Utilization of wall reflection data in the acoustic pyrometry for estimating the gas temperature in a duct section

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Abstract

Estimation of temperature field of the medium in a duct section is required for monitoring and controlling the combustion status of various power systems. Acoustic pyrometry is the most promising technique for this purpose. The usual acoustic pyrometry concept is using the measured retarded time data of sound propagation between multiple sets of acoustic sensors and actuators, which are to be used for the inverse calculation employing a proper basis function. A large number of retarded time data are needed for accurate temperature estimation; however, due to practical limitations, the number of data representing the direct propagation path is usually a small number. This study suggests the additional use of the retarded time data from the wall reflection of sound to append to the direct data. Numerical simulation is conducted for a rectangular duct section having predetermined multiple hot spots of the medium. Compared to the results using the conventional pyrometry, a clear improvement in finding the position of hot spots and estimating the temperature contours can be observed for such a two-dimensional complex temperature field. It is also found that the reconstruction result is better when the separation between temperature hot spots is wide enough.

Keywords: Acoustic pyrometry, inverse problem, temperature measurement, wall reflection
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1 Introduction

It is known that the acoustic pyrometry has an advantage over optical or radiation pyrometry in measuring the sectional temperature distribution in a gas filled duct. In using the acoustic pyrometry, temperature field is reconstructed by solving the inverse problem. Such inverse problem is usually composed of the measured retarded time vector in between wall-mounted sensors and the transfer matrix which approximates the speed of sound within the medium and its coefficients [1,2]. The space-based interpolation technique is adopted for an approximation bearing high spatial resolution to achieve accurate restoration of target field. However, the high spatial resolution is preceded with large number of interpolation points which is dependent on the number of sensors and actuators. In this study, additional retarded times are obtained using the wall reflection of a propagating acoustic pulse generated from the source.

2 Theory for Temperature Field Estimation

2.1 Inverse reconstruction of 2D temperature field from measured data

Consider a two-dimensional rectangular measurement section as shown in Figure 1.

Figure 1: A two-dimensional measurement section with wall-mounted sensor and actuator pair.

The retarded time, $t_d$, of a sound pulse between two points can be defined by the following Radon transform relating the speed of sound and the retarded time [3]:

$$ t_d = L \int_0^1 \frac{ds}{c(u,v)} = L \int_0^1 F(u,v)ds. $$

(1)
Here, $F(u,v)$ is the slowness function ($1/c = \sqrt{yRT}$) defined as an inverse of the speed of sound, which can be obtained from the time required for a sound pulse to travel a unit length, $u, v$ are the normalized coordinates of a point within the domain, and $s, L$ are the normalized and real distance between two points, respectively. The slowness function $F(u,v)$ can be approximately expressed by choosing proper basis function with a finite number of added terms as

$$F(u, v) = \frac{1}{c(u,v)} \approx \sum_{n=1}^{q} A_n \psi_n,$$  \hspace{1cm} (2)

where $\psi_n$ is selected basis function, $q$ the number of added term, $n$ the order of basis function, and $A_n$ the unknown coefficient. The radial basis function is selected to approximate the distribution of sound speed in the target field. The multiquadric function is selected among the various kind of radial basis function because it assumes that the characteristics of target field should be continuous [4]. The definition of multiquadric function is as follows:

$$\psi_n = \sqrt{(u - (u_j)_n)^2 + (v + (v_j)_n)^2 + \sigma^2}.$$  \hspace{1cm} (3)

Here, subscript $j$ is the coordinates of internal interpolation points, $n$ the $n$-th interpolation point, and $\sigma$ the shape factor, which is a weighting on the selected distance. The shape factor can be determined from the Hardy’s formula, which uses the mean distance of the interpolation points [4]. Finally, the approximated retarded time between all possible sensor/actuator pairs can be expressed as

$$t_d = L \int_{0}^{1} \left( \sum_{n=1}^{q} A_n \psi_n \right) ds = \sum_{n=1}^{q} A_n \Psi_n,$$  \hspace{1cm} (4)

$$\Psi_n = L \int_{0}^{1} \sqrt{(u - (u_j)_n)^2 + (v + (v_j)_n)^2 + \sigma^2} ds.$$  \hspace{1cm} (5)

If there are $p$ multiple paths due to multiple sensor/actuator pairs at the wall, a matrix equation can be formulated as
Here, $\Psi$ is the transfer matrix which expresses the distribution of speed of sound on the field from the information of sensor position and acoustic path. The temperature field of the target plane is given by

$$T(u, v) = \frac{1}{\gamma_R} \left( \sum_{n=1}^{q} \left( \Psi_n^* t_d \right) \psi_n \right)^{-2}. \tag{7}$$

The symbol ‘†’ denotes the Moore-Penrose generalized inverse. The temperature distribution of entire field can be spatially interpolated by using Equation (6) with the calculated solution of Equation (5) and the definition of multiquadric function in Equation (3).

### 2.2 Employment of additional retarded time due to wall reflections

If one uses space-based interpolation, it is necessary to obtain the input information, i.e., the retarded time data, it is preferred, not mandatory though, to use the overdetermined data, that is, at least one rank higher than the target number of positions to reduce the effect of measurement noise. The number of retarded times is determined from the number of adopted acoustic sources and sensors. However, practical limitations restrict the number of sensor arrays that can be configured. In this study, additional retarded times are obtained using the wall reflection of a propagating acoustic pulse. It should be noted that a similar concept of using a reflected acoustic pulse is already suggested by Kudo and Minamide [5,6], the technique suggested in the present study is clearly different from the previous works. The previous works starts with the calculation of initial temperature field with direct propagation data. Then, the retarded time associated with the arbitrarily selected reflection path is calculated using the initial calculation result. Therefore, the basic concept of these works is very similar to the interpolation method of regenerating many retarded time data for arbitrary direct propagation paths that is suggested by Kim and Ih [7]. This regeneration technique depends mainly on the accuracy of initial result, so the additional data would be beneficial to reduce the random error. In contrast, the present technique incorporates the measured reflection data into the initial retarded time data set of direct propagation, so the actual increase of the input data is achieved. By this way, it is expected that both the random and bias errors can be reduced.

The acoustic pulse reflected by the wall surface is assumed to be a specular one. When a specular reflection occurs, the reflected pulse propagates through the shortest path due to Fermat’s principle. Thus, the propagation length of the reflection path can be estimated using the geometrical information of the target field. Although, the number of recorded retarded times is easily increased with the measurement of higher order reflections, only the first reflection is considered in this study because it is sufficient to obtain enough signal-to-noise ratio. Also, it is worried that the reflected pulse bandwidth of the multiple reflection signal would be widened, so
the estimation of the retard time would be involved with some additional error due to diffusion. Because only the first reflection is considered, the reflection path should exist between sensors placed in a parallel manner, as shown in Figure 2. Hence, the matrix equation in Equation (5) can be reformulated by using the both direct, \( t_{d,D} \), and reflected retarded time, \( t_{d,R} \), as follows:

\[
\begin{bmatrix}
\{t_{d,D}\} \\
\{t_{d,R}\}
\end{bmatrix} =
\begin{bmatrix}
\Psi_{n,D} \\
\Psi_{n,R}
\end{bmatrix}
\begin{bmatrix}
\{A_{n,D}\} \\
\{A_{n,R}\}
\end{bmatrix}.
\]  

(8)

Here, subscripts D, R represent the elements of matrix equation corresponding to the direct and reflected acoustic paths, respectively. The new distribution of temperature field can be obtained by using the solution of the combined Equations (8) and (7). By adding the retarded time associated with the reflected path into the original equation, it is possible to utilize a large number of interpolation points. In this way, one can expect the enhancement of reconstruction accuracy.

![Figure 2: Ray paths of the single reflection of sound (LHS only). S=source, R=receiver.](image)

### 3 Numerical simulation

#### 3.1 Preparation of the input data

A numerical example is prepared to simulate the reconstruction of the complicated two-dimensional temperature distribution by using the retarded time from direct paths and additional reflected paths. The reference 2D temperature field having 4 local maxima is shown in Figure 3.

![Figure 3: Reference temperature distribution with 4 local hot spots.](image)
A rectangular-shaped target field is prepared to reconstruct the sectional temperature distribution of a given target. The size of the target plane is 1.6 (W) X 1.125 (D) m². In the present study, the accuracy of the reconstructed temperature field is compared with the simulated reference field using without and with the additional retarded time from wall reflections. The root-mean-square error is used to present the approximation error. The retarded times between sensors are calculated using the given reference field. To be similar to the actual situation, the uniform Gaussian noise is added to the original signal without any measurement noise, thus making the signal-to-noise ratio of retarded time vector to be 20 dB.

In total, twelve acoustic sensors, corresponding to 54 direct paths, are deployed in an asymmetric manner to avoid the increase in singularity due to the sensor configuration, as depicted in Figure 4. Also, the positions of additional reflected paths are shown in Figure 5.

Thus, a total of 54 terms are used in the basis function to approximate the temperature distribution if direct paths are used only; if the reflected paths are also used additionally, total 67 terms are used in the basis function. The position of the interpolation points are optimally selected from the uniform distribution of many candidate points using the genetic algorithm. It is expected that optimally selected position of interpolation points can reduce the inherent singularity of the transfer matrix and increase the reconstruction accuracy by minimizing the singularity factor of the transfer matrix, which can be defined as
\[ SF = tr(\Lambda^{-2}) = \sum diag(\Lambda^{-2}). \]  

(9)

The resultant distribution of the interpolation points is illustrated in Figure 6. Figures 6(a) and 6(b) compare the optimally selected interpolation points for both 54-point case and 67-point case. The shape factor is determined by using the Hardy’s formula.

![Figure 6](image)

Figure 6: Positioning of interpolation points: (a) 54 optimally deployed points, (b) 67 optimally deployed points.

### 3.2 Reconstruction results

The reconstruction results by employing two different data schemes, using only direct paths and additionally using the reflected paths, are shown in Figure 7. The reconstruction image with additional reflected paths is far enhanced than the reconstruction result with direct propagation paths only. In Figure 7(a), the number of hot spots is identified as 4, but the positions of upper right and lower left hot spots deviate a bit from the given reference distribution. Although the average position of each hot spot is not very deviated from the reference distribution as shown in Figure 3, the estimated temperature in the center zone is overestimated in comparison with the reference field. Due to the overestimated temperature field, in practice, it is expected that one may be confused in determining the exact number of hot spots within the target field.

![Figure 7](image)

Figure 7: Reconstructed sectional temperature field: (a) Reconstructed with direct path data only (\(ERMS=65\%)\), (b) reconstructed with additional reflected path data (\(ERMS=53\%)\).

Contrastingly, one can see a clear separation of heat sources by applying the additional retarded time data from the wall reflections, as shown in Figure 7(b). Due to the increase of the number of retarded time vector, 13 additional interpolation points can be deployed in the target plane without adding the actual sensors. It is noted that the approximation error is also
decreased by 12%. One can conclude that the employment of additional retarded time vector from wall reflection of a sound pulse can enhance the spatial distribution of field temperature profile and the magnitude of temperature at the field point without adding the sensors.

4 Conclusions

In this study, we have studied about the effect of additional retarded time data on enhancing the reconstruction results of 2D temperature fields without adding any new sensors or sources. It is thought that the difficulties of measuring temperature fields with multiple hot spots, due to a lack of sufficient retarded time data, can be overcome by measuring additional retarded time data from reflection paths. To investigate the feasibility of suggested method, the rectangular-shaped two-dimensional temperature field with multiple hot spots is simulated numerically. It is also expected that an optimal deployment can reduce reconstruction error by minimizing the singularity of the transfer matrix, which is amplified by measurement noise. In the reconstruction result, one can clearly see that the employment of additional retarded time vector from wall reflection enhances the reconstruction accuracy for the complex temperature field. The position and the number of hot spots of reconstruction result are similar to the reference field and the reconstruction error is decreased by 12% in comparison with the reconstruction using the direct paths only. Because the suggested method does not need any additional sensors or sources, this method does not increase maintenance or installation costs.

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References


