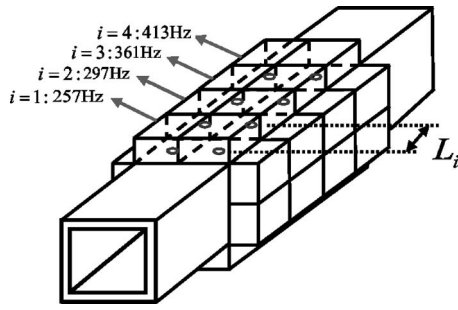


FIG. 3. Panel-type silencer using resonator array ( $L_i$ : distance of resonators).

$$T_{\text{tot}} = T_{257 \text{ Hz}} \cdot T_{\text{tube}} \cdot T_{297 \text{ Hz}} \cdot T_{\text{tube}} \cdot T_{361 \text{ Hz}} \cdot T_{\text{tube}} \cdot T_{413 \text{ Hz}}$$

$$= \begin{bmatrix} 1 & 0 \\ \frac{N}{Z_{r1}} & 1 \end{bmatrix} \cdot T_{\text{tube}} \cdot \begin{bmatrix} 1 & 0 \\ \frac{N}{Z_{r2}} & 1 \end{bmatrix} \cdot T_{\text{tube}} \cdot \begin{bmatrix} 1 & 0 \\ \frac{N}{Z_{r3}} & 1 \end{bmatrix} \cdot T_{\text{tube}} \cdot \begin{bmatrix} 1 & 0 \\ \frac{N}{Z_{r4}} & 1 \end{bmatrix}$$
(8)

where  $N$  denotes the number of resonators in the parallel direction.

Using Eqs. (5) and (8), the total TL of the panel-type silencer can be obtained. Figure 4 compares each resonator's TL with the total TL of the panel-type silencer. As depicted in Fig. 4, the total TL has each resonance frequency's peak and shows the additional effect.

Thus we confirmed that the panel-type silencer can obtain high TL characteristics in the broader band, as noted before. This theoretical result should then be verified experimentally.

### III. EXPERIMENT

#### A. Experimental setup

Figure 5 shows the experimental setup for the TL measurement of the simplified panel-type silencer. The duct's cross-sectional area is  $50 \times 50 \text{ mm}^2$  with a higher cutoff frequency of 3.43 kHz for plane wave propagation. An anechoic termination with a cutoff frequency of about 100 Hz is employed in this experiment to avoid the effects of the reflected waves.

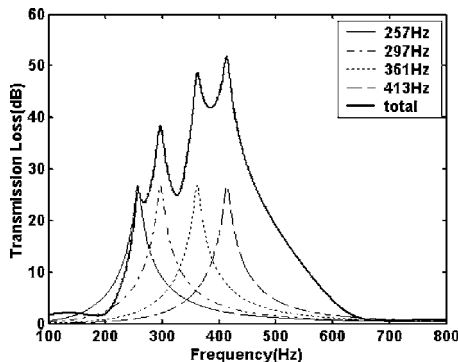


FIG. 4. Transmission loss of panel-type silencer. ( $N$ : 8,  $l$ : 25 mm,  $D$ : 10 mm,  $S$ :  $50 \times 50 \text{ mm}^2$ ,  $L$ : 100 mm).

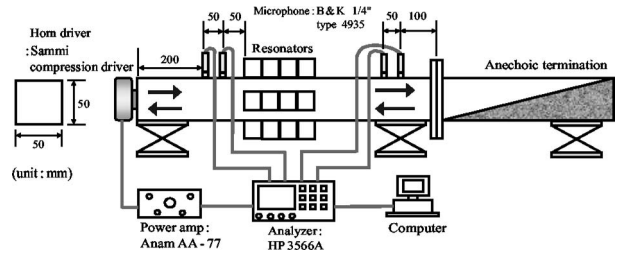


FIG. 5. The experiment setup for measurement of transmission loss.

In this study, TL was measured by a two-microphone method from the Seybert and Ross wave decomposition theory.<sup>10</sup>

### B. Results

Figure 6 illustrates the panel-type silencer using each of four resonators ( $i=1$ ; 257 Hz,  $i=2$ ; 334 Hz,  $i=3$ ; 387 Hz,  $i=4$ ; 447 Hz) in the parallel direction. Figure 7 shows a comparison of the measured and predicted transmission loss.

### IV. EQUIVALENT IMPEDANCE ANALYSIS

#### A. Parallel combination

The panel-type silencer has two important design parameters. Each resonator's resonance frequency and distance between the resonator's hole determine the TL of the panel-type silencer. This means that we have to study the acoustic characteristics of the array resonators to determine an effective resonator arrangement.

Using an equivalent impedance concept that represents the total acoustic characteristics of the silencer, we can explain the acoustic characteristics according to the resonator arrangement. Figure 8 illustrates the equivalent impedance of a silencer using array resonators.

The equivalent impedance ( $Z_{\text{eq}}$ ) for a parallel arrangement is obtained as follows.

As depicted in Fig. 1, the pressure reflection coefficient and the pressure transmission coefficient at  $x=0$  can be represented as follows:

$$R_{\text{coeff}} = \frac{B}{A}, \quad T_{\text{coeff}} = \frac{C}{A}. \quad (9)$$

Considering the number of resonators in the parallel direction ( $N$ ), the transfer matrix between point 1 and point 2 can be written as

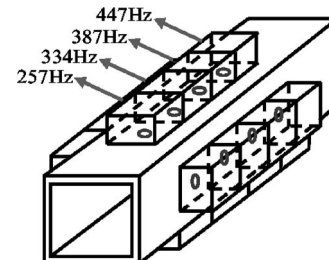


FIG. 6. Panel-type silencer using each of four resonators.

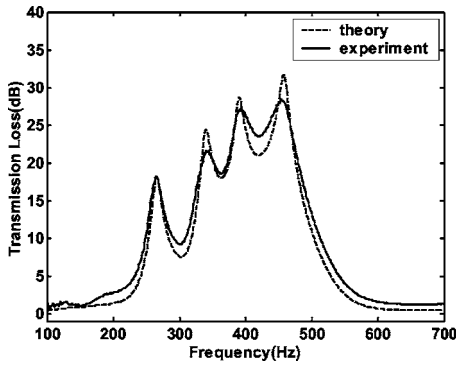


FIG. 7. A comparison between the experimental and theoretical result of the panel type using each of four resonators ( $L$ : 100 mm,  $l$ : 25 mm,  $D$ : 10 mm,  $S$ :  $50 \times 50$  mm<sup>2</sup>).

$$\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{N}{Z_r} & 1 \end{bmatrix} \begin{pmatrix} P_2 \\ U_2 \end{pmatrix}. \quad (10)$$

That is, Eq. (10) can be rearranged as

$$P_1 = P_2, \quad (11a)$$

$$U_1 = \frac{N \cdot P_2}{Z_r} + U_2. \quad (11b)$$

Using Eqs. (1), (9), and (11) at  $x=0$ , the pressure reflection coefficient ( $R_{\text{coeff}}$ ) and the pressure transmission coefficient ( $T_{\text{coeff}}$ ) at the impedance surface can be obtained in terms of the impedance of the resonator ( $Z_r$ ) and the acoustic impedance ( $Z$ ) of the duct:

$$R_{\text{coeff}} = \frac{-N \cdot Z}{N \cdot Z + 2 \cdot Z_r}, \quad (12)$$

$$T_{\text{coeff}} = \frac{2 \cdot Z_r}{N \cdot Z + 2 \cdot Z_r}. \quad (13)$$

Using Eq. (12), the equivalent impedance ( $Z_{\text{eq}}$ ) for a parallel arrangement can be evaluated as

$$Z_{\text{eq}} = \frac{P_1}{U_1} = \frac{Z(A+B)}{A-B} = \frac{Z(1+R_{\text{coeff}})}{1-R_{\text{coeff}}} = \frac{Z \cdot Z_r}{N \cdot Z + Z_r}. \quad (14)$$

Figure 9 shows the change of equivalent impedance ( $Z_{\text{eq}}/Z$ ), which is normalized by the acoustic impedance of the duct while varying the number of resonators in the parallel direction. As the number of resonators increases, the resistance (Re) of the impedance broadens. The reactance (Im) of the impedance has a value of zero at resonance frequency (200 Hz).

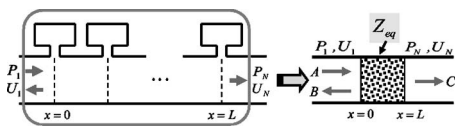


FIG. 8. Equivalent impedance of a silencer using array resonators ( $P$ : sound pressure,  $U$ : volume velocity,  $A$ : incident wave,  $B$ : reflected wave,  $C$ : transmitted wave).

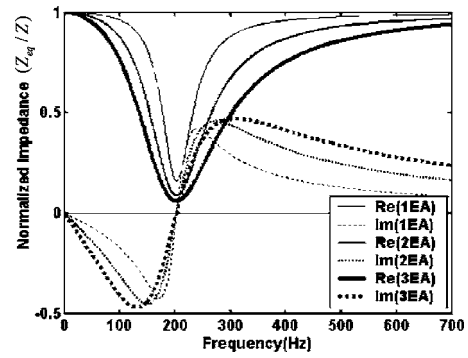


FIG. 9. Impedance change of the parallel arrangement varying the number of resonators (Re: resistance, Im: reactance).

## B. Serial combination

As depicted in Fig. 8, the total transfer matrix can be expressed as follows for a serial array:

$$\begin{pmatrix} P_1 \\ U_1 \end{pmatrix} = T_1 \cdot T_2 \cdot T_3 \cdot \dots \begin{pmatrix} P_N \\ U_N \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{pmatrix} P_N \\ U_N \end{pmatrix}. \quad (15)$$

Equation (15) can be rewritten as

$$P_1 = T_{11} \cdot P_N + T_{12} \cdot U_N, \quad (16a)$$

$$U_1 = T_{21} \cdot P_N + T_{22} \cdot U_N. \quad (16b)$$

Using Eqs. (9) and (16), the pressure reflection coefficient ( $R_{\text{coeff}}$ ) and the pressure transmission coefficient ( $T_{\text{coeff}}$ ) can be obtained in terms of the element of total transfer matrix at  $x=0 \sim L$ :

$$T_{\text{coeff}} = \frac{2}{T_{11} + T_{12}/Z + T_{21} \cdot Z + T_{22}}, \quad (17a)$$

$$R_{\text{coeff}} = \frac{T_{11} + T_{12}/Z - T_{21} \cdot Z - T_{22}}{T_{11} + T_{12}/Z + T_{21} \cdot Z + T_{22}}. \quad (17b)$$

Figure 10 illustrates an electrical analogy<sup>11</sup> of a simple model using two resonators.

First, considering the second resonator and a direct tube between the first and the second resonator, the impedance at  $x=L$  can be expressed as

$$Z_L = \frac{Z \cdot Z_2}{Z + Z_2}. \quad (18)$$

As depicted in Fig. 10, considering the wave propagation having magnitudes  $A$  and  $B$  in the direct tube, the sound pressure can be represented as

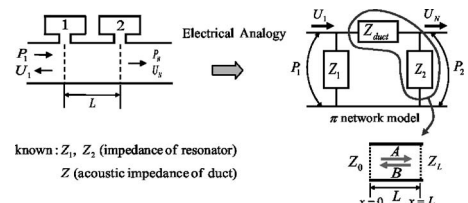


FIG. 10. Electrical analogy of the simple model using two resonators ( $P$ : sound pressure,  $U$ : volume velocity).



$$P(x) = Ae^{-jk(x-L)} + Be^{jk(x-L)}. \quad (19)$$

Using Eq. (19), the impedance at  $x=0$ ,  $x=L$  can be obtained as

$$Z_L = \frac{P(L)}{U(L)} = Z \cdot \frac{A+B}{A-B}, \quad (20)$$

$$Z_0 = \frac{P(0)}{U(0)} = Z \cdot \frac{Ae^{jkL} + Be^{-jkL}}{Ae^{jkL} - Be^{-jkL}}. \quad (21)$$

Using Eqs. (20) and (21), the impedance ( $Z_0$ ) at  $x=0$  can be evaluated as

$$Z_0 = Z \cdot \frac{(Z_L/Z) + j \tan(kL)}{1 + j(Z_L/Z)\tan(kL)}. \quad (22)$$

Finally, considering the first resonator, the equivalent impedance ( $Z_{eq}$ ) of a serial array can be obtained as follows:

$$Z_{eq} = \left( \frac{1}{Z_1} + \frac{1}{Z_0} \right)^{-1}. \quad (23)$$

The change of acoustic characteristics is explained by using the equivalent impedance analysis. Thus, we can determine the effective resonator arrangement for obtaining the TL characteristics for low-frequency band noise reduction.

### C. Characteristics change by the distance between resonators

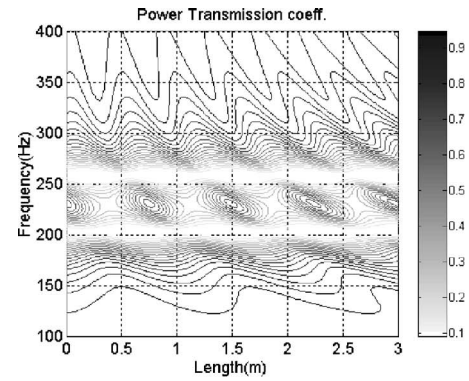
As depicted in Fig. 10, in the case of using two resonators with 202 Hz (front) and 258 Hz (rear), the power transmission coefficient ( $T_{II} = |T_{coeff}|^2$ ) and the power reflection coefficient ( $R_{II} = |R_{coeff}|^2$ ), the absorption coefficient ( $D_{II} = 1 - R_{II} - T_{II}$ ) for varying the distance between resonators can be obtained as the contour graph in Fig. 11.

It has a periodic characteristic of  $\lambda/2$  ( $\lambda$ : wavelength) as a whole and the TL between the first and second resonance frequency has a maximum value when the distance between resonators is  $\lambda/4$  of its wavelength.

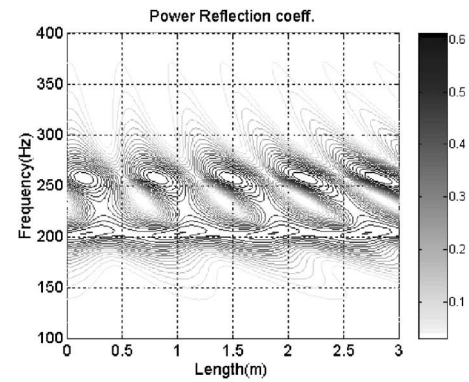
Figure 11 shows that the front resonator reflects sound power uniformly and the rear resonator attenuates sound power dominantly. When the arrangement is changed to 258 Hz (front) and 202 Hz (rear), the power reflection coefficient ( $R_{II}$ ) and the absorption coefficient ( $D_{II}$ ) have a symmetric shape of 202 Hz (front) and 258 Hz (rear). However, the TL is not changed in this case.

Thus, the change of distance between resonators is more important than the arrangement order of the resonators from the viewpoint of TL. These characteristics are identified by the equivalent impedance analysis, as noted earlier.

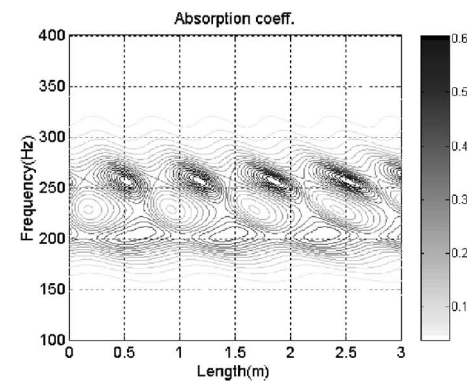
Figure 12 shows the resistance (Re) and reactance (Im) of equivalent impedance that is normalized by the acoustic impedance of the duct while varying the distance between resonators for the case of two resonators, 202 Hz (front) and 258 Hz (rear). The resistance of impedance is related to the energy dissipation and the reactance of impedance is related to the energy reflection. Figure 11 and Fig. 12 show that the absorption coefficient is high at the high resistance and the reflection coefficient is high at the zero-crossing point of the reactance.



(a)



(b)



(c)

FIG. 11. Acoustic characteristics varying the distance between resonators with 202 Hz (front) and 258 Hz (rear). (a) Power transmission coefficient; (b) power reflection coefficient; (c) absorption coefficient.

### D. Effective arrangement of resonators

According to the equivalent impedance analysis, we can determine the optimal distance between resonators to obtain a high TL in the objective frequency band.

In the model shown in Fig. 3 ( $i=1$ ; 257 Hz,  $i=2$ ; 297 Hz,  $i=3$ ; 361 Hz,  $i=4$ ; 413 Hz), the distance between resonator's hole can be determined as

$$L_i = \frac{\frac{\lambda_i}{4} + \frac{\lambda_{i+1}}{4}}{2} = \frac{c \left( \frac{1}{f_i} + \frac{1}{f_{i+1}} \right)}{8}. \quad (24)$$

Figure 13 shows a comparison of TL curves when the distances of the resonators are 100 mm and  $\lambda/4$ .

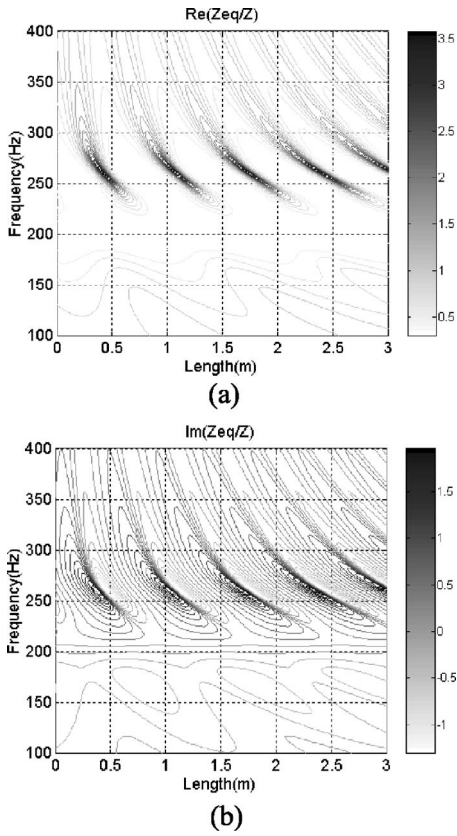


FIG. 12. Equivalent impedance varying the distance between resonators with 202 Hz (front) and 258 Hz (rear). (a) Resistance; (b) reactance.

When the distances between resonators are  $\lambda/4$ , the TL has a higher value in the objective frequency band. But the silencer must have a long length in order to effectively control noise at low frequency because the wavelength is relatively long. Therefore we intend to propose a new design method that optimizes the arrangement of the resonators.

## V. DESIGN PROBLEM

### A. Problem definition

The silencer using array resonators has many design parameters (volume of cavity:  $W \times H \times L_i$ , neck length of resonator:  $l$ , hole diameter of resonator:  $D$ , distance between reso-

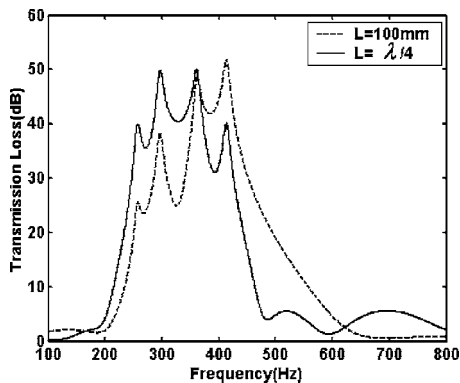


FIG. 13. Transmission loss of panel-type silencer when the distances between resonators are 100 mm and  $\lambda/4$  ( $N:8$ ,  $l:25$  mm,  $D:10$  mm,  $S:50 \times 50$  mm<sup>2</sup>).

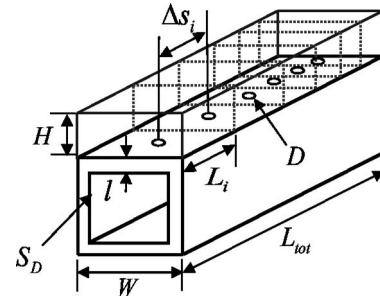


FIG. 14. The silencer model for the minimization of the design parameters ( $W \times H \times L_i$ : volume of cavity,  $l$ : neck length of resonator,  $D$ : hole diameter of resonator,  $\Delta s_i$ : distance between resonators,  $L_{tot}$ : total length of silencer).

nators:  $\Delta s_i$ ). If all design parameters are considered, the silencer model becomes very complex. Therefore, it is necessary to minimize the design parameters. Figure 14 illustrates the silencer model that minimizes the design parameters. Here  $l$ ,  $W$ ,  $H$ , and  $D$  can be fixed by geometric shape.

The resonance frequency ( $f_i$ ) can be expressed as

$$f_i = \frac{c}{2\pi} \sqrt{\frac{\pi D^2/4}{(l + 0.75D)WHL_i}} \quad (i = 1, 2, \dots, n), \quad (25)$$

where  $n$  is the number of resonators in the serial direction.

Thus the function of TL can be defined as

$$TL = \text{fun}(L_i, \Delta s_i). \quad (26)$$

From Eq. (25),  $L_i$  is expressed in terms of  $f_i$  as follows:

$$L_i = \frac{c^2 \cdot D^2}{16\pi(l + 0.75D)W \cdot H \cdot f_i^2} = \frac{A}{f_i^2} \quad \left( A = \frac{c^2 \cdot D^2}{16\pi(l + 0.75D)W \cdot H} \right), \quad (27)$$

where  $A$  is constant.

The distance between holes ( $\Delta s_i$ ) also can be expressed in terms of  $f_i$  on the assumption that the resonator's hole is located at the center of the cavity:

$$\Delta s_i = \frac{L_i + L_{i+1}}{2} = \frac{A/f_i^2 + A/f_{i+1}^2}{2} = \frac{A(f_i^2 + f_{i+1}^2)}{2f_i^2 \cdot f_{i+1}^2}, \quad (28)$$

From Eqs. (26)–(28), the function of TL can be represented as

$$TL = \text{fun}(f_1, f_2, f_3, \dots, f_n). \quad (29)$$

If the total volume of all resonators is given, the constraint of the silencer design can be expressed as

$$\sum_{i=1}^n L_i = L_{tot}. \quad (30)$$

Using Eq. (27), this constraint can be rewritten as

$$\sum_{i=1}^n \frac{A}{f_i^2} = L_{tot}. \quad (31)$$

Thus, we can obtain the multivariable optimization problem in terms of resonance frequency with a total volume constraint.

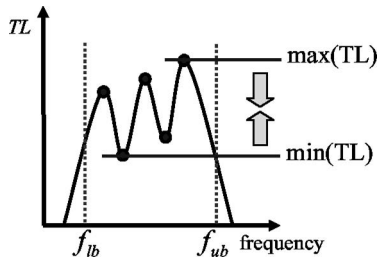


FIG. 15. The objective function for optimization ( $f_{lb}$ : low bound frequency,  $f_{ub}$ : upper bound frequency).

## B. Optimization

For the desired TL, it is necessary to select a suitable objective function. The determination of the objective function for the design of TL shape is very important.

The optimization problem for TL shape having broadband characteristics at low frequency can be expressed as

minimize  $\max(TL) - \min(TL)$ :objective function

subject to  $\sum_{i=1}^n \frac{A}{f_i^2} = L_{tot}$ :equality constraint

$f_{lb} \leq f_i \leq f_{ub}$ :bound constraint. (32)

Figure 15 illustrates the objective function in Eq. (32).

When the bound constraint is  $300 \text{ Hz} \leq f_i \leq 550 \text{ Hz}$  and each parameter is  $S_D = 50 \times 50 \text{ mm}^2$ ,  $W = 70 \text{ mm}$ ,  $H = 30 \text{ mm}$ ,  $D = 10 \text{ mm}$ ,  $l = 10 \text{ mm}$ ,  $L_{tot} = 30 \text{ mm}$ , the results of optimization are as presented in Fig. 16.

Figure 16 shows that each resonance frequency is 336, 342, 358, 385, 414, 448, and 481 Hz, and the difference between the maximum and minimum of total TL is less than 4.7 dB.

From these results, we can obtain various TL characteristics by selecting different objective functions under constraints. Finally, this optimization method shows that it is possible to design a transmission loss shape of a silencer using array resonators.

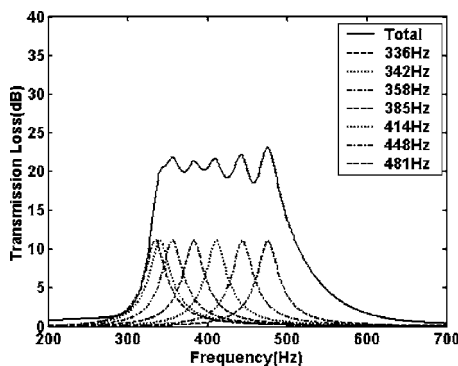


FIG. 16. The result of optimization for the broadband TL shape.

## VI. CONCLUSION

Helmholtz resonators are used to reduce noise in the narrow frequency band. The transmission loss characteristics of a silencer through the use of many resonators are obtained by theoretical and experimental results. The results show that it is possible to design a silencer to have broadband characteristics through serial and parallel arrangements of resonators at low frequency.

The change of acoustic characteristics of the silencer model can be explained by using an equivalent impedance analysis. Considering the equivalent impedance analysis, the change of the distance between resonators is more significant than the arrangement order of resonators from the viewpoint of TL. As a result, it has a periodic characteristic of  $\lambda/2$  as a whole and the transmission loss has a maximum value when the distance between resonators is  $\lambda/4$  of its wavelength.

In this study we propose a new design method that optimizes the arrangement of resonators for a TL shape that has broadband characteristics in low frequency. Thus we can obtain various transmission loss shape by selecting a suitable objective function under given constraints.

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