LETTER

Cut-Off Rate of Multiple Antenna Systems over Frequency-Flat, Fast Fading Channels

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SUMMARY For multilevel-coded modulation, the cut-off rate of multiple antenna systems over frequency-flat, fast fading channels is derived. Following Wozencraft’s approach, a closed-form expression for the cut-off rate is obtained as a function of energy ratio per dimension. It is shown that the maximum value of cut-off rate increases linearly with the number of transmit antennas.

key words: cut-off rate, multi-input multi-output, frequency-flat fast fading

1. Introduction

Multiple antenna systems are attractive owing to their enormous capacity. While many reports dealt with the capacity improvement of multiple antenna systems from the viewpoint of information theory [1], a different approach is chosen here to derive the capacity of multiple antenna systems over frequency-flat, fast fading channels: for multilevel-coded modulation including binary-coded modulation, the cut-off rate, the maximum allowable code rate for reliable communications, was obtained as a function of energy ratio per dimension, following Wozencraft’s approach [2]. To the best of our knowledge, there has never been any report that analyzes the cut-off rate of multiple antenna systems with multilevel-coded modulation over frequency-flat, fast fading channels (Refer to [3] for frequency-flat slow fading channels). Furthermore, the application of Wozencraft’s classical approach to multiple antenna systems is unique to this paper. Here the channel state information (CSI) is assumed to be available only at the receiver [4], [5].

2. System Model

Let us consider a communication link consisting of T transmit antennas and R receive antennas that operates over frequency-flat, fast fading channels. Suppose that the transmitter modulates each K-bit sequence into a $N \times T$ transmitted signal matrix

$$S = \{s_{nt}, \ n = 1, \ldots, N, \ t = 1, \ldots, T\}$$

and, correspondingly, the receiver recovers each K-bit sequence, based on the observation of an $N \times R$ received signal matrix

$$R = \{r_{nr}, \ n = 1, \ldots, N, \ r = 1, \ldots, R\}.$$  

Here $s_{nt}$ is the transmitted signal at time $n$ and transmit antenna $t$, and $r_{nr}$ is the received signal at time $n$ and receive antenna $r$, related by the equation

$$r_{nr} = \sum_{t=1}^{T}s_{nt}h_{tr} + n_{nr}, \ n = 1, \ldots, N, \ r = 1, \ldots, R.$$  

(1)

The channel gain between transmit antenna $t$ and receive antenna $r$ at time $n$ is denoted by $h_{trn}$, and, assuming Rayleigh distribution, is independent (with respect to $t$, $r$ and $n$) with density

$$p(h_{trn}) = (1/\pi)\exp\{-|h_{trn}|^2\}.$$  

The additive white gaussian noise at time $n$ and receive antenna $r$ is denoted by $n_{nr}$, and is independent (with respect to $n$ and $r$) with density

$$p(n_{nr}) = (1/\pi N_0)\exp\{-|n_{nr}|^2/N_0\}.$$  

Note that $S$ is chosen to be one of $M = 2^K$ possible $N \times T$ signal matrices, of which the $m$-th signal matrix is denoted by

$$S_m = \{s_{mnt}, \ n = 1, \ldots, N, \ t = 1, \ldots, T\}.$$  

3. Derivation of Cut-Off Rate

Instead of attempting to find a set of $M$ signal matrices, let us consider the random selection of signal sets and the average error probability over the ensemble of signal sets [2]. Assuming $q$-level-coded modulation, there are $q^{NTM}$ possible signal sets and their corresponding communication systems, consisting of a transmitter, a channel, and an optimum receiver, illustrated in Fig. 1. For simplicity, we consider only the case where the random selection is equally probable, in other words, the probability that any specific signal set is chosen is equal to $1/q^{NTM}$. Denoting an average over the ensemble of signal sets by the overbar, the average error probability $P_E$ is bounded as

$$\overline{P_E} \leq (M - 1)P(S_m, S_m') < M \cdot P(S_m, S_m'),$$  

(2)

where $P(S_m, S_m')$ denotes the pairwise error probability, in
other words, the probability that the receiver incorrectly de-
cides $S_m$ as $S_{m'}$.

The transmitted signal is restricted to a finite number of different values, in other words, each $s_{m,n,t}$ is assigned any of $q$ values equally spaced over the interval $[-\sqrt{E_N/T}, \sqrt{E_N/T}]$, where $E_N$ denotes the energy per dimension. The set of the $q$ values of $\{s_{m,n,t}\}$, signal alphabet, is denoted by $\{a_l\}$, where

$$-\sqrt{E_N/T} = a_1 < a_2 < \cdots < a_q = \sqrt{E_N/T}. \quad (3)$$

Note that

$$P(s_{m,n,t} = a_{l_1}, \ldots, s_{m',n,t} = a_{l_2}) = \prod_{t=1}^{T} P(s_{m,n,t} = a_{l_1}, s_{m',n,t} = a_{l_2}) = \frac{1}{q^n}. \quad (4)$$

since the random selection is equally probable so that

$$P(s_{m,n} = a_{l_1}, s_{m',n} = a_{l_2}) = \frac{1}{q^n}. \quad (5)$$

Based on the derivation of pairwise error probability [4, 5], it follows that

$$P(S_m, S_{m'}) \leq \prod_{n=1}^{N} \left(1 + \frac{\sum_{t=1}^{T} (s_{m,n,t} - s_{m',n,t})^2}{4N_0}\right)^{-R}. \quad (6)$$

Defining the distance $d_{l_1,l_2}$ as

$$d_{l_1,l_2} = |a_{l_1} - a_{l_2}|,$$

it follows that

$$\prod_{n=1}^{N} \left(1 + \frac{\sum_{t=1}^{T} (s_{m,n,t} - s_{m',n,t})^2}{4N_0}\right)^{-R} = \left(\frac{1}{q^n} \sum_{l_1=1}^{q} \cdots \sum_{l_T=1}^{q} \frac{1}{q^T} \left(1 + \frac{\sum_{t=1}^{T} d_{l_1,l_{t_2}}^2}{4N_0}\right)^{-R}\right)^N. \quad (7)$$

Therefore, the cut-off rate $R_0$ is obtained as

$$R_0 = -\log_2 \left\{ \sum \left(1 + \frac{\sum_{t=1}^{T} d_{l_1,l_{t_2}}^2}{4N_0}\right)^{-R} \right\}, \quad (8)$$

and the cut-off rate for $q = 4$ is shown in Fig. 2. Note that we denote the energy ratio per dimension by $E_N/N_0$. Taking into consideration the asymptotic property

$$\lim_{N_0 \to \infty} \left(1 + \frac{\sum_{t=1}^{T} d_{l_1,l_{t_2}}^2}{4N_0}\right)^{-R} = \begin{cases} 0 & \sum_{t=1}^{T} d_{l_1,l_{t_2}}^2 \neq 0 \\ 1 & \sum_{t=1}^{T} d_{l_1,l_{t_2}}^2 = 0 \end{cases}, \quad (9)$$

it follows that

$$\lim_{N_0 \to \infty} R_0 = T \log_2 q, \quad (10)$$

since

$$\sum_{t=1}^{T} d_{l_1,l_{t_2}}^2 = 0 \quad \text{is equivalent to} \quad d_{l_1,l_{t_2}} = d_{l_2,l_{t_2}} = \cdots = d_{l_T,l_{T_2}} = 0. \quad (11)$$

4. Conclusion

The maximum value of $R_0$ increases, not only logarithmically with $q$, the size of signal alphabet, but also linearly with $T$, the number of transmit antennas. Note that, although it is independent of $R$, the number of receive antennas, it is easily shown, from (6), that the cut-off rate $R_0$ itself is increasing with $R$, as shown in Fig. 2. It again implies that multiple antenna systems are capable of achieving enormous capacity, compared with single antenna systems.
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References