FEEDBACK LIMITED OPPORTUNISTIC SCHEDULING AND
ADMISSION CONTROL FOR ERGODIC RATE GUARANTEES
OVER NAKAGAMI-\(m\) FADING CHANNELS

YOORA KIM
School of Electrical Engineering and Computer Science
Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea

GANG UK HWANG
Department of Mathematical Sciences and Telecommunication Engineering Program
Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea

HEA SOOK PARK
Network Research Department
Electronics and Telecommunications Research Institute, Daejeon 305-700, Korea

Abstract. In this paper, we consider downlink transmission in a cellular wireless network, where a base station communicates with multiple mobile stations (MSs) over Nakagami-\(m\) fading channels. MSs are classified into two classes, and each class has its own minimum ergodic rate requirement. We propose an opportunistic scheduling and admission control scheme that aims at guaranteeing minimum ergodic rates for all MSs in the network. In order to maintain fairness among MSs in the same class and reduce the feedback load on the uplink of the network, our proposed scheme uses normalized SNR thresholds and exploits multiuser diversity with limited feedback. In our analysis, we give a formula by which we can easily check whether an incoming MS, who requests to join a class in the network, can be accepted or not. For accepted MSs in the network, we obtain the values of thresholds with which all MSs in the network can be guaranteed respective minimum ergodic rate requirements. Through numerical studies and simulations, we confirm the validity of our scheme and analysis, and show the usefulness of our scheme.

1. Introduction. In wireless communication networks, the shared wireless medium is spatiotemporally varying due to e.g., multipath propagation, user mobility, and non-stationary clutter [7]. As a result, different users in the network usually experience different channel conditions at the same time. Multiuser diversity comes from independent channel variations among multiple users [11], [16]. To efficiently utilize the scarce radio spectrum, opportunistic scheduling schemes exploit multiuser diversity rather than combating the channel variations. For instance, MaxSNR scheme allocates the wireless channel to the user with the best instantaneous received signal-to-noise ratio (SNR) at each scheduling instant. Such scheduling scheme can maximize the total information-theoretic capacity of a wireless network [11], [15].

2000 Mathematics Subject Classification. Primary: 90B18; Secondary: 90B36.
Key words and phrases. Admission control, ergodic rate guarantee, feedback reduction, multiuser diversity, Nakagami-\(m\) fading, normalized SNR threshold, opportunistic scheduling.
Even though opportunistic scheduling scheme has an advantage of capacity enhancement over non-opportunistic scheduling schemes such as the round-robin scheduling, it has several crucial drawbacks compared with them. First, it can cause unfairness among users with heterogeneous channel conditions by allocating the channel most of the time to the users with the strongest channel conditions in average. Moreover, it necessitates the scheduler to know the channel conditions of all users at every scheduling instant, which substantially increases the feedback load of a wireless network [8].

In this paper, we consider downlink transmission in a cellular wireless network, where a base station (BS) communicates with multiple mobile stations (MSs) over Nakagami-\(m\) fading channels. We use the Nakagami-\(m\) model because the Nakagami-\(m\) model represents a wide range of fading channels [1], [14]: it becomes the Rayleigh fading channel when \(m = 1\) and can closely approximate the Ricean fading channels when \(m > 1\). To reduce the feedback load on the uplink of the network, we propose an opportunistic scheduling scheme that exploits multiuser diversity with limited feedback by using thresholds. In order to maintain fairness among MSs with heterogeneous channel conditions, our proposed scheduling scheme uses normalized SNR thresholds as in [18].

For quality-of-service (QoS) provisioning, we assume in this paper that the network provides two different service classes, denoted by class-\(n\) (\(n = 1, 2\)), and that class-\(n\) consists of MSs with minimum ergodic rate requirement \(R_n\) (b/s/Hz). Here, the ergodic rate is defined by the Shannon capacity per unit bandwidth, which is frequently used as a throughput measure in the literature [3]. In this paper, normalized SNR threshold values are obtained for the purpose of guaranteeing minimum ergodic rate requirement \(R_n\) for all MSs in class-\(n\). In order to provide minimum ergodic rate guarantees in case of dynamic user population, where MSs enter or leave the network over time, we also propose an admission control scheme that decides whether an incoming MS can be accepted or not. Our proposed admission control scheme has the following benefits. First, our admission control scheme incorporated with our proposed opportunistic scheduling scheme can reduce the feedback load on the uplink of the network. Moreover, by the use of the rate region (defined later), our proposed admission control scheme requires relatively low computational cost for checking the admissibility of an incoming MS.

In our analysis, we give a closed-form formula for the rate region with which we can easily check whether an incoming MS, who requests to join a class in the network, can be accepted or not. For accepted MSs in the network, we derive a formula to obtain the values of thresholds with which all MSs in the network can be guaranteed respective minimum ergodic rate requirements.

Through numerical studies and simulations, we confirm the validity of our scheme and analysis, and show the usefulness of our scheme. Numerical and simulation results show that our proposed scheme can guarantee minimum ergodic rate requirement \(R_n\) for all MSs in class-\(n\). In addition, the results also show that our proposed scheme can maintain fairness among MSs in the same class, even though MSs are subject to heterogeneous channel conditions.

The remainder of this paper is organized as follows. In Section 2, we describe the system model and propose our scheduling and admission control scheme. In Section 3, we formulate the problems considered in this paper. We solve the problems in Section 4 and provide numerical and simulation results in Section 5. Finally, we give our conclusions in Section 6.
2. **System model and proposed scheme.** We consider downlink transmission in a cellular wireless network, where a BS communicates with multiple MSs over Nakagami-m fading channels. The network provides two different service classes, and each MS in the network belongs to one of service classes. We assume that class-$n$ consists of MSs with minimum ergodic rate requirement $R_n$. For convenience, we denote the $i$th MS in class-$n$ by $\text{MS}_{n,i}$ throughout this paper.

2.1. **Wireless channel model.** In this subsection, we describe the wireless channel model from the BS to an MS considered in this paper. We assume a single-input single-output (SISO) system, and accordingly the received signal $y_{n,i}(t)$ of $\text{MS}_{n,i}$ at time $t$ can be written as follows:

$$y_{n,i}(t) = g_{n,i}(t)b(t) + w_{n,i}(t)$$

(1)

where $g_{n,i}(t) \in \mathbb{C}$ is the fading channel gain from the BS to $\text{MS}_{n,i}$, $b(t) \in \mathbb{C}$ is the signal broadcasted from the BS using a (normalized) transmit power $|b(t)|^2 = 1$, and $w_{n,i}(t) \in \mathbb{C}$ is the additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2$ in the channel of $\text{MS}_{n,i}$.

From (1), the instantaneous received SNR of $\text{MS}_{n,i}$ at time $t$ can be defined by $\gamma_{n,i}(t) = |g_{n,i}(t)|^2/\sigma^2$. The corresponding normalized received SNR of $\text{MS}_{n,i}$ at time $t$ can be defined by $h_{n,i}(t) = \gamma_{n,i}(t)/\bar{\gamma}_{n,i}$, where $\bar{\gamma}_{n,i} = \mathbb{E}[\gamma_{n,i}(\infty)]$ denotes the average received SNR of $\text{MS}_{n,i}$ in the steady state. In this paper, we assume that each MS is subject to independent Nakagami-m fading. Hence, the probability density function (pdf) of $h_{n,i}$ is given by [14]

$$f_{h_{n,i}}(x) = \frac{m}{\Gamma(m)} (mx)^{m-1} \exp(-mx), \quad x \geq 0,$$

(2)

where $m$ is the Nakagami fading parameter ($m \geq 1/2$), and $\Gamma(m)$ is the Gamma function defined by $\Gamma(m) = \int_0^\infty x^{m-1} \exp(-x)dx$. From (2), the cumulative density function (cdf) of $h_{n,i}$ is derived as

$$\Pr(h_{n,i} \leq x) = \frac{\gamma(m, mx)}{\Gamma(m)}, \quad x \geq 0,$$

where $\gamma(m, t) = \int_0^t x^{m-1} \exp(-x)dx$ is the lower incomplete Gamma function.

2.2. **Proposed opportunistic scheduling and admission control scheme.** In this subsection, we propose an opportunistic scheduling and admission control scheme that aims at guaranteeing minimum ergodic rate requirement $R_n$ for all MSs in class-$n$. Each MS in class-$n$ is assigned a priori the same normalized SNR threshold value $h_{th}$. At time $t$, $\text{MS}_{n,i}$ is allowed to feed back its instantaneous channel quality, i.e., the normalized SNR value $h_{n,i}(t)$, to the BS only when it satisfies the following feedback condition:

Feedback Condition: $h_{n,i}(t) \geq h_{th}$.  

(3)

Otherwise, $\text{MS}_{n,i}$ remains silent at time $t$. It is assumed that the estimation of the instantaneous channel quality is perfect and that the feedback delay is negligible.

At each scheduling instant, the scheduler considers only eligible MSs, who satisfy the feedback condition (3), and selects only one MS whose normalized SNR value is the best. Then, the BS transmits data to the selected MS at time $t$. Note that our proposed scheduling scheme performs identically to the proportional fair scheduling
(PFS) scheme if \( h_{th_1} = h_{th_2} = 0 \) [8]. When there is no eligible MS, network resources may be wasted, but for a large number of MSs, this has a vanishing effect [5].

Suppose that an incoming MS requests to join class-\( n \). Our proposed admission control scheme allows the incoming MS to join class-\( n \) only when the acceptance of the incoming MS does not violate minimum ergodic rate guarantees for all MSs in the network. If any one of MSs can not be guaranteed its minimum ergodic rate requirement, then the incoming MS is rejected to join class-\( n \). Note that, in our proposed admission control scheme, the values of normalized SNR thresholds are allowed to be adapted, if needed, to accept the incoming MS. In the subsequent sections, we analyze our system and provide a criterion to accept an incoming MS.

### 3. Problem formulation.

In this section, we focus on the ergodic rate performances of MSs under our proposed scheduling and admission control scheme, and formulate the problems considered in this paper.

For a given average received SNR value \( \bar{\gamma}_{n,i} \), the ergodic rate of MS \( n,i \) is determined by both the class sizes \( \vec{N} \triangleq (N_1, N_2) \) and the values of normalized SNR thresholds \( \vec{h}_{th} \triangleq (h_{th_1}, h_{th_2}) \), and can be expressed as follows:

\[
C_{n,i}(\vec{N}, \vec{h}_{th}) = \int_{h_{th_1}}^{\infty} \log_2(1 + \bar{\gamma}_{n,i} x) \cdot \Pr(\max_{k,j} \{h_{k,j} \cdot I(h_{k,j} \geq h_{th_k})\} = x \mid h_{n,i} = x) f_{h_{n,i}}(x) dx
\]  

where \( I(\cdot) \) denotes the indicator function, and \( f_{h_{n,i}}(x) \) denotes the pdf of \( h_{n,i} \) given in (2). Here, the first term is the Shannon capacity per unit bandwidth with the normalized SNR \( h_{n,i} = x \). The second term represents the conditional probability that MS\( n,i \) is selected by the scheduler given that its normalized SNR \( h_{n,i} \) is \( x \). The ergodic rate of MS\( n,i \) is obtained by averaging the Shannon capacity per unit bandwidth over the distribution of the normalized SNR \( h_{n,i} \) multiplied by the probability of being scheduled [10].

The objective in the design of our admission control and scheduling scheme is to regulate the class sizes \( \vec{N} \) and determine the values of normalized SNR thresholds \( \vec{h}_{th} \) with which minimum ergodic rate requirement \( R_n \) is guaranteed for all MSs in class-\( n \). To clarify the problems considered in this paper, we introduce the following definition.

**Definition 1.** A vector \( \vec{N} \) of class sizes is called **admissible** if there exists a corresponding vector \( \vec{h}_{th} \) of normalized SNR thresholds that realizes minimum ergodic rate guarantees for all MSs in each class, i.e.,

\[
C_{n,i}(\vec{N}, \vec{h}_{th}) \geq R_n, \quad n = 1, 2, \quad i = 1, \ldots, N_n.
\]  

Such normalized SNR thresholds \( \vec{h}_{th} \) are called the **admissible thresholds**.

For given class sizes \( \vec{N} \), Definition 1 says that if any one of MSs in the network can not be guaranteed its minimum ergodic rate requirement for any choice of thresholds \( \vec{h}_{th} \), then the class sizes \( \vec{N} \) are not admissible. For admissible class sizes \( \vec{N} \), all MSs in classes are guaranteed respective minimum ergodic rate requirements by using admissible thresholds \( \vec{h}_{th} \).

The admissibility condition in (5) can be rewritten as follows:

\[
\min_{i} C_{n,i}(\vec{N}, \vec{h}_{th}) \geq R_n, \quad n = 1, 2.
\]
Since our proposed scheduling scheme uses normalized SNR thresholds, the conditional probability on the right-hand side of (4) is equal for all MSs in the same class, and accordingly we obtain for $L(n)$ the following:

$$C_{n,L(n)}(\vec{N}, \vec{h}_{th}) = \min_i C_{n,i}(\vec{N}, \vec{h}_{th}).$$

Based on this fact, we hereafter focus on the ergodic rate performance of MS$_{n,L(n)}$ and call MS$_{n,L(n)}$ the tagged MS in class-$n$. For the tagged MSs, we introduce the rate region in the following definition.

**Definition 2.** For given class sizes $\vec{N}$, the region in $\mathbb{R}^2$ formed by the ergodic rates of all tagged MSs is called the *rate region* and denoted by $\mathbb{R}_{\text{rate}}^2(\vec{N})$, i.e.,

$$\mathbb{R}_{\text{rate}}^2(\vec{N}) \triangleq \left\{ \left( C_{1,L(1)}(\vec{N}, \vec{h}_{th}), C_{2,L(2)}(\vec{N}, \vec{h}_{th}) \right) \in \mathbb{R}^2 \middle| 0 \leq h_{th,n} < \infty, n = 1, 2 \right\}.$$

By using the rate region $\mathbb{R}_{\text{rate}}^2(\vec{N})$, the admissibility of given class sizes $\vec{N}$ can easily be checked as follows: if the rate vector $\vec{R} \triangleq (R_1, R_2)$ is in the rate region $\mathbb{R}_{\text{rate}}^2(\vec{N})$, then the class sizes $\vec{N}$ are admissible. Otherwise, the class sizes $\vec{N}$ are not admissible.

The first problem considered in this paper is to derive a closed-form formula for obtaining the rate region $\mathbb{R}_{\text{rate}}^2(\vec{N})$. Hence, we can easily check the admissibility of given class sizes $\vec{N}$. For given admissible class sizes $\vec{N}$, it remains to determine admissible threshold values $\vec{h}_{th}$. The second problem considered in this paper is to obtain admissible threshold values $\vec{h}_{th}$ for given admissible class sizes $\vec{N}$.

4. **Analytic results.** In this section, we solve the problems formulated in Section 3. For this, we first derive closed-form formulas for the ergodic rates of tagged MSs in Section 4.1. Then, by using the formulas derived in Section 4.1, we obtain the rate region $\mathbb{R}_{\text{rate}}^2(\vec{N})$ and admissible thresholds $\vec{h}_{th}$ in Section 4.2. In Section 4.3, we derive the channel access probability of an MS in the network and show that our proposed scheduling and admission control scheme can maintain fairness among MSs in the same class, even though MSs are subject to heterogeneous channel conditions.

4.1. **Ergodic rates of tagged MSs.** In this subsection, we derive closed-form formulas for the ergodic rates of tagged MSs. For this, we first define a random variable $\hat{h}_n \triangleq \max_i h_{n,i}$. Based on the independent assumption of $h_{n,i}$, we next derive the cdf of $h_n$ and the conditional probability of $h_n = x$ given that $h_{n,L(n)} = x$ as follows:

$$\Pr(\hat{h}_n \leq x) = \prod_{i=1}^{N_n} \Pr(h_{n,i} \leq x) = \left\{ \frac{\gamma(m, mx)}{\Gamma(m)} \right\}^{N_n}.$$

(6a)

$$\Pr(\hat{h}_n = x | h_{n,L(n)} = x) = \prod_{i=1, i \neq L(n)}^{N_n} \Pr(h_{n,i} \leq x) = \left\{ \frac{\gamma(m, mx)}{\Gamma(m)} \right\}^{N_n-1}.$$

(6b)
where \( \hat{h}_{th} \triangleq \max_n h_{th,n} \). Here, the first term on the right-hand side of (7) represents the case in which MS(s) in class-1 is the only eligible MS(s), and MS\(_{1,L(1)}\) has the best instantaneous normalized SNR value among the eligible MS(s). Hence, the scheduler selects MS\(_{1,L(1)}\) for downlink transmission in this case. The second term on the right-hand side of (7) represents the case in which there are eligible MSs in class-2 as well as in class-1, but MS\(_{1,L(1)}\) has the best instantaneous normalized SNR value among the eligible MSs. Hence, the scheduler selects MS\(_{1,L(1)}\) for downlink transmission in this case. For the other cases, MS\(_{1,L(1)}\) can not be selected by the scheduler.

By substituting the functions on the right-hand sides of (6a) and (6b) into (7), we obtain

\[
\overline{C}_{1,L(1)}(\vec{N}, \hat{h}_{th}) = \int_{h_{th}}^{\infty} \log_2(1 + \gamma_{1,L(1)} x) \cdot \Pr(\hat{h}_2 < h_{th2}) \Pr(\hat{h}_1 = x | h_{1,L(1)} = x) f_{h_{1,L(1)}}(x) \, dx \\
+ \int_{h_{th}}^{\infty} \log_2(1 + \gamma_{1,L(1)} x) \Pr(h_{th2} \leq h_2 < x) \cdot \Pr(\hat{h}_1 = x | h_{1,L(1)} = x) f_{h_{1,L(1)}}(x) \, dx
\]

(7)

Note that for \( m \in \mathbb{N} \), the lower incomplete Gamma function can be expressed as [6]

\[
\gamma(m, x) = \Gamma(m) \left[ 1 - \exp(-x) \sum_{k=0}^{m-1} \frac{x^k}{k!} \right].
\]

(9)

By substituting (9) into (8) and then using Binomial expansion, we obtain

\[
\overline{C}_{1,L(1)}(\vec{N}, \hat{h}_{th}) = \sum_{l=0}^{N_1+N_2-1} \binom{N_1+N_2-1}{l} \frac{(-1)^l}{\Gamma(m)} \sum_{k_l=0}^{m-1} \cdots \sum_{k_1=0}^{m-1} \\
\cdot \frac{I(m \hat{h}_{th}, \infty, \gamma_{1,L(1)}/m, l + 1, m - 1 + \sum_{j=1}^{l} k_j)}{(k_1)!(k_2)!, \ldots, (k_l)!} \\
+ \left\{ \frac{\gamma(m, m \hat{h}_{th2})}{\Gamma(m)} \right\} \sum_{l=0}^{N_1-1} \binom{N_1-1}{l} \frac{(-1)^l}{\Gamma(m)} \sum_{k_l=0}^{m-1} \cdots \sum_{k_1=0}^{m-1} \\
\cdot \frac{I(m \hat{h}_{th1}, m \hat{h}_{th}, \gamma_{1,L(1)}/m, l + 1, m - 1 + \sum_{j=1}^{l} k_j)}{(k_1)!(k_2)!, \ldots, (k_l)!}
\]

(10)

where \( I(a, b, \alpha, \beta, n) \triangleq \int_a^b \log_2(1 + ax) \exp(-\beta x) x^n \, dx \), and \( k_j \)'s are defined to be zero when \( l = 0 \). By a similar argument as above, we can obtain the ergodic rate.
the ergodic rate of the tagged MS in class-1 (resp. in class-2) over Nakagami-\(m\) fading channel by combining Proposition 1, Theorem 1 and (10) (resp. (11)). Note that when \(\arg \max_n h_{thn} = l\), the ergodic rate of the tagged MS in class-\(l\) depends only on the value of its
normalized SNR threshold $h_{th}$ for the fixed class sizes $\bar{N}$. Hence, we hereafter use $\mathcal{C}_{1,L(1)}(\bar{N}, h_{th})$ to denote $\mathcal{C}_{1,L(1)}(\bar{N}, h_{th})$ when $\max_n h_{th,n} = l$.

For a special case, we consider the ergodic rates of tagged MSs over Rayleigh fading channel. Then, from (10) and (11) with $m = 1$, we have

$$\mathcal{C}_{1,L(1)}(\bar{N}, \bar{h}_{th}) = \sum_{l=0}^{N_1+N_2-1} \left( \frac{N_1}{l} N_2 - 1 \right) (-1)^l I(\hat{h}_{th}, \infty, \mathcal{R}_{1,L(1)}, l+1, 0)$$

$$+ \{ \gamma(1, h_{th2}) \} N_2 \sum_{l=0}^{N_1-1} \left( \frac{N_1}{l} - 1 \right) (-1)^l I(h_{th1}, \hat{h}_{th}, \mathcal{R}_{1,L(1)}, l+1, 0)$$

$$\mathcal{C}_{2,L(2)}(\bar{N}, \bar{h}_{th}) = \sum_{l=0}^{N_1+N_2-1} \left( \frac{N_1}{l} N_2 - 1 \right) (-1)^l I(\hat{h}_{th1}, \hat{h}_{th}, \mathcal{R}_{2,L(2)}, l+1, 0)$$

$$+ \{ \gamma(1, h_{th}) \} N_1 \sum_{l=0}^{N_2-1} \left( \frac{N_2}{l} - 1 \right) (-1)^l I(h_{th2}, \hat{h}_{th}, \mathcal{R}_{2,L(2)}, l+1, 0).$$

It can easily be checked that the above results are identical to the results obtained in [9], which partially verifies the validity of our derivation.

4.2. Rate region and admissible thresholds. By using the formulas for the ergodic rates $\mathcal{C}_{n,L(n)}(\bar{N}, \bar{h}_{th}) (n = 1, 2)$ in (10) and (11), we can obtain the rate region $\mathbb{R}^2_{\text{rate}}(\bar{N})$ as in the following theorem.

**Theorem 2.** For given class sizes $\bar{N}$, the rate region $\mathbb{R}^2_{\text{rate}}(\bar{N})$ is given by

$$\mathbb{R}^2_{\text{rate}}(\bar{N}) = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq B, 0 \leq y \leq f(x)\}$$

where

$$B \triangleq \lim_{h_{th2} \to \infty} \mathcal{C}_{1,L(1)}(\bar{N}, (0, h_{th2}))$$

$$\begin{align*}
= \sum_{l=0}^{N_1-1} \left( \frac{N_1}{l} - 1 \right) \frac{1}{m} \sum_{k_1=0}^{m-1} \sum_{k_l=0}^{m-1} I(0, \infty, \mathcal{R}_{1,L(1)}, l+1, m-1 + \sum_{j=1}^{l} k_j) \\
\quad \times \frac{k_1!(k_2)!, \ldots, (k_l)!}{(k_1)(k_2), \ldots, (k_l)!}
\end{align*}$$

represents the upper bound of $\mathcal{C}_{1,L(1)}(\bar{N}, \bar{h}_{th})$, and the function $f(x)$ is defined by

$$f(x) \triangleq \max_{\mathcal{C}_{1,L(1)}(\bar{N}, \bar{h}_{th}) = x} \mathcal{C}_{2,L(2)}(\bar{N}, \bar{h}_{th})$$

$$= \begin{cases} 
\mathcal{C}_{2,L(2)}(\bar{N}, (h_{th1,x}, 0)), & 0 \leq x \leq \mathcal{C}_{1,L(1)}(\bar{N}, 0), \\
\mathcal{C}_{2,L(2)}(\bar{N}, h_{th2,x}^*), & \mathcal{C}_{1,L(1)}(\bar{N}, 0) < x \leq B.
\end{cases}$$

Here, $h_{th1,x}$ and $h_{th2,x}^*$ are uniquely determined from $\mathcal{C}_{1,L(1)}(\bar{N}, h_{th1,x}) = x$ and $\mathcal{C}_{1,L(1)}(\bar{N}, (0, h_{th2,x}^*)) = x$, respectively.

**Proof.** By a similar argument as in [10, Appendix A], we can prove Theorem 2 and omit detailed derivations.

There are no closed-form expressions for $h_{th1,x}$ and $h_{th2,x}^*$ in Theorem 2. However, they can be solved numerically.

For given admissible class sizes $\bar{N}$ and minimum ergodic rate requirements $\bar{R} = (R_1, R_2)$, i.e., $\bar{R} \in \mathbb{R}^2_{\text{rate}}(\bar{N})$, our proposed scheduling scheme can guarantee minimum ergodic rates $R_1$ and $R_2$ for all MSs in class-1 and in class-2, respectively.
However, it remains to determine admissible threshold values $\tilde{h}_{th}$. In Theorem 3, we give a method to obtain admissible threshold values $\tilde{h}_{th}$ for given admissible class sizes $\bar{N}$ and minimum ergodic rate requirements $\bar{R}$.

**Theorem 3.** For given admissible class sizes $\bar{N}$ and minimum ergodic rate requirements $\bar{R}$, we have

$$\hat{h}_{th} = \begin{cases} h_{th1}, & \text{if } R_1 \leq \overline{C}_{1,L}(\bar{N}, 0) \text{ and } R_2 \geq \overline{C}_{2,L}(\bar{N}, h_{th}^*), \\ h_{th2}, & \text{otherwise,} \end{cases}$$

where $h_{th}^*$ is uniquely determined from $\overline{C}_{1,L}(\bar{N}, h_{th}^*) = R_1$ for $R_1 \leq \overline{C}_{1,L}(\bar{N}, 0)$.

Let $l \triangleq \arg\max_n h_{th_n}$ and $m \triangleq \arg\min_n h_{th_n}$. Then, there exists unique solution $\tilde{h}_{th}^* = (h_{th1}, h_{th2})$ to the following system of equations:

$$\overline{C}_{l,L}(\bar{N}, h_{th1}) = R_l,$$

$$\overline{C}_{m,L}(\bar{N}, h_{th1}, h_{th2}) = R_m.$$  

According to Definition 1, the solution $\tilde{h}_{th}^*$ becomes admissible thresholds.

**Proof.** From the proof of Theorem 2, our theorem follows immediately. \qed

**4.3. Fairness analysis.** We define the fairness metric for $MS_{n,i}$ by the probability of $MS_{n,i}$ being served by the BS [2], which is called the channel access probability and denoted by $P_{ac,n,i}(\bar{N}, \tilde{h}_{th})$ throughout this paper. From (4), the channel access probability of $MS_{n,i}$ can be expressed as follows:

$$P_{ac,n,i}(\bar{N}, \tilde{h}_{th}) = \int_{h_{th,n}}^{\infty} \Pr(\max_{k,j} \{h_{k,j} \cdot I(h_{k,j} \geq h_{th_k})\} = x \mid h_{n,i} = x) f_{h_{n,i}}(x) \, dx.$$  

By a similar argument used in Section 4.1, we can derive the channel access probability of $MS_{n,i}$ as in the following proposition.

**Proposition 2.** For given class sizes $\bar{N}$ and normalized SNR thresholds $\tilde{h}_{th}$, the channel access probability of $MS_{n,i}$ is given by

$$P_{ac,n,i}(\bar{N}, \tilde{h}_{th}) = \sum_{l=0}^{N_1+N_2-1} \binom{N_1 + N_2 - 1}{l} \frac{(-1)^l}{\Gamma(m)}$$

$$\cdot \sum_{k_1=0}^{m-1} \cdots \sum_{k_l=0}^{m-1} \frac{J(mh_{th_1}, \infty, l + 1, m - 1 + \sum_{j=1}^l k_j)}{(k_1)!(k_2)!, \ldots, (k_l)!}$$

$$+ \left\{ \frac{\gamma(mh_{th_1}, mh_{th_2}/h_{th_2})}{\Gamma(m)} \right\}^{N_1N_2/N_n} \sum_{l=0}^{N_n-1} \binom{N_n - 1}{l} \frac{(-1)^l}{\Gamma(m)}$$

$$\cdot \sum_{k_1=0}^{m-1} \cdots \sum_{k_l=0}^{m-1} \frac{J(mh_{th_1}, mh_{th_2}, l + 1, m - 1 + \sum_{j=1}^l k_j)}{(k_1)!(k_2)!, \ldots, (k_l)!}$$

where $J(u_1, u_2, \mu, k)$ is defined by $J(u_1, u_2, \mu, k) \triangleq \int_{u_1}^{u_2} \exp(-\mu t) t^k \, dt$ for $u_1, u_2, \mu > 0$, $k \in \mathbb{N} \cup \{0\}$, and given in (15).

In Proposition 2, we observe for $n = 1, 2$ that

$$P_{ac,n,1}(\bar{N}, \tilde{h}_{th}) = \ldots = P_{ac,n,N_n}(\bar{N}, \tilde{h}_{th})$$
which verifies that our proposed scheduling and admission control scheme can maintain fairness among MSs in the same class, even though MSs are subject to heterogeneous channel conditions.

5. Simulation results. In this section, we provide numerical results and show the usefulness of our proposed scheduling and admission control scheme. To confirm the validity of our analysis, we also provide simulation results by using the Nakagami-$m$ fading simulation model in [17].

Throughout this section, the rate vector $\vec{R}$ is set to $(R_1, R_2) = (0.70, 0.50)$ (b/s/Hz), and the Nakagami fading parameter is assumed to be $m = 1$ (i.e., the Rayleigh fading channel). Note that the Rayleigh fading channel is frequently used in the literature (see, e.g., [4], [12]) to model multipath fading environment with no direct line-of-sight path [13].

Suppose that the network consists of (initially) eight MSs with the following parameters:

- Class sizes: $\{N_1, N_2\} = \{3, 5\}$
- Average received SNRs of MSs in class-1:
  $\{\bar{\gamma}_{1,1}, \bar{\gamma}_{1,2}, \bar{\gamma}_{1,3}\} = \{15, 10, 17\}$ (dB)
- Average received SNRs of MSs in class-2:
  $\{\bar{\gamma}_{2,1}, \bar{\gamma}_{2,2}, \bar{\gamma}_{2,3}, \bar{\gamma}_{2,4}, \bar{\gamma}_{2,5}\} = \{17, 14, 13, 16, 20\}$ (dB)

For the tagged MSs (i.e., MS$_{1,2}$ and MS$_{2,3}$ under this setting), the rate region $R^2_{\text{rate}}(3, 5)$ is plotted in FIGURE 1 by using Theorem 2. In the figure, we observe that the rate vector $\vec{R}$ is in the rate region $R^2_{\text{rate}}(3, 5)$. Hence, we can obtain admissible thresholds $h_{th}$ by using Theorem 3, and the results are given by $(h_{th_1}, h_{th_2}) = (0.96, 2.05)$.

FIGUREs 2 (a) and (b) show the ergodic rates of MSs in class-1 and in class-2, respectively, which are obtained from analysis and simulation by using admissible
Figure 2. Ergodic rates of MSs (a) in class-1 and (b) in class-2 by using admissible thresholds $(h_{th1}, h_{th2}) = (0.96, 2.05)$. In the figures, we show the 95 percent confidence intervals for the simulation results. We observe that all MSs in class-1 as well as in class-2 can achieve respective minimum ergodic rate requirements $R_1$ and $R_2$. In addition, we also observe that the achieved ergodic rates of MSs in each class are proportional to their average received SNR values.

FIGUREs 3 (a) and (b) show the channel access probabilities of MSs in class-1 and in class-2, respectively, which are obtained from analysis and simulation by
using admissible thresholds \((h_{th1}, h_{th2}) = (0.96, 2.05)\). In the figures, we show the 95 percent confidence intervals for the simulation results. As shown in the figures, MSs in the same class can be served by the BS with the same channel access probability. The results in FIGUREs 3 (a) and (b) imply that our proposed scheduling and admission control scheme can maintain fairness among MSs in the same class, even though they are subject to heterogeneous channel conditions.
Suppose that MS_{2,6} with the average received SNR \( \gamma_{2,6} = 10 \) (dB) requests to join class-2 in the network as given in FIGURE 1. Then, tagged MS in class-2 is now changed to MS_{2,6} because it has the lowest average received SNR value among MSs in class-2. For the tagged MSs (i.e., MS_{1,2} and MS_{2,6} under this setting), the rate region \( R_{\text{rate}}^{(3,5)} \) is plotted in FIGURE 4 by using Theorem 2. In the figure, we observe that the rate region shrinks from \( R_{\text{rate}}^{(3,5)} \) in FIGURE 1 to \( R_{\text{rate}}^{(3,6)} \) in FIGURE 4 after the acceptance of MS_{2,6}, and consequently the rate vector \( \vec{R} \) is out of the rate region \( R_{\text{rate}}^{(3,6)} \). Hence, the incoming MS_{2,6} is rejected to join class-2 under our admission control scheme.

6. **Conclusion.** In this paper, we propose an opportunistic downlink scheduling and admission control scheme that aims at guaranteeing minimum ergodic rate requirement \( R_n \) for all MSs in class-\( n \) over Nakagami-\( m \) fading channels. In order to maintain fairness among MSs in the same class and reduce the feedback load on the uplink of the network, our proposed scheme uses normalized SNR thresholds and exploits multiuser diversity with limited feedback.

In our analysis, we give a formula by which we can easily check whether an incoming MS, who requests to join a class in the network, can be accepted or not. For accepted MSs in the network, we obtain admissible thresholds with which all MSs in the network can be guaranteed respective minimum ergodic rate requirements. Numerical and simulation results show that our proposed scheme can guarantee minimum ergodic rate requirement \( R_n \) for all MSs in class-\( n \). In addition, our analytic results are shown to be well matched with the simulation results, which partially verifies the validity of our analysis.

**Appendix [Proof of Theorem 1].** To prove Theorem 1, we need the following lemma.
Lemma 1. For $a, b, \alpha, \beta > 0$ and $n \in \mathbb{N}$, we have
\[ I_a^b \frac{\exp(-\beta x) x^n}{1 + \alpha x} \, dx = - \frac{\ln(2)}{\alpha} G(\alpha, \beta, n, x)|_{x=a}^{x=b} \]  
(12)
where
\[ G(\alpha, \beta, n, x) \triangleq \frac{1}{\alpha^n \ln(2)} \exp \left( \frac{\beta}{\alpha} \right) \left\{ (-1)^n E_1 \left( \frac{\beta}{\alpha} + \beta x \right) + \exp \left( - \left( \frac{\beta}{\alpha} + \beta x \right) \right) \sum_{k=1}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) (-1)^{n-k} \sum_{s=0}^{k-1} \frac{(k-1)! (1 + \alpha x)^s}{s! (\beta/\alpha)^{k-s}} \right\}. \]

Proof. Let $t \triangleq 1 + \alpha x$. By the change of variables and Binomial expansion, we obtain
\[ I_a^b \frac{\exp(-\beta x) x^n}{1 + \alpha x} \, dx = \frac{1}{\alpha^{n+1}} \exp \left( \frac{\beta}{\alpha} \right) \sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) (-1)^{n-k} \int_{1+\alpha a}^{1+\alpha b} \exp \left( - \frac{\beta}{\alpha} t \right) t^{k-1} dt. \]
Suppose $k = 0$. Then, the integral on the right-hand side of (13) becomes
\[ \left. \int_{1+\alpha a}^{1+\alpha b} \exp \left( - \frac{\beta}{\alpha} t \right) \frac{1}{t} \, dt \right|_{x=a}^{x=b} = - \frac{\ln(2)}{\alpha} G(\alpha, \beta, n, x)|_{x=a}^{x=b}. \]
(14)
Note that for $u_1, u_2, \mu > 0$ and $k \in \mathbb{N} \cup \{0\}$ [6]
\[ \left. \int_{u_1}^{u_2} \exp(-\mu t) t^k \, dt \right|_{x=u_1}^{x=u_2} = - \exp(-\mu x) \sum_{s=0}^{k} \frac{k!}{s!(\beta/\alpha)^{k-s}} \frac{x^s}{x^{k+1}} \right|_{x=u_1}^{x=u_2}. \]
(15)
Hence, for $k = 1, \ldots, n$, the integral on the right-hand side of (13) becomes
\[ \int_{1+\alpha a}^{1+\alpha b} \exp \left( - \frac{\beta}{\alpha} t \right) t^{k-1} dt = - \exp \left( - \left( \frac{\beta}{\alpha} + \beta x \right) \right) \sum_{s=0}^{k-1} \frac{(k-1)! (1 + \alpha x)^s}{s! (\beta/\alpha)^{k-s}} \right|_{x=a}^{x=b}. \]
(16)
By combining (14) and (16), we obtain
\[ I_a^b \frac{\exp(-\beta x) x^n}{1 + \alpha x} \, dx = - \frac{\ln(2)}{\alpha} G(\alpha, \beta, n, x)|_{x=a}^{x=b}. \]
\[ \Box \]

We are now ready to prove Theorem 1. We first differentiate $F(\alpha, \beta, n, x)$ with respect to $x$ and obtain
\[ \frac{\partial}{\partial x} F(\alpha, \beta, n, x) = \frac{\alpha}{\ln(2)} \exp(-\beta x) x^n - \beta F(\alpha, \beta, n, x) + n F(\alpha, \beta, n-1, x). \]
(17)
We next integrate (17) from $x = a$ to $x = b$ and obtain
\[ F(\alpha, \beta, n, x)|_{x=a}^{x=b} = \frac{\alpha}{\ln(2)} \int_{a}^{b} \frac{\exp(-\beta x) x^n}{1 + \alpha x} \, dx - \beta I(a, b, \alpha, \beta, n) + n I(a, b, \alpha, \beta, n-1). \]
(18)
From (18), we can express $I(a, b, \alpha, \beta, n)$ as follows:
\[ I(a, b, \alpha, \beta, n) = - \frac{1}{\beta} \left\{ F(\alpha, \beta, n, x)|_{x=a}^{x=b} - \frac{\alpha}{\ln(2)} \int_{a}^{b} \frac{\exp(-\beta x) x^n}{1 + \alpha x} \, dx - n I(a, b, \alpha, \beta, n-1) \right\}. \]
(19)
By substituting (12) into (19), we finally obtain

\[ I(a, b, \alpha, \beta, n) = -\frac{1}{\beta} \left\{ \left[ F(\alpha, \beta, n, x) + G(\alpha, \beta, n, x) \right]_{x=b}^{x=a} - nI(a, b, \alpha, \beta, n - 1) \right\}. \]

Acknowledgments. This work was supported by the Korea Science and Engineering Foundation (KOSEF) grant funded by the Korea government (MOST) (No. R01-2007-000-20053-0).

REFERENCES


Received August 2008; 1st revision November 2008; 2nd revision April 2009.

E-mail address: jd_salinger@kaist.ac.kr
E-mail address: guhwang@kaist.edu
E-mail address: parkhs@etri.re.kr