Twist effect on spectral properties of two-mode fiber acousto-optic filters

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Abstract: The splitting and the shift of resonance peaks in the output spectrum of two-mode fiber acousto-optic devices caused by twist perturbation are described. An elliptical-core two-mode fiber and a two-mode photonic-crystal fiber are used for the analysis. The splitting of the resonance peaks are found to be almost linearly proportional to the twist angle of the fiber with the slopes of about 0.58 nm/(rad/m) for the elliptical-core fiber and about 0.29 nm/(rad/m) for the photonic crystal fiber. The experimental results compare well with the theoretical predictions.

OCIS codes: (060.2310) Fiber optics; (230.1040) Acousto-optical devices; (060.5295) photonic crystal fiber.

References and links

1. Introduction

All-fiber acousto-optic tunable filters (AOTFs) are of interest for their applications to optical communications and sensing with their advantages of low insertion loss, wide tuning range, and fast switching speed [1-3]. The effect of external perturbations such as bending, twisting, longitudinal tension, and temperature change are studied and utilized for designing devices with new functions and improved performance [4-7]. As an example, a 180 degree-twist of an elliptic-core (e-core) two-mode fiber (TMF) in an AOTF resulted in the improvement of side mode suppression from -9.7 dB to -15.5 dB [5].

In this paper, we describe in detail our observation of the splitting and the shift of resonance peaks in the output spectra of TMF AOTFs under fiber twist of up to multiple turns. Two types of TMFs were used for the experiment; one with an e-core fiber and the other with a photonic crystal fiber (PCF). The e-core TMF supports only one of the LP$_{11}$ mode lobe orientations along the major axis of the core ellipse, whereas the PCF supports both the lobe orientations over the wavelength range of our interest. We show that the resonance peaks of the AOTF split as the fiber is twisted and the separation is almost linearly proportional to the twist angle. We also show that a circularly polarized acoustic wave can select one of the split peaks. Theoretical calculations based on the coupled-mode theory provide an accurate explanation of the experimental observations.

2. Theory

TMF AOTFs are based on acousto-optic (AO) coupling between the LP$_{01}$ and the LP$_{11}$ core modes of TMFs utilizing the lowest-order flexural acoustic wave [8, 9]. In the following analysis we assume the fibers have acoustic birefringence due to small ellipticity in the outer cladding. It is worth noting that the propagation velocity of the flexural acoustic wave depends on the fiber diameter [10] and the ellipticity results in a linear acoustic birefringence [11]. Accordingly, the flexural acoustic wave traveling along the acoustically birefringent fiber has two orthogonal vibration components along the minor (x-) and the major (y-) axes of the cladding ellipse that are defined as eigen axes for the linear acoustic birefringence. We also assume that the polarization state of the flexural acoustic wave traveling along the fiber at a position, $A(z)$, can be simply represented in the Jones matrix format as follow.

$$A(z) = \begin{pmatrix} A_x(z) \\ A_y(z) \end{pmatrix} = \begin{pmatrix} \exp(i\beta_x z) & 0 \\ 0 & \exp(i\beta_y z) \end{pmatrix} \begin{pmatrix} A_x(0) \\ A_y(0) \end{pmatrix}$$

(1)

where, $z$ is the propagation length, $A_x$ and $A_y$ are the Jones vector components of the acoustic wave along the minor (x-) and the major (y-) axes of the cladding ellipse, respectively, $\beta_x$ and $\beta_y$ are the acoustic propagation constants of the acoustic linear polarization modes. When the fiber is twisted, the polarization state of the flexural acoustic wave along the twisted fiber can be expressed as the following in a reference frame rotating with the fiber cladding ellipse [5].

$$A(z) = \begin{pmatrix} A_x(z) \\ A_y(z) \end{pmatrix} = \exp(i\beta_0 z) \begin{pmatrix} \cos X - i \Delta \beta_z \sin X X \\ \tan \tau \sin X X \\ X \cos X + i \Delta \beta_z \sin X X \\ X \cos X + i \Delta \beta_z \sin X X \end{pmatrix} \begin{pmatrix} A_x(0) \\ A_y(0) \end{pmatrix}$$

(2)

where, $\beta_0 = 2\pi / \Lambda_0 = (\beta_x + \beta_y) / 2$, $\Delta \beta_z = (\beta_y - \beta_x) / 2$, and $X = (\Delta \beta_z^2 + \tau^2)^{1/2}$. $\tau$ is the twist angle per unit length. Positive sign of $\tau$ represents counterclockwise rotation as seen by an observer toward whom the acoustic wave
is propagating. Under our current assumption of high twist rate \((\tau^2 \gg \Delta \beta^2)\), the Eq. (2) can be simplified as

\[
A(z) = \begin{pmatrix}
A_x(z) \\
A_y(z)
\end{pmatrix} = \exp \left( \frac{2\pi z}{\Lambda_0} \begin{pmatrix}
\cos \tau z & -\sin \tau z \\
\sin \tau z & \cos \tau z
\end{pmatrix} \right) \begin{pmatrix}
A_x(0) \\
A_y(0)
\end{pmatrix} \tag{3}
\]

This equation simply states that the acoustic wave propagates without changing its vibration direction in the laboratory frame. We experimentally confirmed this feature by directly measuring the acoustic amplitude along the fiber. The details of the experiment are included in section 3. It can be interpreted as the high fiber twist rate suppresses the effect of small acoustic birefringence. On the other hand, the lobe orientations of the LP_{11} mode of the optical wave follow the fiber twist under the condition that the fiber twist period is much longer than the beatlength between the two orthogonal LP_{11} mode lobe orientations (the even and the odd LP_{11} modes) as in the case of an e-core fiber. The even and the odd LP_{11} modes are defined as those with the higher and the lower mode effective refractive indices, respectively. Since the flexural acoustic wave couples the LP_{01} mode to the LP_{11} modes with the lobe orientation in the same direction as the acoustic vibration, the coupling strength for a particular LP_{11} mode will be modulated along the twisted fiber length. For a uniform twist rate, the modulation of coupling strength is sinusoidal as a function of \(z\).

The even and odd LP_{11} mode lobes of an e-core TMF are oriented along the major and minor axes of the core, respectively, and each guided mode (the LP_{01} and the LP_{11} modes) has distinct two polarization modes that are also linearly polarized in the direction of the major and minor axes of the core. For the e-core TMF used in our study, the principal axes of the core ellipse are well aligned with those of the cladding ellipse that causes acoustic linear birefringence, which was verified by using a fiber geometry analyzer. This alignment is the result of the fabrication process of the e-core fiber, and simplifies the acousto-optic mode coupling characteristics.

![Diagram](image-url) Fig. 1. Illustration of the variation of each acoustic polarization component along the even and the odd axes when a linearly polarized flexural acoustic wave is applied along the even axis of twisted fiber at the input (The even and odd axes represent the lobe directions of the even and the odd LP_{11} mode, respectively.)
Figure 1 depicts the variation of the acoustic polarization components along the major and the minor axes of a twisted e-core fiber core when a linearly polarized acoustic wave is excited along the major axis of the fiber core at the input. As noted above, the acoustic polarization is maintained along the interaction region because the twist rate is much greater than the magnitude of the acoustic linear birefringence. The optical birefringence between two polarization components in each mode of the e-core TMF is much bigger than the circular birefringence induced by the twist and therefore the fiber behaves as a polarization maintaining fiber under the twist [12, 13]. Over the wavelength range around 1300nm where our experiment was carried out, the fiber guides only the LP01 and the even LP11 modes in the core, further simplifying the situation. When a linearly polarized acoustic wave is applied along the twisted e-core TMF, the acoustic amplitude component contributing to the coupling to the even LP11 mode can be written from the Eq. (3) by

$$A_{\text{even}}(z) = A_{\text{ave}} \exp \left( i \frac{2\pi}{\Lambda_0} z \right) \cos \left( \tau z - \phi \right) = A_{\text{ave}} \left( \exp \left( i \frac{2\pi}{\Lambda_0} z - \phi \right) + \exp \left( - i \frac{2\pi}{\Lambda_0} z + \phi \right) \right)$$

where, $A_{\text{ave}} = \sqrt{A_x(0)^2 + A_y(0)^2}$, $\sin \phi = A_y(0) / A_{\text{ave}}$, $\cos \phi = A_x(0) / A_{\text{ave}}$.

This equation implies that the guided light experiences two different periods of acoustic perturbations ($\exp(i(2\pi/\Lambda_0 + \tau)z)$ and $\exp(i(2\pi/\Lambda_0 - \tau)z)$) along the even axis of the twisted fiber regardless of the acoustic vibration direction. This results in two resonance wavelengths, thereby two notches in the output spectrum. In this case, the phase matching conditions are given by $2\pi/L_0 = 2\pi/\Lambda_0 + \tau$ and $2\pi/L_0 = 2\pi/\Lambda_0 - \tau$. $L_0$ is the optical beatlength between the LP01 and the even LP11 modes. As a result, the original resonance peak for an untwisted fiber is split into two identical peaks on both sides of the original position under the fiber twist for the linear acoustic polarization state. In the experiment, we launched a linearly polarized acoustic wave along the major axis ($A_x(0) = 0$) of the twisted e-core TMF. The phase matching condition for non-twist case is $2\pi/L_0 = 2\pi/\Lambda_0 + \Delta \beta_a$, which is easily obtained from the Eq. (2) with the values of $\tau = 0$, and $A_x(0) = 0$. Therefore, the resonance peak for untwisted fiber case is not exactly in the middle of the two split resonance peaks. Another interesting feature is that only one resonance peak can be produced when circularly polarized acoustic waves are launched along the twisted fiber. For calculation convenience, we set the initial acoustic amplitude components as $A_x(0) = A(0)$, $A_y(0) = \pm iA(0)$ without overall phase term. The upper and lower signs represent the left- and right-circular acoustic polarizations, respectively. From the Eq. (3), the acoustic amplitude component contributing to the mode coupling, $A_{\text{even}}(z)$, becomes,

$$A(z)_{\text{even}} = -i \cdot A(0) \cdot \exp \left( i \frac{2\pi}{\Lambda_0} + \tau \right) z$$

$$A(z)_{\text{even}} = i \cdot A(0) \cdot \exp \left( i \frac{2\pi}{\Lambda_0} - \tau \right) z$$

where the Eqs. (5a) and (5b) correspond to the acoustic amplitude components contributing to the coupling to the even LP11 mode for the case of the right- and left-circular acoustic polarization, respectively. The equations indicate that the phase matching conditions are $2\pi/L_0 = 2\pi/\Lambda_0 + \tau$ for the right-circular acoustic polarization, and $2\pi/L_0 = 2\pi/\Lambda_0 - \tau$ for the left-circular acoustic polarization, leading to single resonance peaks for each input acoustic polarization. This feature leads to tuning of a resonance peak without splitting in proportion to the fiber twist rate.

The situation becomes more complex for the two-mode PCF used in the experiment, where both the even and the odd LP11 modes are supported in the fiber core over the wavelength of our interest. Moreover, in contrast to the e-core TMF, the LP11 mode lobe...
orientations do not coincide with the axes of cladding ellipse [14]. Nevertheless, the Eq. (3) is valid for this case with the x and y notations representing the odd and the even mode axes as long as the high fiber twist rate suppresses the effect of small acoustic birefringence. When a linearly polarized acoustic wave is launched along the twisted PCF, the acoustic amplitude components contributing to the mode coupling to the even and the odd LP_{11} modes can be written as

$$
\begin{pmatrix}
A^{\text{odd}}(z) \\
A^{\text{even}}(z)
\end{pmatrix}
= A_{\text{ave}} \cdot \exp \left( i \frac{2\pi}{\Lambda_0} z \right) \begin{pmatrix}
-\sin(\tau z - \phi) \\
\cos(\tau z - \phi)
\end{pmatrix}
$$

$$
= \frac{A_{\text{ave}}}{2} \cdot \exp(-i\phi) \cdot \exp \left( i \frac{2\pi}{\Lambda_0} z \right) \begin{pmatrix}
-e^{i\tau z} - e^{-i\tau z}
\end{pmatrix}
$$

where, $A_{\text{ave}} = \sqrt{A^{\text{odd}}(0)^2 + A^{\text{even}}(0)^2}$, $\sin \phi = A^{\text{odd}}(0) / A_{\text{ave}}$, $\cos \phi = A^{\text{even}}(0) / A_{\text{ave}}$

As can be seen in this equation, there are 4 phase matching conditions; two from the fiber twist rate as in the case of the e-core fiber for each of the odd and the even modes. They are $2\pi/L_B^{\text{LP}_01-\text{LP}_{11\text{odd}}} = 2\pi/\Lambda_0 \pm \tau$, and $2\pi/L_B^{\text{LP}_01-\text{LP}_{11\text{even}}} = 2\pi/\Lambda_0 \pm \tau$. This means that two resonance spectral peaks under no twist will split into four peaks when the fiber is twisted. For the experimental demonstration of the resonance peak splitting in the PCF, we applied the acoustic wave along the even axis ($A_{\text{ave}}(0) = 0$) of the twisted PCF at the input. As in the case of the e-core fiber, the magnitude of resonance splitting is also determined by the amount of the twist angle and the beatlength dispersion relation of the couplings between the LP_{01} mode and the even and the odd LP_{11} modes. It should be noted that the optical birefringence between two polarization components in each mode of the PCF is much smaller than the circular birefringence induced by the twist. As a result, the effect of intrinsic linear optical birefringence could be neglected when the fiber is twisted by a large twist rate [14].

3. Experimental results and analysis

3.1 Elliptical-core TMF AOTF

We fabricated an e-core TMF AOTF as shown in Fig. 2.

![Fig. 2. The schematic configuration of the acousto-optic interaction part: TMF: two-mode fiber; SMF: single-mode fiber, MS: mode stripper](image-url)

The acousto-optic filter consists of an acoustic transducer and an e-core TMF (with core diameter of 8 μm × 12 μm and an NA of 0.16) mounted on two rotation stages. The input acoustic polarization state was adjusted with an acoustic transducer comprising two PZT sections by manipulating the amplitude and phase of the each PZT section. The twist angle of the fiber was controlled by the two rotation stages. Broadband unpolarized light from a 1300-nm LED was launched into the TMF and the LP_{11} mode was stripped by tightly bending a section of the input fiber. Specific phase-matched wavelength components were converted to the even LP_{11} mode by the AOTF and the coupled even LP_{11} mode is removed by mode
strip at the filter output. A standard single mode fiber spliced to the TMF transmitted the remaining \( LP_{01} \) mode in the TMF to an optical spectrum analyzer (OSA). The AO interaction length and the applied acoustic frequency were 25.5 cm and 3.4 MHz, respectively.

The output spectrum for non-twist case is shown in Fig. 3(a).

Fig. 3. Transmission spectra of the AOTF (a) without fiber twist, (b), at the twist angle of 5.5\( \pi \) where the applied acoustic polarizations are \( y \)-linear,

The resonance wavelength shown in the figure is 1297.3 nm, which was determined by the phase matching condition of \( 2\pi/L_{g0} = 2\pi/\Lambda_0 + \Delta\beta_{g} \). For the demonstration of the resonance wavelength splitting and shift, we measured the transmission spectrum of the AOTF as the fiber was twisted by the angle ranging from \( 2\pi \) to 5.5\( \pi \) for the given interaction length. The absolute value of \( \Delta\beta_{g} \) is calculated to be about 0.9\( \pi \), so that the twist angle of more than 2\( \pi \) (= \( \tau \cdot l \)) is enough to satisfy the assumption of \( (\tau^2 \gg \Delta\beta_{g}^2) \). Figure 3(b) shows the results for the \( y \)-linearly polarized acoustic wave at the twist angle of 5.5\( \pi \). The \( y \)-linearly polarized acoustic wave splits the resonance wavelength into two, up- and down- shifted ones, having equal coupling efficiency. The split resonance wavelengths were measured to be 1281.6 nm and 1319.3 nm, which were determined by the phase matching condition of \( 2\pi/L_{g0} = 2\pi/\Lambda_0 \pm \tau \).

The notch depth of about -16 dB in Fig. 3(a) indicates about 97% mode coupling. The notch depth of -3 dB (50% mode coupling) in Fig. 3(b) is due to the magnitude of acoustic amplitude reduced in a half as can be seen in Eq. (4). The coupling efficiency is determined by the acoustic amplitude.

Figure 4 shows the experimental and the calculation results of the resonance peak splitting by the linearly polarized acoustic wave as a function of the twist angle. For the calculation of dispersion relation between the \( LP_{01} \) and the \( LP_{11} \) modes, we used an approximate empirical formula relating an e-core TMF to an equivalent circular core TMF [15]. The relationship

\[
\Delta_{\beta_{g}} 
\]
between the notch splitting and the twist angle was almost linear in the twisting range from $2\pi$ to $5.5\pi$, and the ratio was 0.039 nm/degree (2.18 nm/rad) for the given interaction length of 25.5 cm. The experimentally achievable maximum notch splitting was 37.8 nm at the twist angle of $5.5\pi$.

The right-circularly and left-circularly polarized acoustic waves shift the resonance wavelength to 1319.3 nm and 1281.6 nm as shown in Figs. 5(a) and 5(b), respectively.

![Graph showing transmission spectra of the AOTF at the twist angle of $5.5\pi$](image)

Considering that the wavelength satisfying the condition of $2\pi/L_0 = 2\pi/\Lambda_0$ is 1300.2 nm, the resonance wavelength shifts are 18.6 nm (down shift) and 19.1 nm (up shift), respectively. The difference of 0.5 nm between the wavelength shifts is found to be from the slight nonlinearity in the dispersion curve of the beat length over the wavelength span of 40 nm.

2. Two-mode photonic crystal fiber AOTF

We carried out similar experiment with a solid-core two-mode PCF. The fiber is a commercially available PCF, which has inversion and six-fold rotation symmetry of the air-hole lattice surrounding a silica core as shown in Fig. 6.

![Image of the PCF](image)

Figure 7 shows the filtering spectra of the PCF AOTF with a 26-cm-long interaction length at the applied acoustic frequency of 3.24 MHz for a broadband unpolarized input light from the 1300-nm LED when a linearly polarized acoustic wave is launched along the minor axis of the cladding ellipse of the untwisted PCF. In this case, the phase matching conditions are $2\pi/L_{\text{LP01-LP11}}^{\text{odd}} = 2\pi/\Lambda_{\text{minor}}$, $2\pi/L_{\text{LP01-LP11}}^{\text{even}} = 2\pi/\Lambda_{\text{minor}}$, and the resonance wavelengths satisfying the phase matching conditions were experimentally found to be about 1266.8 and 1292.8 nm, respectively. The resonance amplitude difference between the even and the odd modes in the figure is due to the angular misalignment between the acoustic and the optical eigen axes [14].
Fig. 7. Transmission spectrum of the PCF AOTF with a 26-cm-long interaction length at
applied acoustic frequency of 3.24 MHz for a broadband unpolarized input light from a LED.
(Solid line: experiment, Dashed line: calculation)

For the verification of the assumption that the high fiber twist rate suppresses the effect of
small acoustic birefringence that led to Eq. (3), we measured the acoustic oscillation
amplitudes along the fiber length as shown in Fig. 8(a). Simple fiber interferometers formed
by the probe fiber end-face and the outer surface of the fiber under test are used for the
measurement [16]. At the launching point, the acoustic wave was at 45° angle with respect to
the minor axis (x-axis in the Fig. 8(a)) of the acoustic birefringence. Acoustic amplitudes at 0°
and 135° angles with respect to the x-axis were measured as a function of the positions along
the fiber length. Figure 8(b) is for the case with no fiber twist and shows that the acoustic
amplitude at 0° along the fiber length is constant except for attenuation, whereas that at 135°
follows a sine curve. This is anticipated for a fiber with linear acoustic birefringence. Figure
8(c) is for the case with 2π fiber twist and shows that the acoustic amplitudes at 0°, 45°, and
135° show no sine dependence on fiber length. This indicates that the intrinsic linear acoustic
birefringence is suppressed at the twist angle of 2π for the given interaction length of 26 cm.
Also note that the acoustic amplitude orthogonal to the initial launching polarization angle
stays at zero through the fiber length, demonstrating the suppression of the linear
birefringence is practically complete. The results shown here justifies the assumption made
for Eq. (3) in section 2.

Fig. 8. (a). Experimental setup for measurement of the acoustic oscillation amplitude along the
two different directions of 0° and 135°. The variation of the acoustic amplitudes (b) along the
fiber without twist and (c) along the fiber with the twist angle of 2π

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As was discussed in section 2, the two resonance peaks in Fig. 7 are expected to be split into four in the presence of twist perturbation, and the corresponding phase matching conditions are 

\[ \frac{2\pi}{L_D} L_{01}^{\text{LP01-LP11odd}} = \frac{2\pi}{A_0} + \tau, \quad \frac{2\pi}{L_D} L_{01}^{\text{LP01-LP11even}} = \frac{2\pi}{A_0} - \tau. \]

Figure 9 shows the filtering spectra of the AOTF at four different cases of \( \tau = 2\pi, 4\pi, 5\pi, \) and \( 6\pi \) for the given acousto-optic interaction length of 26 cm. In this case, the twist angle of more than \( 2\pi \) satisfies the assumption of \( (\tau^2 \gg \Delta \beta_2)^2 \). Note that the absolute value of \( \Delta \beta_2/l \) was found to be about \( 0.9\pi \).

The split resonance peaks have almost the same notch depth of near -3dB. The magnitude of peak splitting at the four different twist angles of \( \tau = 2\pi, 4\pi, 5\pi, \) and \( 6\pi \) are about 7.9, 14.1, 17.2, 20.3 nm, respectively. The interesting feature in Fig. 9 is that the peak splitting for the even LP\(_{11} \) mode coupling is slightly larger than that for the odd LP\(_{11} \) mode coupling.

![Transmission spectra of the two-mode PCF AOTF at the four different twist angles of 2\(\pi\), 4\(\pi\), 5\(\pi\), and 6\(\pi\) (Acoustic frequency: 3.24 MHz)](image)

**Fig. 9.** Transmission spectra of the two-mode PCF AOTF at the four different twist angles of \( 2\pi, 4\pi, 5\pi, \) and \( 6\pi \). (Acoustic frequency: 3.24 MHz)

Figure 10 depicts the reason with the beatlength dispersion curves and the corresponding resonance wavelengths for a given twist perturbation.

![Calculated beatlength curves between the LP\(_{01} \) and the even and the odd LP\(_{11} \) modes of the PCF used in the experiment.](image)

**Fig. 10.** Calculated beatlength curves between the LP\(_{01} \) and the even and the odd LP\(_{11} \) modes of the PCF used in the experiment.
The four resonance wavelengths in the Fig. 9 are plotted as a function of twist angle in Fig. 11(a).

The wavelengths satisfying the phase matching conditions of \( \frac{2\pi}{L_0} \lambda_{LP01-LP11_{\text{odd}}} = \frac{2\pi}{\Lambda_0} \) and \( \frac{2\pi}{L_0} \lambda_{LP01-LP11_{\text{even}}} = \frac{2\pi}{\Lambda_0} \) are designated by the dotted line in the Fig. 11(a). The experimentally observed notch splitting for the even LP\(_{11}\) mode coupling is plotted as a function of the twist angle in Fig. 11(b). The relationship between the notch splitting and the twist angle was almost linear over the each wavelength span of 20 nm. For the even mode, the notch splitting rate was calculated to be about 1.1 nm/rad over the twist angle range from 2\(\pi\) to 6\(\pi\) for the given interaction length of 26 cm. The calculation was carried out based on the assumption of the high fiber twist rate (\(\Delta\beta_\theta \approx 0\)). The slight discrepancy between theoretical and experimental results around relatively small fiber twist rate in the Fig. 11(b) is thought to be due to the non-negligible linear acoustic birefringence of the fiber.

5. Conclusion

We investigated experimentally and theoretically the resonance spectral splitting and shift of the AOTF formed with an e-core TMF and a solid-core two-mode photonic crystal fiber under a large-angle twist perturbation. Possibilities of wavelength tuning by fiber twist are demonstrated. Theoretical predictions explain well the experimental results.