Rigid-plastic finite-element analysis of sheet-metal forming processes using a selective membrane/shell formulation

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Industrial Summary

In most sheet-metal forming processes a large part of the deforming material exhibits stretch-dominant characteristics and the bending effect is found to exist only in a limited portion of the total deforming region. Thus the use of shell or continuum elements for the whole region is not economic. A selective membrane/shell (M/S) scheme is proposed which combines the layered degenerated shell elements with membrane elements. In the proposed scheme, membrane elements and layered degenerated shell elements are employed selectively according to the characteristics of the deformation, using the suggested criterion to consider the bending effect. The selective M/S scheme is introduced into rigid-plastic finite-element analysis for plane-strain, axisymmetric and three-dimensional problems. The computational results are then compared with the results of experiment, the comparison showing that with the selective M/S scheme the bending effect can be considered effectively whilst the computational time is reduced significantly.

Notation

A finite-element assembly operator

\[ B_{Li}, B_{NLi} \] matrix relating Lagrangian strain with nodal displacement for linear and non-linear parts, respectively

\[ E_{ij} \] covariant component of the Lagrangian strain tensor

\[ H_1 \] shape function matrix for coordinates

\[ H_2 \] shape function matrix for displacements

\[ K \] stiffness matrix

\[ m \] index used in Hill's new yield criterion

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I. Introduction

In sheet-metal working, the effect of bending is usually small as compared with the effect of stretching and many sheet-metal working processes have been analysed successfully by membrane analysis, in which the bending effect is neglected. In the deep-drawing process, bending occurs near to the die corner as the material slides into the die cavity along the curved profile of the die corner and then as it is straightened after have passed over the curved profile. Therefore, in analysing such a process, the effect of bending should be considered properly.

The rigid-plastic finite element method is now one of the most powerful numerical techniques for the analysis of sheet-metal forming, due to its numerical efficiency. Kim and Kobayashi [1,2] introduced the finite-element method into the analysis of axisymmetric sheet-metal forming processes. Later, the three-dimensional rigid-plastic finite-element method based on membrane theory was applied successfully to the analyses of various sheet-metal forming processes [3–8]. Recently, some analyses of three-dimensional processes by the elastic-plastic finite-element method based on membrane theory have been reported [9–15]. On the other hand, elastic-plastic analyses using shell elements [16–19] or continuum elements [20–22], in which the bending effect is considered, have been reported.

Most finite-element analyses have employed only one type of element amongst membrane-, continuum- and shell-elements in the finite-element mesh system, depending on the problem. Recently, Onate and Saracibar [23] and Sosnowski et al. [24] introduced a selective bending/membrane approach based on the viscous shell formulation. They applied the scheme to hemispherical punch stretching and superplastic forming under the plane-strain condition. They introduced the parameter of energy ratio as a criterion of change from bending elements to membrane elements.
Kawka and Makinouchi [25] simulated the square-cup drawing test without a blank holder by the combined use of membrane-, shell- and continuum-elements. In their analysis, change of element type during deformation does not occur.

In the present study, a selective M/S scheme is introduced for the rigid-plastic finite-element analyses of plane-strain, axisymmetric and three-dimensional problems. In the proposed method, layered degenerated shell elements are employed in the region where the bending effect cannot be neglected, in combination with membrane elements in the region where the bending effect is virtually negligible. The criterion for the change from membrane elements to shell elements is introduced according to the nature of deformation.

Plane-strain stretching by a rectangular punch and cylindrical-cup deep-drawing are analysed to examine the validity of the present method and the computed results are compared with available results [19]. As a three-dimensional case, the deep-drawing of a square cup from a circular blank is analysed also. The application and the limitation of the proposed method are then discussed.

2. Theory

2.1. Rigid-plastic finite-element method

Kim and Yang [26] derived a general incremental formulation for the large deformation problem of rigid-plastic materials. The method incorporates geometric and material non-linearity based on the membrane theory and employs Hill's quadratic yield theory. The approach was extended to cover planar anisotropy in Hill's quadratic yield theory [27] and Hill's new yield theory by Chung et al. [7]. Yang et al. [8] treated the contact condition by using the skew boundary condition. In this study, the foregoing formulations [7,8] are used as a basis for further development. Hill's new yield criterion for a normal anisotropic sheet metal is given by

\[
2(1 + r)|\sigma|^m = (1 + 2r)|\sigma_1 - \sigma_2|^m + |\sigma_1 + \sigma_2|^m. 
\]  

(1)

A boundary problem is considered which determines the displacement field of a rigid-plastic sheet metal in three-dimensional space under the given boundary conditions. The configuration of the sheet surface and the distribution of the effective strain are known at time \( t \). The necessary and sufficient condition for the stress field for the rigid plastic sheet metal to be in equilibrium at time \( t + \Delta t \) is given from the virtual work principle [28]:

\[
\delta W = \int_{\Omega} S^i j \delta E_{ij} \, dV, 
\]  

(2)

where \( S^i j \) is a contravariant component of the second Piola–Kirchhoff stress tensor and \( E_{ij} \) is the Lagrangian strain tensor during one step. During one step, the assumption of a minimum energy path is introduced by using the principal convected
coordinate system. For a rigid-plastic incompressible sheet metal, the necessary and sufficient condition for the stress field to be in equilibrium at time $t + \Delta t$ can be given as follows [7]:

$$\delta W = \int_{V_0} \sigma \delta (\Delta \varepsilon) \, dV.$$  \hfill (3)

2.2. Degenerated shell element formulation

The element behavior is based on the assumption that lines that are originally normal to the mid-surface of a shell element remain straight during element deformation, transverse normal stress being neglected (Fig. 1). In addition to these common assumptions for shell elements, the transverse shear stress is assumed to be absent (Fig. 2). To take into account the change of the material property through the thickness direction, the elements are divided into several layers. In other words, a state of plane stress is assumed at each layer.

The coordinates of a material point at time $t$ and $t + \Delta t$ are given, respectively, in the following form [29]:

$$X = \sum_{k=1}^{m} h_k x^k + \frac{\theta^3}{2} \sum_{k=1}^{m} a_k h_k V_n^k,$$

$$x = \sum_{k=1}^{m} h_k x^k + \frac{\theta^3}{2} \sum_{k=1}^{m} a_k h_k u_n^k,$$  \hfill (4, 5)

where $a_k$ is the thickness at node $k$. The displacement $u$ during $\Delta t$ is given by

$$u = x - X.$$  \hfill (6)
Substituting Eqs. (4) and (5) into Eq. (6),

\[ u = \sum_{k=1}^{m} h_k u_k + \frac{\theta^3}{2} \sum_{k=1}^{m} \alpha_k h_k (v_n^k - V_n^k), \]  

(7)

where \( V_n^k \) and \( v_n^k \) are unit vectors normal to the shell mid-surface in the direction \( \theta^3 \) at the \( k \)th nodal point at \( t \) and \( t + \Delta t \), respectively.

Let \( \beta_1^k \) and \( \beta_2^k \) be the rotations of the corresponding normal vectors about the vectors \( V_1^k \) and \( V_2^k \) from the configuration at time \( t \) to the configuration at time \( t + \Delta t \). Then the following approximation is made for small incremental angles \( \beta_1^k \) and \( \beta_2^k \),

\[ v_n^k - V_n^k = -V_2^k \beta_1^k + V_1^k \beta_2^k. \]  

(8)

Substituting Eq. (8) into Eq. (7), the incremental displacements for an internal element are obtained in terms of incremental displacements and rotations of a nodal point,

\[ u = \sum_{k=1}^{m} h_k u_k + \frac{\theta^3}{2} \sum_{k=1}^{m} \alpha_k h_k ( -V_2^k \beta_1^k + V_1^k \beta_2^k). \]  

(9)

\( X_i \) and \( x_i \) are the orthogonal Cartesian coordinates of a material point at time \( t \) and \( t + \Delta t \), \( u_i \) are the orthogonal Cartesian components of the displacement vector of the material point during \( \Delta t \), as shown in Fig. 3.

Since the foregoing expressions are based on an arbitrary convected coordinate system and the natural coordinates are also a convected coordinate system [7], the natural coordinates \((\theta_1, \theta_2)\) in Fig. 4 can be used as the convected coordinates without
loss of generality. In the natural convected coordinate system, the Lagrangian strain is then given by

$$E_{\alpha \beta} = \frac{1}{2} \left( \frac{\partial u_\alpha}{\partial \theta^\beta} \frac{\partial X_i}{\partial \theta^\alpha} + \frac{\partial u_\beta}{\partial \theta^\alpha} \frac{\partial X_i}{\partial \theta^\beta} + \frac{\partial u_\alpha}{\partial \theta^\beta} \frac{\partial u_\beta}{\partial \theta^\alpha} \right).$$

(10)

The Cartesian coordinates and displacements of a material point are expressed, respectively, in the following matrix form:

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = H_1 \cdot \bar{X},$$

(11)

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = H_2 \cdot \bar{U},$$

(12)

where $H_1$ is the shape function for the coordinates and $H_2$ is the shape function for the displacements, and $\bar{X}$ and $\bar{U}$ are nodal point coordinates and displacement vectors, expressed as follows:

$$\bar{X}^T = \{X_1, X_2, X_3, \ldots, X_1^n, X_2^n, X_3^n\}^T,$$

$$\bar{U}^T = \{u_1, u_2, u_3, \beta_1^n, \beta_2^n, \ldots, u_1^n, u_2^n, u_3^n, \beta_1^n, \beta_2^n\}^T.$$
The variation of the Lagrangian strain can be expressed as follows:

\[
\delta E = \begin{bmatrix}
\delta E_{11} \\
\delta E_{22} \\
\delta E_{12}
\end{bmatrix} = \begin{bmatrix}
\delta \bar{U}^T (B_{L1} \bar{X} + B_{NL1} \bar{U}) \\
\delta \bar{U}^T (B_{L2} \bar{X} + B_{NL2} \bar{U}) \\
\delta \bar{U}^T (B_{L3} \bar{X} + B_{NL3} \bar{U})
\end{bmatrix}.
\]  

(13)

\(B_{L1}, B_{L2}, B_{L3}\) are non-symmetric matrices, given by

\[
B_{L1} = \frac{\partial H_1^T}{\partial \theta^1} \frac{\partial H_1}{\partial \theta^1},
\]

\[
B_{L2} = \frac{\partial H_2^T}{\partial \theta^2} \frac{\partial H_1}{\partial \theta^2},
\]

\[
B_{L3} = \frac{1}{2} \left( \frac{\partial H_2^T}{\partial \theta^1} \frac{\partial H_1}{\partial \theta^2} + \frac{\partial H_2^T}{\partial \theta^2} \frac{\partial H_1}{\partial \theta^1} \right).
\]  

(14)

\(B_{NL1}, B_{NL2}, B_{NL3}\) are all symmetric matrices, given by

\[
B_{NL1} = \frac{\partial H_1^T}{\partial \theta^1} \frac{\partial H_2}{\partial \theta^1},
\]

\[
B_{NL2} = \frac{\partial H_2^T}{\partial \theta^2} \frac{\partial H_2}{\partial \theta^2},
\]

\[
B_{NL3} = \frac{1}{2} \left( \frac{\partial H_2^T}{\partial \theta^1} \frac{\partial H_2}{\partial \theta^2} + \frac{\partial H_2^T}{\partial \theta^2} \frac{\partial H_2}{\partial \theta^1} \right).
\]  

(15)
The effective strain increment $\Delta \bar{\varepsilon}$ during one step is expressed as a function of invariants of the Lagrangian strain tensor using the assumption for a deformation path. From the principle of virtual work and applying it to the procedure of degenerated shell element formulation, the following linearized finite-element equation is obtained and is solved by the well-known Newton–Raphson method

$$K\Delta U = R.$$  \hspace{2cm} (16)

Since the iteration procedure is numerically ill-conditioned due to the assumption of rigid plastic material when the effective strain increment at some part is too small as compared with the rest and since unloading during deformation cannot be treated by the rigid–plastic finite-element method, the method by Osakada et al. [30] is employed for the proper rigid-body treatment. In this method, the effective stress is expressed as follows:

$$\bar{\sigma} = \frac{\sigma_0}{\sqrt{(d\bar{\varepsilon}^2 + d\bar{\varepsilon}_0^2)}} \cdot d\bar{\varepsilon},$$ \hspace{2cm} (17)

where $d\bar{\varepsilon}_0$ is a small constant compared to $d\bar{\varepsilon}$ and $\sigma_0$ is chosen as the effective stress at $\bar{\varepsilon}_0 = \bar{\varepsilon}_{i-1} + 5 d\bar{\varepsilon}_0$, where $\bar{\varepsilon}_{i-1}$ is the effective strain at the previous step.

2.3. Membrane element formulation

The coordinates and displacements of a material point are expressed in the following matrix form:

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = H_1 \cdot \bar{X},$$ \hspace{2cm} (18)

$$u = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = H_1 \cdot \bar{u}. \hspace{2cm} (19)$$

where $H_1$ is the shape function for the coordinates and displacements, and $\bar{X}$ and $\bar{u}$ are nodal point coordinates and displacement vectors, expressed as follows:

$$\bar{X} = \{X^1_1, X^1_2, X^2_1, \ldots, X^4_1, X^4_2, X^4_3\}^T,$$

$$\bar{u} = \{u^1_1, u^1_2, \ldots, u^4_1, u^4_2, u^4_3\}^T.$$  

Unlike degenerated shell element formulation, the shape function for the coordinates is the same as that for the displacements. The variation of the Lagrangian strain for the membrane element is derived easily by substituting $H_1$ for $H_2$ in the degenerated shell element formulation.
2.4. Selective membrane/shell formulation

In the proposed selective M/S scheme, membrane elements and layered degenerated shell elements are used in a selective manner in accordance with the local feature of sheet deformation at each deforming step; i.e., membrane elements are used in zones where the bending effect is negligible, whereas in zones where the bending effect is predominant layered degenerated shell elements are chosen. The difficulty of the present formulation lies in the assemblage of the global stiffness matrix, treatment of rotation of the transition nodes shared by different type of elements and definition of selection criterion for element type.

2.4.1. Definition of the selection criterion

The bending effect increases as the curvature increases relative to the thickness. For a ratio of thickness to radius of curvature smaller than 1/20, the bending effect appears to become negligible [10]. Onate and Saracibar [23] introduced the energy ratio parameter (ratio of membrane energy to total energy) as the criterion of change from the bending formulation to the membrane formulation. In the present study a non-dimensional geometric parameter that helps to select the type of elements according to the nature of deformation is employed. To define the criterion for change of element type, the following thickness-modified curvature at the ith element is introduced,

$$\zeta_i = t_i \sqrt{(\kappa_1)_i^2 + (\kappa_2)_i^2}. \quad (20)$$

where \( t_i \) is the thickness at the center of the ith element and \((\kappa_1)_i\) and \((\kappa_2)_i\) are principal curvatures at the center of the ith element of the surface constructed from the finite-element meshes of the sheet. Physically, \( \zeta_i \) is defined as a thickness-modified curvature and indicates the effect of thickness with respect to curvature.

If \( \zeta_i \geq \zeta_{cr} \), where \( \zeta_{cr} \) is the critical thickness-modified curvature, the effect of bending is then significant and the layered degenerated shell formulation is chosen for the region of chosen shell elements. If \( \zeta_i < \zeta_{cr} \), the effect of bending may be negligible and the membrane formulation is used. Usually, \( \zeta_{cr} \) is taken to be about one-half of 1/20, since the principal curvatures of the surface constructed from meshes are somewhat underestimated.

2.4.2. Treatment of rotation of transition nodes shared by different elements

For a better understanding of the treatment, two-dimensional sections are shown in Fig. 5, instead of the real three-dimensional sheet surface.

As shown in Fig. 5, the displacement vector of the transition node \( i \) with respect to a membrane element is \( U_M = [u_1, u_2, u_3] \) and that of the transition node \( i \) with respect to a layered degenerated shell element is \( U_S = [u_1, u_2, u_3, \beta_1, \beta_2] \). The displacement vectors at that node are not coincident with each other. Then, the rotational degrees of freedom at the transition node must be frozen for rotational continuity. This can be simply accomplished by deleting the rows and columns corresponding to the nodal rotations in the element stiffness matrix. This treatment could not introduce any
appreciable error unless the type of elements is changed frequently within the short range of the computational region.

2.4.3. Numerical Procedure

The numerical procedure of the selective M/S scheme is as follows:

(i) At the start of the deformation process the membrane elements are used for all elements in the sheet.

(ii) At each step, thickness-modified curvatures for all of the elements are calculated and the selection criterion of Eq. (20) is checked for every element.

(iii) If a change of element type is required according to the criterion at the current deformation step, the equation numbering, integration-point numbering and deformation data are adjusted according to the changed element type.

(iv) The element stiffness matrix for membrane elements, $k^e_{d}$, and for shell elements, $k^e_{s}$, are different in size. The global stiffness matrix is then assembled as follows:

$$K = \sum_{e=1}^{n_{el}} A (k^e_{d}) + \sum_{s=1}^{n_{el}} A (k^e_{s}), \quad (21)$$

where $n_{el}$ is the total number of elements, $n_{elm}$ is the total number of membrane elements and $n_{els}$ is the total number of shell elements ($n_{el} = n_{elm} + n_{els}$). After assembling, static condensation of rotational degrees of freedom is carried out at the transition nodes shared by different type of elements.

(v) The procedure continues shown in Fig. 6.
3. Results and discussion

3.1. Plane-strain stretching by a rectangular punch


These analyses showed that the effect of bending becomes increasingly larger as the ratio of the punch (or die) radius to the sheet thickness becomes small.

In the present study, plane-strain punch stretching is analyzed using the selective M/S scheme based on the rigid-plastic FEM, a corresponding experiment being carried out. Fig. 7 shows the schematic diagram of the geometry of the punch and the die.

The material and process variables used in the simulation are as follows: sheet material, cold-rolled steel: stress-strain characteristics, $\sigma = 595.15(\varepsilon)^{0.216}$ Mpa;
Lankford value for normal anisotropy, $r = 1.66$; Coulomb coefficient of friction, $\mu = 0.24$; sheet thickness, $t = 0.742$ mm.

The Coulomb coefficient of friction used in the present analysis was obtained from the wrapping-friction test [32]. During the experiment no lubricant was used between the tool and the sheet metal and the metal flow into the die cavity was limited to the minimum by using a bead. Along the central section perpendicular to the longer sides of the rectangular punch, the condition is approximately plane-strain.

In the present rigid-plastic finite-element analysis, 19 four-node membrane and layered degenerated shell elements with three layers were used. The predictions of the membrane, shell, and selective M/S analysis were compared with the results of experiment in order to show the validity of the selective M/S formulation.

Each of these analyses was carried out for a punch travel of 12 mm with 1.0 mm step size. Taking the initial deceleration factor for the Newton–Raphson method to be 0.25, the solutions for each of the analyses generally converged within 6–10 iterations for a single step, with the fractional norm of $5 \times 10^{-5}$. The computational time ratios of the membrane, shell, and selective M/S analyses were 5.75: 100.0: 29.9, respectively.

In the selective M/S analysis the membrane elements are used for the whole region at the start of the deformation process and the element type is changed for 6 elements at the punch and die corner out of the total 19 elements, at a punch displacement of 12 mm.

Fig. 8 shows the distribution of thickness-modified curvature and the deformed configuration at a punch travel of 12 mm. As shown in the figure, the thickness-modified curvature exceeds the proposed critical value, so that the element type is
changed accordingly from the membrane element to the shell. The proposed selection criterion is thus shown to be effective for any intermediate step of deformation.

Fig. 9 shows the strain distributions obtained by three kinds of analyses and the measured strain distribution at a punch displacement of 12 mm. In the membrane
analysis a peak strain does not appear at the punch edge, but for the shell analysis as well as for the selective M/S analysis, a peak strain is apparent at this location. As shown in Fig. 9, the selective M/S analysis is in very close agreement with the shell analysis, showing almost the same value of peak strain.

3.2. Deep drawing of a cylindrical cup

Deep drawing is one of the most basic processes in sheet-metal working and involves very complicated deformation mechanics. In deep drawing, most of the sheet material undergoes stretching, whilst some part of the sheet, especially around the die and punch corner profiles, is subjected to bending and unbending during the forming process. In order to examine the effect of bending in deep drawing, the deep drawing of a cylindrical cup is analysed and the corresponding experiment has been carried out. The material and process variables used in the simulations are as follows: sheet material, aluminum-killed steel; stress–strain characteristics, $\bar{\sigma} = 508.8(\varepsilon)^{0.247}$ MPa; Lankford value for normal anisotropy, $r = 1.867$; sheet thickness, $t = 0.8$ mm; blank diameter, $\phi 120.0$ mm; Coulomb coefficient of friction, $\mu = 0.24$; blank-holding force, 9800 N.

The schematic view of the process is shown in Fig. 10(a): during the experiment, the blank-holding force was held constant. Fig. 10(b) shows the finite-element mesh
Fig. 11. Distribution of thickness-modified curvature and deformed configuration in the deep drawing of a cylindrical cup ($\varepsilon_{cr} = 0.03$).

Fig. 11 shows the distribution of thickness-modified curvature and the deformed configuration along the current radial distance from the center, when the punch displacement is 30.4 mm. As shown in the figure, the thickness-modified curvature exceeds the proposed critical value, the element type being changed from membrane to layered degenerated shell according to the nature of deformation. Unbending takes place when an element passes around the die shoulder and the element type is changed automatically from membrane to shell and again from shell to membrane. The proposed selection criterion is thus shown to be effective for any intermediate step of deformation.

Figs. 12 and 13 show the summed surface-strain ($\varepsilon_1 + \varepsilon_2 = -\varepsilon_t$) distributions along the radial distance from the center obtained by the three kinds of analyses together with the measured strain distribution when the punch displacements are 30.4 and 46.0 mm, respectively. Both in the analysis using shell elements and in the selective M/S analysis, the summed surface-strain distribution differs along a different layer of layered degenerated shell. As shown in the figure, the predictions for the three kinds of analysis are almost coincident with each other for the flange part and at the punch head. The computational results are in good agreement with the results of experiment at the punch head. However, in the neighbourhood of the punch shoulder and the die shoulder, thinning appearing in the experiment can not be predicted by the
membrane analysis. The predictions of the shell analysis and the selective M/S analysis are practically coincident over the whole region and the strain distributions along the top layer at the punch shoulder are in good agreement with the results of experiment.
3.3. Deep drawing of a square cup from a circular blank

Square-cup drawing is one of the most common sheet-metal forming processes as well as being an important formability test for sheet metal. Various finite-element simulations, including rigid-plastic [4,8] and elastic-plastic [33,34] finite-element analyses of square-cup drawing have been reported. The analysis of the same process has not yet been attempted by rigid-plastic finite-element analysis using degenerated shell elements. In the present study, deep drawing of a square cup from a circular blank is analysed by the selective M/S finite-element method as well as by the shell approach for comparison with the corresponding experiment being carried out. Circular grids of 0.1 in (2.54 mm) diameter are printed by electro-chemical etching onto the sheet surface to enable the measurement of the surface strain. The material and process variables used in the simulation are as follows: sheet material, cold-rolled steel; stress–strain characteristics, $\sigma = 577.1(0.0009 + \varepsilon)^{0.274}$ MPa; index used in Hill's new yield criterion, $m = 2.0$; Lankford value for normal anisotropy, $r = 1.769$; sheet thickness, $t = 0.7$ mm; blank diameter, 90 mm; dimension of the punch, 40 mm x 40 mm; punch radius, 5 mm; punch corner radius, 3.2 mm; die opening, 42.5 mm x 42.5 mm; die radius, 5 mm; Coulomb coefficient of friction, $\mu = 0.24$ on the punch and $\mu = 0.10$ on the die; blank-holding force, 7840 N.

During the experiment no lubricant was used for the punch whilst vinyl sheet was used as the lubricant between the die and the sheet metal. The blank-holding force was held constant during deep drawing. In the present analysis, the blank-holding force is assumed to act on the nodes located at the outer rim of the flange, Coulomb forces being computed and assigned to these nodes. A schematic view of the process is shown in Fig. 14, whilst Fig. 15 shows the finite-element mesh employed, consisting of 441 nodes and 400 elements for the membrane, shell and selective M/S analyses. Each

Fig. 14. Geometry of the tooling for the deep drawing of a square cup.
Fig. 15. Finite-element mesh used in the analysis of the deep drawing of a square cup from a circular blank.

element of the layered degenerated shell is divided into three layers. Taking the initial deceleration factor for the Newton–Raphson method to be 0.25, the solution generally converged within 15 iterations for a single step, with a fractional norm of $5 \times 10^{-5}$. Analysis was carried out to a punch displacement of 20.8 mm, in order to arrive at this punch displacement, 47 steps being required for the membrane analysis and 45 steps for the shell analysis. The ratio of computation time for the shell and selective M/S analyses is 100:52.

Fig. 16 shows the distribution of thickness-modified curvature and the deformed configuration when the punch displacement is 20.8 mm. As shown in the figure, the elements near to the punch and die corner profiles have been changed from initial membrane elements to shell elements when exceeding the critical value ($\zeta_{cr} = 0.03$) in thickness-modified curvature. As in cylindrical deep drawing, unbending takes place when an element passes over the die and punch corner profiles and the element type is changed automatically from membrane to shell and again from shell to membrane. The proposed selection criterion is thus shown to be effective for any intermediate step of deformation.

Figs. 17 and 18 show the major surface-strain distributions on the outer surface along the transverse and diagonal directions of the square cup obtained by two kinds of analyses together with the measured strain distribution, when the punch displacement is 12.9 and 20.8 mm, respectively.

As shown in Fig. 17, the major-strain distribution along the transverse direction agrees reasonably well with the results of experiment, but it shows more deviation from the results of experiment than does the diagonal distribution. This is due to the sensitive response to other factors, such as friction and blank-holding force, along this
Fig. 16. Distribution of thickness-modified curvature and deformed configuration in the deep drawing of a square cup ($c_{rev} = 0.03$).

direction with less influence from geometric constraint, as compared with the diagonal direction:

Fig. 18 shows the distribution of major surface-strain along the diagonal direction for two values of punch travel. The computed surface-strain distribution agrees quite well with the experimental distribution at the punch stroke of 12.9 mm, but shows some deviation of the peak-strain value at the punch stroke of 20.8 mm: however, there is a good prediction of the position of the peak strain, which is considered to be due to the fracture of the vinyl sheet at the later stage of punch travel, resulting in greater surface friction between the sheet material and the tool. Fig. 18 shows that the agreement in tendency between the computation and experiment for the diagonal direction is generally better than that for the transverse direction, this being attributable to the geometric constraint more dominantly affecting the deformation than does the frictional coefficient or the bending effect.
Fig. 17. Comparison of the computed results with the results of experiment for the major surface-strain distribution on the outer surface along the transverse direction in the deep drawing of a square cup from a circular blank.

Fig. 18. As for Fig. 17 for the diagonal direction.
4. Conclusions

A selective M/S formulation has been proposed and applied to the plane-strain stretching, cylindrical-cup deep-drawing and square-cup deep-drawing processes. A selection criterion, defined as a non-dimensional geometric parameter that helps to select the type of elements according to the nature of deformation, has been introduced. Computations are performed for plane-strain, cylindrical-cup deep-drawing and square-cup deep-drawing. Computations are also carried out for pure membrane analysis as well as for pure shell analysis, for the sake of comparison, the corresponding experiments having been carried out in order to verify the numerical results. From the comparison of the results of experiment with the predictions of the three kinds of analyses, the shell and selective M/S analyses in plane-strain stretching have shown better agreement with the results of experiment in terms of strain distribution, whilst the predictions of the shell and selective M/S analyses for deep drawing have shown reasonable agreement with the results of experiment.

It has thus been shown that the bending effect is accommodated by the proposed formulation, with the computational time being reduced by over 50%, as compared with the computational time for the full shell analysis.

References


