Moving frame technique for planar acoustic holography

Hyu-Sang Kwon and Yang-Hann Kim

Center for Noise and Vibration Control, Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Science town, Taejon, 305-701, Korea

(Received 24 October 1996; accepted for publication 22 December 1997)

Acoustic holography is one of the best methods to visualize sound fields. The quality of the visualized sound is primarily determined by the size of the hologram, its microphone spacing, and the number of microphones. This paper describes a way to virtually increase the hologram size and the spatial resolution of the holograph. For a stationary sound field, the method continuously sweeps the sound field by a line array of microphones. For moving sound sources, radiating sound is measured by using a line array of microphones fixed in space. In both cases, the measured signals have Doppler effects. The theoretical formulation has been systematically addressed by employing a moving coordinate which has relative motion between the measurement coordinate and the hologram coordinate. Simulations and experiments support the proposed theory. The drawback is that the method is only applicable to discrete frequencies. © 1998 Acoustical Society of America.

PACS numbers: 43.20.Fn, 43.35.Sx [ANN]

INTRODUCTION

The beauty of the acoustic holograph method is that one can construct an entire sound field by only measuring sound pressure on a hologram plane. This enables one to see every detail of the sound field of interest. These include not only the pressure distribution but also all other acoustic variables such as velocity, kinetic/potential energy, and intensity distribution over any surface of interest. However, limitations also exist. These are mainly due to the finite size of the hologram plane and microphone spacing; for details, see Refs. 1 and 2. The former can be regarded as a two dimensional window effect on the hologram, and the latter is associated with spatial sampling. These limitations cannot be completely avoided.

In order to get a better holograph, one simply needs a larger hologram size and smaller microphone spacing. For stationary sound fields, one can increase the size of the hologram as well as the microphone spacing by simply scanning an array microphone over the hologram plane. An efficient method of this kind was introduced by Hald.3 It uses a reference microphone or microphones, depending on the number of coherent sources, and a line array of microphone which scans over the hologram plane. This method works only for a non-moving sound source which is emitting a stationary sound field. The sound field must be stationary for each measurement set. The scanning in this case cannot be performed continuously. This is done at discrete positions which one could call ‘‘step-by-step’’ scanning.

This paper addresses a scanning method which is similar to that of the original acoustic holograph4 and the step-by-step scanning technique but generalizes them so that one can also use it for the visualization of a sound field induced by moving noise sources. For a stationary sound field, the method continuously scans over a desired measurement plane. The line array microphone travels over the measurement plane at a constant speed. For moving sources, the method measures sound pressure by a line array which is fixed in space. In both of these cases there is relative velocity between the array and sources. This velocity will introduce a Doppler shift to the measured sound pressure signals.

This paper addresses a procedure to handle this well known effect on acoustic holograms. The moving frame technique is suggested for this purpose. This essentially takes into consideration three different coordinate systems; the absolute coordinate, the hologram coordinate, and the microphone coordinate.

I. THEORETICAL BACKGROUND OF MOVING FRAME TECHNIQUE

A general configuration of hologram measurement by the moving frame technique is depicted in Fig. 1. Three co-

---

FIG. 1. Coordinate systems (absolute coordinate, microphone coordinate, and hologram coordinate) and their relative motions which are determined by the locations of source and microphone array.
ordinates are employed in this measurement system. One is the “absolute coordinate” which is fixed in space, the other one is the “microphone coordinate” which is allowed to move in space, and the last one is the “hologram coordinate.” The hologram coordinate must move with the same speed as the sound sources.

We denote these coordinate systems as

\[(x, y, z): \text{absolute coordinate system},\]
\[(x_m, y_m, z_m): \text{microphone coordinate system},\]
\[(x_h, y_h, z_h): \text{hologram coordinate system}.\]

Sound pressure on each coordinate system can be written as

\[p(x, y, z; t), \quad p_{\text{mic}}(x_m, y_m, z_m; t), \quad \text{and} \quad p_{\text{hol}}(x_h, y_h, z_h; t).\]

The subscript mic and hol express microphone and hologram coordinate system respectively. We assume that all coordinates are in parallel and that their origins are at the same location at \(t = 0\). Then, at arbitrary time \(t\),

\[
y = y_m = y_h, \quad (1a) \\
z = z_m = z_h, \quad (1b) \\
x = u_m t + x_m, \quad (1c) \\
x = u_h t + x_h. \quad (1d)
\]

The relative velocity between the microphone coordinate and the hologram coordinate; \(u_{m/h}\), is

\[u_{m/h} = u_m - u_h.\]  \tag{2}

Then, Eqs. (1c) and (1d) give the relation between \(x_h\) and \(x_m\). That is

\[
\begin{align*}
\text{FIG. 2. Procedure to calculate the hologram from the measured pressure signal in moving frame technique.}
\end{align*}
\]

\[
\text{FIG. 3. Simulation configuration; microphone moves in the } x \text{ direction, 0.3 m away from the source plane.}
\]

If one assumes that the microphone array is rigidly attached to the microphone coordinate as shown in Fig. 1, then the pressures measured by the microphone and hologram coordinate will be

\[ p_{\text{mic}}(x_m, y_m, z_H; t) = p_{\text{hol}}(x_m + u_m/t, y_h, z_H; t). \]  

Note that one can always make \( x_m \) zero in most cases, therefore Eq. (4) can be simplified to

\[ p_{\text{mic}}(0, y_m, z_H; t) = p_{\text{hol}}(u_m/t, y_h, z_H; t). \]  

This equation essentially allows us to construct acoustic pressure on the hologram plane.

A hologram expresses a spatial distribution of pressure with respect to frequency; that is, the temporal Fourier transform of \( p_{\text{hol}}(x_h, y_h, z_H; t) \). This cannot be obtained by taking the temporal Fourier transform of Eq. (5). Equation (5) provides the relationship between measured pressure signals with regard to the microphone and hologram coordinates. It is noteworthy that the first argument of pressure on the hologram is time dependent. In other words, the measured signal \( p_{\text{mic}} \) is Doppler shifted. This effect can be easily dealt with if sound pressure propagates in one direction only.

However, it will not be straightforward if the pressure field of interest is more than two dimensional. For example, directivity pattern of sources could be dipole or quadrupole even if these have the same temporal frequency.

Progress comes from the realization that the wave number spectrum could express this effect in a more explicit form. This is simply because the wave number transform does not explicitly have anything to do with time; it has to do with space. Mathematical derivations associated with this idea are presented in the following.

First one could write,

\[ p_{\text{hol}}(u_m/t, y_h, z_H; t) \]

\[ = F_F^{-1}\{p_{\text{hol}}(u_m/t, y_h, z_H; f_h)\} \]

\[ = \int_{-\infty}^{\infty} p_{\text{hol}}(u_m/t, y_h, z_H; f_h) e^{-j2\pi f_h t} \, df_h. \]  

Then the temporal Fourier transform of Eq. (6), in other words, the hologram is...
Fouriers transforms signal around 440 Hz.

Equation (9) can be simplified by using the property of the delta function, that is

\[ F_{\hat{R}}\{ p_{\text{hol}}(u_{mlt}, y_h, z_H; t) \} = \frac{1}{u_{mlt}} \int_{-\infty}^{\infty} \hat{P}_{\text{hol}}(\frac{2\pi(f_{hi}-f)}{u_{mlt}}, y_h, z_H; f_h) \, df_h. \]  

This equation forms the basis of the proposed “moving frame technique” method. The left-hand side of Eq. (10) is the temporal Fourier transform of sound pressure with respect to the microphone coordinate, or measured signals [Eq. (5)]. The integrand of the right-hand side of the equation is the Doppler shifted wave number spectrum in the z direction. The inverse Fourier transform of the wave number spectrum is what we want. This is the sound pressure distribution of a selected frequency on a plane of interest. Therefore, one can get a desired hologram at \( z = z_H \) by using Eq. (10).

However, one must note that the equations are valid only for discrete frequency. Therefore, this method has very severe drawbacks compared with other conventional method. This is what the method has to pay for greatly reducing the number of microphones. This method uses a line array, for example, \( N \) sensors, instead of \( N \times N \) sensors as the conventional method does. Next, the issues associated with the discrete frequency will be addressed.

If one has a sound field generated by a discrete frequency; single frequency \( f_{hi} \), then Eq. (10) can be simply written as

\[ F_{\hat{R}}\{ p_{\text{hol}}(u_{mlt}, y_h, z_H; t) \} / \Delta f_h = \frac{1}{u_{mlt}} \hat{P}_{\text{hol}}(\frac{2\pi(f_{hi}-f)}{u_{mlt}}, y_h, z_H; f_{hi}) . \]  

Figure 2 illustrates the way in which Eq. (11) could be implemented in practice. If one has multiple, discrete frequency components, then Eq. (10) reduces to

\[ F_{\hat{R}}\{ p_{\text{hol}}(u_{mlt}, y_h, z_H; t) \} / \Delta f_h = \frac{1}{u_{mlt}} \sum_{i=1}^{N} \hat{P}_{\text{hol}}(\frac{2\pi(f_{hi}-f)}{u_{mlt}}, y_h, z_H; f_{hi}) . \]
As illustrated in Fig. 2, it is possible to have a frequency band which is common to the discrete frequencies. In other words, side band overlapping can occur (Fig. 2). The way to avoid this undesirable, side band overlapping follows.

Let's assume that we have \( N \) frequencies whose magnitude distribution is:

\[
\begin{align*}
 f_{h_1} &< f_{h_2} < \cdots < f_{h_{i-1}} < f_{h_i} < f_{h_{i+1}} < \cdots < f_{h_N}.
\end{align*}
\]

Corresponding wave numbers will be

\[
k_i = \frac{2 \pi f_{h_i}}{c},
\]

where \( c \) is the speed of sound propagation.

It is well known that the wave number in the \( x \) direction, \( k_x \), has to be less than two times the free space wave number to avoid spatial aliasing, that is

\[
-2k_i \leq k_x \leq 2k_i \quad (13)
\]

or,

\[
-4 \pi f_{h_i} \leq k_x \leq 4 \pi f_{h_i} \quad (14)
\]

and the relation between \( k_i \) and frequencies for the moving frame system is

\[
\frac{2 \pi (f_{h_i} - f)}{u_m/h} = k_i.
\]

Therefore Eq. (13) can be rewritten as

\[
-4 \pi f_{h_i} \leq \frac{2 \pi (f_{h_i} - f)}{u_m/h} \leq 4 \pi f_{h_i} \quad (15)
\]

Rearranging this equation with respect to \( f \) gives

\[
(1 - 2M)f_{h_i} \leq f \leq (1 + 2M)f_{h_i} \quad (16)
\]

where \( M = \frac{u_m/h}{c} \) is a Mach number.

For the adjacent discrete frequency components, one can write Eq. (16) as

\[
(1 - 2M)f_{h_{i-1}} \leq f \leq (1 + 2M)f_{h_{i-1}}
\]

and

\[
(1 - 2M)f_{h_{i+1}} \leq f \leq (1 + 2M)f_{h_{i+1}}.
\]
Equations (16) and (17) essentially express how the discrete frequency component spreads out in the frequency domain due to the Doppler shift. It is now clear that the upper boundary of frequency spreading by $f_{hi}$ must not be overlapped by the lower boundary of $f_{hi+1}$, and the lower boundary by $f_{hi}$ is not also allowed to be overlapped by that due to $f_{hi-1}$. In other words

$$(1 - 2M)f_{hi+1} > (1 + 2M)f_{hi}$$

and

$$(1 - 2M)f_{hi} > (1 + 2M)f_{hi-1}$$

or,

$$f_{hi+1} > \frac{1 + 2M}{1 - 2M} f_{hi} \quad \text{and} \quad f_{hi-1} < \frac{1 - 2M}{1 + 2M} f_{hi}.$$ 

One should also note that Eq. (18) determines a Mach number to be less than 0.5. This restriction comes from the condition to avoid spatial aliasing (Eq. (13)). For example, for a noise source moving with the speed of $M=0.1$, which is about 123 km/h, Eq. (18) determines

$$f_{hi+1} > 1.5f_{hi} \quad \text{and} \quad f_{hi-1} < 0.67f_{hi}$$

FIG. 8. Predicted results of moving array experiments by using 2 identical speakers which radiate a pure tone of 500 Hz. Comparison with conventional hologram measurement techniques; spatial distribution of pressure magnitude on source plane, including that of $y=0$ line. (a) Simultaneous measurement method, $6 \times 5$ points (spacing spacing 10 cm $(x)$, 10 cm $(y)$). (b) Step-by-step scanning method, $16 \times 16$ points (sampling spacing 3 cm $(x)$, 3 cm $(y)$). (c) Moving frame technique (velocity=0.204 m/s), $16 \times 16$ points (sampling spacing 3 cm $(x)$, 2.80 cm $(y)$). (d) Moving frame technique (velocity=0.448 m/s), $16 \times 16$ points (sampling spacing 3 cm $(x)$, 2.80 cm $(y)$).
which is an acceptable requirement in practice.

We now summarize this section and briefly address unsolved problems which will affect the performance of the proposed method when it is applied to practical problems.

We have found the relationship between sound pressure on the hologram coordinate and the microphone coordinate [Eq. (10)]. This enables one to either sweep a microphone array over a stationary sound field or to allow sound sources to move with constant speed by keeping the array stationary in space. The desired acoustic pressure on the hologram can be obtained in both cases. A condition to avoid undesirable errors which are associated with side band overlapping due to the Doppler effect has been obtained in Eq. (18). When one has a discrete spectrum which violates the condition, or a discrete spectrum with continuous background spectrum, the proposed method will be plagued by errors. These errors may be controlled by the relative magnitude between the discrete frequency component’s and that of side band’s. Further theoretical development is required to apply the method to more general cases.

II. SIMULATIONS AND EXPERIMENTS

Numerical simulations have been performed to demonstrate what we have proposed in the previous section. Monopole and dipole sources have been used for the simulations. We assume that the sources are located on the source plane which is defined as \( z = 0 \), at the fixed coordinate, \((x, y, z)\) (see the details in Fig. 3). As illustrated in the same figure, a microphone was assumed to be located 0.3 m away from the source plane and be moving along the \( x \) axis with Mach number, 0.01. A hologram was constructed based on the pressure measured by the moving microphone for 1 s: The sampling rate was 1/2048 s, providing 2048 data points.

In the first instance, a monopole is located at the \((0, m, 0, m)\) position and radiates the sound of 340 Hz. Figure 4(a) shows the variation of pressure magnitude and frequency due to the Doppler effect. The Fourier transform of this signal with respect to time is shown in Fig. 4(b). As explained in the previous section, we have the signal distributed over a band of frequency, [Fig. 4(b) and (c)], whose center frequency is 340 Hz. If one examines the frequency representation of the signal [Fig. 4(c)], then one may realize that it has a similar shape as the wave number spectrum of the monopole sound source, shifted by 340 Hz. This is an expression of Eq. (10), which says that the temporal Fourier transform of the measured signal will be same as its wave number spectrum but shifted by \( f_h \), due to the Doppler effect.

To have general confidence on the proposed method, a more complicated case was also examined (Fig. 5). Four types of sources having different frequencies construct the pressure signal:

- a monopole of 340 Hz located at \((0, 0, m, 0, m)\),
- an \( x \)-polarized dipole of 390 Hz located at \((0, 0, m, 0, m)\),
- a \( z \)-polarized dipole of 680 Hz located at \((0, 0, m, 0, m)\), and
- two monopoles of 440 Hz located at \((0, m, 0, m, 0)\), and \((0.5, m, 0.5, m, 0)\).

Figure 5(a) and (b) shows the time signal, depicted by the moving microphone, and its temporal Fourier transform. As one can see, the Doppler shifted spectrum are so well separated that frequency band overlappings are not possible. It is also interesting to note that the details of each Doppler shifted spectrum [Fig. 5(c) to (f)] mimic corresponding wave number spectra.

Based on the moving frame technique, the holograms were constructed and compared with true holograms. As one would expect, a perfect match is obtained. Some of the typical results are shown in Fig. 6.

The experiment, which is illustrated in Fig. 7, was conducted to compare the proposed moving microphone method with other existing methods: the conventional holograph method and the step-by-step scanning method [i.e., Spatial Transformation of Sound Fields (STSF)]. The microphone array (Fig. 7) moves along the array guide rail (Fig. 7) with speed which is controlled by an electric motor (Fig. 7). A string, pulley system (Fig. 7) provides the desired, vertical movement of the array. Ten photo electric sensors, each separated by 5 cm, are used to measure the velocity of the array. Two identical speakers which have no outer casing (Fig. 7) were positioned. Then the speakers were driven by 500 Hz pure tone. Typical experimental results are summarized in Fig. 8. Four different types of holograms were obtained for comparison. One is the hologram, \( 6 \times 5 \) points measurement, in the \( x \) and \( y \) directions respectively [Fig. 8(a)]. The others were the holograms created by using the step-by-step scanning method [Fig. 8(b)], and two cases were obtained by the moving frame method [Fig. 8(c) and (d)]. Compared with Fig. 8(a) the other methods give better resolution. This is simply because the methods increase the effective number of microphones, therefore increase spatial resolution. The difference between Fig. 8(b) and (c) and (d) is that the former measures sound fields by conventional scanning techniques, but the latter two use the moving array method. Comparing the magnitude of pressure along \( y = 0 \) lines clearly shows that the moving frame method recovers the desired hologram from the Doppler shifted wave number spectrum [Eq. (10)].

III. SUMMARY AND CONCLUDING REMARKS

The moving frame technique to measure the hologram was introduced. This method considers the relative movement between the source/hologram and the measuring microphone array. Theoretical formulation demonstrates how the idea of utilizing the wave number spectrum constructs the desired hologram. Numerical simulations and experiments demonstrate that the proposed method is valid and has great potential in various applications.

ACKNOWLEDGMENT

This work has been partially supported by the grant of KOSEF (Korea Science and Engineering Foundation).