Nonlinear particle behavior during cross-type optical particle separation

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The effects of varying the ratio of the optical force to the viscous drag force, termed $S$, on cross-type optical particle separation were investigated experimentally to test previous theoretical predictions. The experiments were performed for various flow velocities, powers of the laser beam, and radii of the laser beam waist and the particles. The behaviors of the particles during optical separation were examined by measuring the retention distances and analyzing the particle trajectories. For small values of $S$, the particles move with constant velocity in the flow direction and the retention distance increases linearly with $S$. However, the particles accelerate and decelerate within the laser beam and the retention distance increases nonlinearly with $S$ when $S$ increases further. © 2009 American Institute of Physics. [doi:10.1063/1.3276548]

Since Ashkin’s pioneering studies of optical levitation and the trapping of micron-sized objects by optical forces,1,2 optical force has been widely applied in many research fields. The change in the momentum of photons when a laser beam passes through an object means that an optical force is exerted on the object. According to its direction, the optical force is dependent on the gradient force, which acts in the direction of the intensity gradient of the laser beam, and the scattering force, which acts parallel to the direction of propagation of the laser beam.3 For optical trapping to occur, the gradient force must be dominant and then a high numerical aperture (NA) lens can be used to capture objects near the focal point of the lens. Optical trapping is now widely used in biological research, such as into the manipulation and interrogation of individual molecules, the dynamics of molecular motors, and the mechanical properties of biological molecules.4 On the other hand, optical levitation requires a low NA lens and a dominant scattering force so that objects are pushed in the direction opposite to that of gravitation. Optical levitation has been applied in optofluidic biological cell sorting.5

Optical levitation typically uses a stationary medium, but can be developed further by using a moving medium in optical separation. In optical separation, particles with different sizes,6 refractive indexes,7 or a combination of both, i.e., with different optical mobilities,8 can be sorted. There are several configurations used in optical separation.6–11 In the present study, cross-type optical particle separation was used.10,11 In cross-type optical separation, the laser beam is illuminated in a direction perpendicular to that of a fluid flow, and then the particles can be sorted in a continuous manner.13 In previous studies of cross-type optical separation, separations according to various properties of particles such as size, refractive index, and optical mobility have been demonstrated theoretically14 and experimentally.10 In cross-type optical separation, when the width of the laser beam is much larger than the radius of the particles, the gradient force can be ignored. The retention distance, which is the deviation of particle position from its initial position, can be predicted by using an approximate linear theory.11

However, it has recently been shown theoretically that the linear theory of cross-type optical particle separation breaks down when a significant gradient force is present.12 Due to the gradient force, particles in the cross-type optical separator can experience acceleration and deceleration when they approach and leave the center axis of the laser beam, respectively. This nonlinear behavior of particles can enhance the retention distance. To account for the effects of both the gradient and scattering forces in cross-type optical particle separation, a nondimensional number, $S$, which is the ratio of the optical force to the viscous drag force, has been derived:12

$$S = \frac{\text{optical force}}{\text{viscous drag}} = \frac{(n_0 P/c)(r_p^2/\omega_0^2)}{6\pi \mu r_p U}, \quad (1)$$

where $n_0$ is the refractive index of the medium, $P$ is the incidence power of the laser beam, $c$ is the speed of light in a vacuum, $r_p$ is the radius of the particles, $\omega_0$ is the waist radius of the laser beam, $U$ is the velocity of the fluid flow, and $\mu$ is the dynamic viscosity of the medium. In the present study, the effects of varying $S$ on the nonlinear behavior of the particles and on the retention distance in cross-type optical particle separation were investigated experimentally. To this end, the particle behaviors were observed and the retention distances were measured and compared with the theoretical predictions for various values of $S$.

Figure 1 shows a schematic diagram of the experimental

![FIG. 1. (Color online) Schematic diagram of the experimental setup.](image-url)
setup. A polydimethylsiloxane microchannel with a width of 600 μm and a height of 100 μm was fabricated by using conventional soft lithography. The laser beam from a Nd:YAG laser operating at 532 nm was focused onto the microchannel by using a plano-convex lens. Two plano-convex lenses (f=50 mm and 75 mm, CVI Optics) were used to change the waist radius of the laser beam. Two samples of polystyrene latex (PSL) (Duke Scientific Corp.) spherical particles with diameters of 5.00±0.05 and 2.50±0.025 μm were used. The PSL particles were suspended in deionized water and fed into the microchannel through a polytetrafluoroethylene tube. The flow velocity was controlled by using a syringe pump (pump 11, Harvard Apparatus). Since the particles were injected apart from the channel wall to prevent the effects of shear layer, they remained in almost uniform flow field during the separation. Images were collected with a complementary metal-oxide semiconductor (CMOS) camera (1200hs, PCO). To prevent the scattering of light from the laser, an interference filter (F10–632.8–4-2.00, CVI Optics) was used; a red LED was adopted as the light source for the CMOS camera. To measure the waist radius of the laser beam in the microchannel, rhodamine B (Junsei Chemical Co., Ltd) was diluted in deionized water and a fluorescent image of the laser beam was captured and analyzed by using a computer programmed image processing tool. Two groups of experimental conditions were used. In both groups, the nondimensional particle radius, \( r_p = r_p / \omega_0 \), was fixed at 0.1, but the radius of the particles (group 1: \( r_p = 1.25 \) μm and group 2: \( r_p = 2.5 \) μm) and the laser beam waist were varied between the groups (group 1: \( \omega_0 = 12.5 \) μm and group 2: \( \omega_0 = 25 \) μm).

According to the linear theory, the retention distance, \( y_{\text{max}} \), is proportional to \( S \):

\[
y_{\text{max}}^* = \frac{y_{\text{max}}}{\omega_0} = S f(m),
\]

where \( f(m) \) is a function of the relative refractive index, \( m \), i.e., the ratio of the refractive index of the particle to that of the medium,

\[
f(m) = \sqrt{2} \frac{3^2}{\pi^2} Q^*(m) \text{erf}(\sqrt{2}).
\]

Here, \( Q^* \) is the nondimensional coefficient of the scattering force for small particles, which depends only on the relative refractive index, and erf is an error function. Based on Eq. (2), the \( y \)-directional maximum deviation of the particle position, i.e., the retention distance, can easily be determined from the known values of \( S \) and the relative refractive index.

However, Eq. (2) is derived under the assumption that the gradient force can be disregarded. A recent theoretical study has shown that this assumption can break down as \( S \) becomes large and that the retention distance can increase nonlinearly with \( S \) when the value of \( S \) is larger than a certain value of \( S \), which can thus be used as a criterion to determine whether the retention distance will be in accord with the linear theory. The value of \( S \) that determines the validity of the linear theory is determined by the ratio of the radius of the particles to the waist radius of the laser beam. In the present study, the ratio is 0.1 and the corresponding value of \( S \) is 5.

The experimental results and the theoretical predictions for the retention distance are shown for various values of \( S \) in Fig. 2. For a given particle radius and laser beam waist, \( S \) is varied by varying the flow velocity in the range 30 ~200 μm/s and the power of the laser beam in the range 0.25 ~ 2 W. The theoretical predictions were obtained by solving the particle dynamics equations without approximation. The results of the linear theory are also plotted for comparison. When the particles are smaller than \( S \), the linear theory is valid but deviates significantly from the experimental measurements in the region where \( S \) is larger than 5, as expected from the theoretical predictions. The measured retention distance increases nonlinearly with increases in \( S \), as predicted from the solutions of the particle dynamic equations, and are in good agreement with the theoretical predictions. When the values of \( S \) are the same for different systems, i.e., for systems with different flow velocities, particle sizes, and powers of the laser beam, the retention distances can be the same. For example, as shown in Fig. 2, the two different groups have the same value of \( S = 9 \) (group 1: \( r_p = 1.25 \) μm, \( \omega_0 = 12.5 \) μm, \( U = 37 \) μm/s, and \( P = 0.5 \) W; group 2: \( r_p = 2.5 \) μm, \( \omega_0 = 25 \) μm, \( U = 37 \) μm/s, and \( P = 1 \) W) and the same retention distance even though their conditions are completely different.

To determine the behaviors of the particles for different values of \( S \), the particle trajectories were monitored for time intervals of 0.2 s, as shown in Fig. 3. The values of \( S \) in Figs. 3(a) (Media 1, enhanced online) and (b) (Media 2, enhanced online) are 9 and 4 respectively. The experimental conditions were as follows: \( r_p = 2.5 \) μm, \( \omega_0 = 12.5 \) μm, \( U = 70 \) μm/s, and \( P \) = 1.9 W for Fig. 3(a), and \( r_p = 2.5 \) μm, \( \omega_0 = 12.5 \) μm, \( U = 100 \) μm/s, and \( P \) = 1.2 W for Fig. 3(b). In both cases, the particles move in the direction of the fluid flow with a constant velocity before they cross the laser beam but are deflected in the laser beam propagating direction when they pass through the laser beam. For \( S = 9 \), a particle is decelerated due to the gradient force when it crosses the center axis of the laser beam and is deflected near the center axis of the laser beam, as shown in Fig. 3(a). On the other hand, for \( S = 5 \), the gradient force is negligible and the particle moves with a constant velocity in the fluid flow direction. Therefore, each particle is deflected across the whole width of the laser beam. To examine the acceleration and deceleration of the particle quantitatively, the \( x \)-directional (flow direction) position of the particle was obtained as a function of time from Fig. 3; Fig. 4 shows the results. Here, the nondimennenments.
sional $x$-directional position and time are defined as $x^* = x/\omega_0$ and $t^* = Ut/\omega_0$, respectively. As shown in Fig. 4, for $S=4$ the variation of the $x$-directional position of the particle is uniformly distributed, i.e., the particle moves with a constant velocity. However, for $S=9$, the particle is accelerated when it approaches the laser beam and decelerates when it leaves the laser beam. After release from the laser beam, the particle moves with a constant velocity. Due to this deceleration, it takes more time to pass across the laser beam for $S=9$ than for $S=4$, as shown in Fig. 4. These different particle behaviors during cross-type optical particle separation are in good agreement with the theoretical predictions. These findings demonstrate that for large values of $S$ the nonlinear increment of the retention distance can improve the resolution of the cross-type optical particle separation with additional increment of the separation time. In the present study, the effects of varying the ratio of the optical force to the viscous drag force on cross-type optical particle separation have been determined. This ratio is termed $S$, and can be used in a criterion for the validity of the previously derived linear theory. For small values of $S$, the retention distance of the particles is in accord with the linear theory, however, for large values of $S$, the particles experience acceleration and deceleration due to the gradient force and the retention distance increases nonlinearly. Although only cross-type optical particle separation was considered in the present study, this analysis can be extended to other kinds of optical separation. The present results do not apply to objects with irregular shapes. However, the present analysis can be extended to objects with arbitrary shapes by applying theoretical methods to the determination of the optical force, such as the T matrix method, the finite difference time domain method, and the finite elements method.

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