Numerical models for prestressing tendons in containment structures

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Abstract

Two modified stress–strain relations for bonded and unbonded internal tendons are proposed. The proposed relations can simulate the post-cracking behavior and tension stiffening effect in prestressed concrete containment structures. In the case of the bonded tendon, tensile forces between adjacent cracks are transmitted from a bonded tendon to concrete by bond forces. Therefore, the constitutive law of a bonded tendon stiffened by grout needs to be determined from the bond–slip relationship. On the other hand, a stress increase beyond the effective prestress in an unbonded tendon is not section-dependent but member-dependent. It means that the tendon stress unequivocally represents a uniform distribution along the length when the friction loss is excluded. Thus, using a strain reduction factor, the modified stress–strain curve of an unbonded tendon is derived by successive iterations. In advance, the prediction of cracking behavior and ultimate resisting capacity of prestressed concrete containment structures using the introduced numerical models are succeeded, and the need for the consideration of many influencing factors such as the tension stiffening effect, plastic hinge length and modification of stress–strain relation of tendon is emphasized. Finally, the developed numerical models are applied to prestressed concrete containment structures to verify the efficiency and applicability in simulating the structural behavior with bonded and/or unbonded tendons.

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1. Introduction

In contrast with structural damage in infra-structures such as bridges, buildings, tunnels and storage tanks, which can be repaired or strengthened with time and causes no additional serious problems, damage in a nuclear power plant (NPP) may result in serious and enduring problems. Accordingly, the design and construction of a NPP are strictly guided by related design codes, and containment is also placed outside of the NPP to cope with internal accidents such as loss of coolant accident (LOCA with pressure and temperature increase in the containment) or an external event such as an aircraft crash, explosions or earthquakes. Further, in advance, the containment structure constitutes an ultimate barrier against the dissemination of fissile products towards the general public (De Boeck, 1993). In particular, the accidents at Three Mile Island NPP in 1979 and at Chernobyl NPP in 1986 underline the necessity to perform failure analyses and calculate the ultimate pressure capability of the containment for the safety assessment of nuclear power plants (Amin et al., 1993). Approximately 95% of recently developed containments are shell-type concrete structures composed of three parts, a dome, a wall and the foundation. These parts are reinforced with prestressing tendons in the wall and the dome, and their structural types are deeply associated with the adopted reactor systems. The pressurized water reactor (PWR) and the Canada deuterium uranium (CANDU) are two representative containments constructed in Korea. In spite of the differences in the containment types, however, the design specifications commonly require that the containment has the capacity not only to sustain no tensile stress in the interior face under normal operating conditions but also to mitigate the consequences of an accident to an acceptable level in the case of an internal accident or an external event accompanying leakage of radioactive substance with a rupture in the primary coolant system because of high pressure and temperature. Hypothesizing that internal pressure may increase without limit, accordingly, a series of structural responses according to an increase of external loads need to be evaluated to reserve the...
safety of the containment structures, and comprehensive experimental and analytical studies have been conducted in this regard (fib, 2001).

The objectives of experimental studies, which generally focus on internal pressurization tests, are to obtain data on the structural response of the model to pressure loading beyond a design basis accident in order to validate analytical modeling, to find the model’s pressure capacity and to observe its response and failure mechanics. Ultimate capacity pressure tests on a 1/14 scale model of Gentilly-2 in Canada (Rizkalla et al., 1984), a 1/10 scale model of Sizewell-B in UK (Twidle and Crowder, 1991), a 1/4 scale model of Ohi-3 in the US (Hessheimer et al., 2003) and MAEV A mock up in France (Kevrokian et al., 2005) can be regarded as representative experimental studies. In parallel with experimental studies, the development of reliable analytical models has also proceeded (Sandia National Laboratories, 2000) with the recognition that experiments are time-consuming and costly and often do not precisely simulate the loading and support conditions of the actual structure.

In spite of a lot of numerical studies for the prestressed concrete containments (Yonezawa et al., 2002; Sandia National Laboratories, 2000), however, still there is no generally accepted analysis method that takes into account the three-dimensional configuration and a lot of non-linear effects for PSC containment. Since the internal bonded tendons are reinforced as the main reinforcement in CANDU containments, any special attention has not been given in numerical modeling of structure on the basis of the assumption that the non-linear behavior of each material is well defined and there is no difference between tendon and mild steel in modeling the material properties (Collins and Mitchell, 1991). However, the non-linear behavior of the prestressed concrete containment structure is strongly governed by the tendon behavior. An accurate estimation of the ultimate resisting capacity and accompanying cracking behavior requires the consideration of the tension stiffening effect for the bonded tendon. Nevertheless, any tension stiffening model that takes into account the mechanical properties and the slip behavior of tendon has not been introduced for the non-linear analysis of CANDU containments with bonded internal tendons.

Accordingly, so as to effectively simulate the post-cracking behavior and to exactly estimate the ultimate resisting capacity of a prestressed concrete containment vessel (PCCV) with bonded tendons, a modified stress–strain curve of tendon is introduced in this paper on the basis of the bond mechanism between tendon and concrete. Obviously, the constitutive law of a bonded tendon stiffened by grouting is different from that of a bare tendon and also from that of mild steel (Kwak and Song, 2002).

On the other hand, the structural behavior of PSC structures with unbonded tendons is member-dependent instead of section-dependent, and the stress in unbonded tendons depends on the deformation of the entire member and is assumed to be uniform at all sections along the span length. This means that the stress cannot be directly determined from a cross-section analysis with the conventional strain compatibility condition as in the case of bonded tendons. To determine the resisting capacity of PSC structures with unbonded internal tendons, accordingly, an exact prediction of the tendon force must be preceded, and consideration of the slip effect along the tendon sheath as well as the stress relaxation with time is emphasized.

In order to consider the interaction between the concrete and tendon, a one-dimensional link element, where one end is connected to the tendon node while the other is connected to the concrete, is generally used (CEB, 1996). In addition, a bond-zone element that describes the slip behavior of the contact surface between steel and concrete can also be used (Kwak and Seo, 2002). However, the use of a link element or a bond-zone element in the finite element analysis of PSC structures imposes the following restrictions: (1) the finite element mesh must be arranged in such a way that the tendon is located along the edge of a concrete element and (2) a double node is required to represent the relative slip between the tendon and concrete. In a complex containment, particularly in three-dimensional models, these requirements lead to a considerable increase in the number of degrees of freedom, not only because of doubling the number of nodes along the tendons but also because the mesh has to be refined so that the bars pass along the edges of concrete elements. The complexity of the mesh definition and the large number of degrees of freedom has discouraged researchers from including the slip effect in many previous studies.

These difficulties in considering the slip effect especially increase in the non-linear analysis of complex large structures with full modeling of the entire structure using commercialized softwares, such as ABAQUS (2004), ADINA (2002) and DIANA (TN0, 2002), because the tendon forces are determined in these programs through a section analysis on the basis of the a perfect bond assumption. Nevertheless, the slip effect must be taken into account in PSC structures with unbonded internal tendons in order to reach to an accurate estimation of the ultimate resisting capacity of the structure. The resisting capacity of a PSC structure with unbonded tendons is surely less than that of a structure with bonded tendons (Collins and Mitchell, 1991).

Therefore, an improved tendon model is introduced in this paper. The introduced model can consider the slip effect of tendon in commercialized software without using any link element that requires double nodes. Instead of utilizing double nodes, sequential iteration and correction procedures are introduced to satisfy the member-dependent properties of the unbonded internal tendons, and the efficiency of the introduced model is verified through correlation studies between experimental data and numerical results.

Especially in design practice, engineers usually use general-purpose programs which can analyze the non-linear behavior of prestressed concrete structures. Therefore, the easy implementation of the introduced numerical model into these programs is also emphasized. In addition, correlation studies between analytical results and experimental values for 1/14 scale mock-up model of Gentilly-2 with bonded tendons and 1/4 scale mock-up model of Ohi-3 with unbonded tendons are conducted. Load–displacement relations of the containment structures following the loading history are then evaluated using commercialized software to verify the soundness of the proposed model and to show its applicability.
2. Material properties

Since three different materials of concrete, steel reinforcing bars and prestressing tendons are used in the prestressed concrete containment structure, the material properties and the constitutive relationships for each material need to be defined in order to simulate the structural responses according to the loading history. In this paper, the numerical analyses are conducted using DIANA 8.1, in which many numerical models are already included and a user defined material model can also be implemented with ease. Accordingly, explanations and characteristics of the adopted constitutive models are briefly discussed in this paper, and more details for each material model can be found in the technical manual of DIANA 8.1 (TNO, 2002).

2.1. Concrete

Under combinations of biaxial stress, concrete exhibits strength and stress–strain behavior that are different from those under uniaxial loading conditions by the effects of Poisson’s ratio and microcrack confinement. The continuous line in Fig. 1 shows the biaxial strength envelope of concrete under proportional loading. In the biaxial compression state of stress, concrete exhibits an increase in compressive strength of up to 25% of the uniaxial compressive strength, when the stress ratio $\frac{\sigma_1}{\sigma_2}$ is 0.5. When the stress ratio $\frac{\sigma_1}{\sigma_2}$ is equal to 1.0, the concrete compressive strength is approximately $1.16 f_c'$, where $f_c'$ is the uniaxial concrete compressive strength. Under a combination of tension and compression, the compressive strength decreases almost linearly with increasing principal tensile stress.

In order to simulate the change of material properties according to the stress state, it is necessary to define the biaxial strength envelope. In contrast to a wall subject to shear forces, where the main part experiences biaxial stress combinations in the tension–compression region (Kwak and Kim, 2004a, b), most of a beam or slab experiences a biaxial stress combination in the tension–tension or compression–compression region and only small portions near the supports lie in the tension–compression region. In particular, a PCCV subject to internal pressure is dominantly affected by the material behavior in the tension–tension region. Accordingly, the biaxial strength envelope in the tension–compression region could be disregarded. The biaxial strength envelope proposed by Kupfer and Gerstle (1973) is modified, and the dotted line in Fig. 1 shows the simplification adopted in this paper. If the same simplification of the strength envelope is used for the analysis of shear walls, then reliable numerical results cannot be expected.

In describing the uniaxial stress–strain relation of concrete, the model of Hognestad (1951) is used with the equivalent concrete compressive strength determined from the biaxial strength envelope of concrete and the corresponding tensile stress defined by Maekawa et al. (2003) (see Fig. 2). To simulate the stress state of concrete under biaxial loading, the plasticity model is adopted in this study for its simplicity and computational efficiency. In DIANA, the combined Rankine/Drucker–Prager yield criterion is applicable in the compressive region and the Rankine criterion bounds the tensile stresses. The used concrete model accounts for progressive cracking and changes in the crack direction by assuming that the crack is always normal to the total principal strain direction (the rotation crack model).

2.2. Steel reinforcing bar

Steel reinforcing bar is usually modeled as linear elastic, linear strain-hardening material with yield stress $f_y$. However, when reinforcing bars are surrounded by concrete, as in membrane elements, the average behavior of the stress–strain relation is quite different, as shown in Fig. 3. The most variant feature is the
shown in Fig. 3, the average stress, this paper to revise the monotonic envelope curve of steel. As by Belarbi and Hsu (1994) from experimental data, is used in linearized average stress–strain relation, which was introduced needs to be defined. Considering these factors, the following stress–strain relations, the average stress–strain relation of steel and the behavior of cracked concrete is represented by average distributed over some tributary area within the finite element in which the local displacement discontinuities at cracks are

**lowering of the yield stress below** $f_y$. Yielding of an RC panel occurs when the steel stress at the cracked section reaches the yield strength of the bare bar. However, the average steel stress at a cracked element still maintains an elastic stress less than the yield strength, because the concrete matrix located between cracks is still partially capable of resisting tensile forces, owing to the bond between the concrete and reinforcement. Determination of the element stiffness on the basis of the yielding of steel at a cracked section at which a local stress concentration appears in the steel may cause an overestimation of the structural response at the post-yielding range. As this phenomenon is accelerated with an increase of the deformation, the analysis of RC panels subject to cyclic loading accompanying relatively large deformations requires the use of average stress–strain relations (Kwak and Kim, 2004a,b).

Accordingly, to trace the cracking behavior of RC panels up to the ultimate limit state by using the smeared crack model, in which the local displacement discontinuities at cracks are distributed over some tributary area within the finite element and the behavior of cracked concrete is represented by average stress–strain relations, the average stress–strain relation of steel needs to be defined. Considering these factors, the following linearized average stress–strain relation, which was introduced by Belarbi and Hsu (1994) from experimental data, is used in this paper to revise the monotonic envelope curve of steel. As shown in Fig. 3, the average stress, $\sigma_c$, is a linear function of the parameter $B = (f_{c,\text{eff}} f_{y})^{1.5}/\rho$ limited by the boundary strain $\varepsilon_y$ for the yielding of steel, where $\rho$ is the percentage of the steel ratio and must be greater than 0.5%. More details for the average stress–strain relation of steel can be found elsewhere (Belarbi and Hsu, 1994).

3. Numerical modeling of tendons

3.1. Bonded tendon

In a cracked cross-section of an RC structure, all tensile forces are balanced by the steel encased in the concrete matrix only. However, between adjacent cracks, tensile forces are transmitted from the steel to the surrounding concrete by bond forces. This effect is called the tension stiffening effect. To verify the bond–slip mechanism, accordingly, many experimental and numerical studies have been conducted (ibid, 2006; Kwak and Kim, 2002). In early studies, the characterization itself of the tension stiffening effect due to the non-negligible contribution of cracked concrete was the main objective. Recently, following the introduction of non-linear fracture mechanics in RC theory (CEB, 1996), more advanced analytical approaches have been conducted (CEB, 1996), and many numerical models that can implement the tension stiffening effect into the stress–strain relation of concrete have been proposed (Maekawa et al., 2003; Kwak and Kim, 2001).

In addition to the use of an average stress–strain relation of concrete that includes the strain softening branch in the tension region, modification of the stress–strain relation for steel must also be accomplished for an accurate assessment of the tension stiffening effect (Kwak and Kim, 2004a,b). Reinforcing steel is usually modeled as a linear elastic, linear strain-hardening material with yield strength $f_y$. However, when reinforcing bars are surrounded by concrete, the average behavior of the stress–strain relation is quite different. The most different feature is the lowering of the yield stress below $f_y$. Even though the steel stress reaches the yield strength of a bare bar at a cracked section, the average steel stress at a cracked element still maintains an elastic stress less than the yield strength. This is because the concrete matrix located between cracks is still partially capable of resisting tensile forces, owing to the bond between the concrete and reinforcement (see Fig. 4). Determination of the element stiffness on the basis of the yielding of steel at a cracked section at which a local stress concentration appears in the steel may cause an overestimation of the structural response at the post-yielding range.

Accordingly, to trace the cracking behavior of a RC structure up to the ultimate limit state by using the smeared crack model, in which the local displacement discontinuities at cracks are distributed over some tributary area within the finite element and the behavior of cracked concrete is represented by average stress–strain relations, the average stress–strain relation of steel needs to be defined together with the introduction of the strain softening branch in the stress–strain relation of concrete (Kwak and Song, 2002). Considering these factors, a few modified average stress–strain relations of steel have been proposed and are popularly employed in the numerical analysis of RC structures (Belarbi and Hsu, 1994; Kwak and Kim, 2004a,b). Nevertheless, direct application of these models into the numerical analysis of PSC structures may not be appropriate because the bond characteristics between tendon and concrete are different from those between reinforcing steel and concrete. Therefore, the average stress–strain relation of a bonded tendon is introduced in this paper on the basis of the bond properties in PSC structures.

The internal tendons incased in a metal sheath are unified with the concrete matrix through grouting. Since the grout material, which consists of a mixture of cement and water (water/cement ratio of about 0.5) together with a water-reducing admixture and an expansive agent, has sufficient strength to bond the tendons to the surrounding concrete, it can be equally considered as concrete while modeling PSC structures with finite elements. On the other hand, the bond characteristics of prestressing tendons
present numerous differences with the bond characteristics of reinforcing bars. To define the bond stress–slip relation between a prestressing tendon and grout material, a formula introduced by Balázs (1992) on the basis of experimental studies is used in this paper.

\[
f_b(x) = \psi c \sqrt{\frac{f'_c}{2}} (\frac{x}{d})^b = \psi c \sqrt{\frac{f'_c}{E_p}} (\frac{x}{d})^b
\]  

where \( c = 2.055 \text{ MPa}^{1/2}, \psi \) the constant defining the upper and lower limits of bond stress and has the average value of 1.0, \( f'_c \) the compressive strength of the grout material, \( d_p \) the diameter of the tendon and \( b \) in the range of \( 0 < b < 1 \) represents the tendon type and has a value of 0.25 in the case of seven-wire strands.

From the strain distribution along the tendon, the local slip \( s(x) \) can be defined as the total difference in elongations between the tendon and the concrete matrix measured over the length between a distance \( x \) from the separation point between the tendon and the concrete matrix and a crack force in Eq. (4) bounded by two adjacent cracks; that is, \( s(x) = \left( \varepsilon_{c}(x) - \varepsilon_{p}(x) \right) d_{f} \). In advance, on the basis of the force equilibrium, the following very well-known governing differential equation for the bond–slip normalized with respect to \( d_p \) can be obtained (Balázs, 1992; Kwak and Song, 2002).

\[
\delta^b(\xi) - K_b f_b(\delta(\xi)) = 0
\]  

where the normalized slip \( \delta(\xi) = s(\xi)/d_{f} \) is the normalized length from the crack surface, \( \xi = x/d_{f} \) is the normalized length from the crack face, \( n = E_p/E_c \), the steel ratio \( \rho_p = A_p/A_c = 1 + \rho_p00/E_c(\varepsilon_p + \varepsilon_{c}) \), and \( E_p \) and \( \rho_p \) are Young’s modulus and sectional area of the tendon, respectively.

The general solution of Eq. (2) is obtained by applying the boundary condition at the crack face and at the center of the cracked region like the anchorage region of Balázs (1992). After obtaining the general solution for the bond–slip, the corresponding bond stress along the steel axis is successively calculated using the force equilibrium and the compatibility condition at an arbitrary location. Moreover, the tendon stress at an arbitrary location between two adjacent cracks can be inferred from the superposition of the bond stress, transformed to the sectional location between two adjacent cracks can be inferred from the stress difference will be smaller than that calculated by Eq. (6).

\[
\sigma_p(x) = \sigma_p0 + \left( \frac{S/2}{d_p} \right) \int_0^x f_b(\delta(\xi)) d\xi = \sigma_p0 + \psi c \sqrt{\frac{f'_c}{E_p}} (\frac{x}{d})^b
\]

where \( \sigma_p0 \) is the bond stress at the center of the cracked region (see Fig. 4).

The stress distribution of a bonded tendon along the length shows a different shape from that of a reinforcing steel, which is represented by a cosine shape with zero slope at the crack face (Belarbi and Hsu, 1994). In addition, in contrast with the bond characteristics in RC members, which show a decrease of bond stress with an increase of slip after reaching the maximum bond stress, the bond stress between tendon and concrete maintains its maximum value up to a larger bond–slip range at the cracked region.

Since the value of \( \sigma_{p0} \) is in the range of 0–0.1 and \( b=0.25 \) for the seven-wire strands, the variation of \( B = \psi c \sqrt{\frac{f'_c}{E_p}}(\xi^3/E_{p})^{1/3} \) MPa is relatively small, and Eq. (3) can be simplified by using the representative value of \( B \).

\[
\sigma_p(x) = \sigma_p0 + 8.5 \psi c \sqrt{\frac{f'_c}{E_p}} (\xi^3/E_{p})^{1/3}
\]

On the basis of the bond stress distribution at the cracked region in Fig. 4, the bond stress difference between the stress at the crack face and the average stress uniformly distributed within the two adjacent cracks can be calculated by

\[
\sigma_p(x) = \sigma_p0 + \left( \frac{S/2}{d_p} \right) \int_0^x f_b(\delta(\xi)) d\xi \]

The average value of \( x = 200 \text{ mm} \) and \( E_p = 200,000 \text{ MPa} \) are used. Eq. (4) is obtained by applying the boundary condition at the crack face and at the center of the cracked region like the anchorage region of Balázs (1992). After obtaining the general solution for the bond–slip, the corresponding bond stress along the steel axis is successively calculated using the force equilibrium and the compatibility condition at an arbitrary location. Moreover, the tendon stress at an arbitrary location between two adjacent cracks can be inferred from the superposition of the bond stress, transformed to the sectional stress by multiplying 4\( \psi \), on the tendon stress \( \sigma_p0 \) at the center of the cracked region (see Fig. 4).

\[
\sigma_p(x) = \sigma_p0 + 4\psi c \sqrt{\frac{f'_c}{2}} (\frac{x}{d})^b = \sigma_p0 + \psi c \sqrt{\frac{f'_c}{E_p}} (\frac{x}{d})^b
\]

The stress distribution of a bonded tendon along the length shows a different shape from that of a reinforcing steel, which is represented by a cosine shape with zero slope at the crack face (Belarbi and Hsu, 1994). In addition, in contrast with the bond characteristics in RC members, which show a decrease of bond stress with an increase of slip after reaching the maximum bond stress, the bond stress between tendon and concrete maintains its maximum value up to a larger bond–slip range at the cracked region.
The yield stress difference in Table 1 has a maximum value of 81.1 MPa corresponding to about 5% of the yield strength of the tendon. However, this appears to be relatively small in comparison with that of mild steel, which shows an approximate 25% difference between the yield strength \( \sigma_y \) and apparent yield stress \( \sigma_{py} \) (Belarbi and Hsu, 1994). The yield stress difference will also be smaller in real PSC structures where a tendon ratio of more than 0.5% is generally applied. Thus, no remarkable difference in the stress–strain relations of bare tendon and embedded tendon is expected. Accordingly, only slight modification of the stress–strain relation for a bare tendon, rather than deriving a new relation, is sufficient in defining the stress–strain relation of an embedded tendon. Fig. 5 shows the modification of the stress–strain relation. Since the linear elastic range is the same as that of the bare tendon, point A in Fig. 5, which represents the minimum steel ratio of 0.5% is assumed to maximize the yield stress difference. The values in parentheses in Table 1 are calculated and compared in Table 1. The yield stress difference in Fig. 5 is a curve connecting three points A–B–C, where A denotes the upper limit for the linear elastic behavior, B is the point of testing the apparent yield stress, and C is the point of testing the apparent yield stress considering the bond characteristics of the embedded tendon bar. This means that the yield stress \( \sigma_{py} \) corresponding to the strain of 0.01 needs to be revised to \( \sigma_{py}^* \). From Eqs. (6) and (8), the yielding point \( \sigma_{py}^* \) of an embedded tendon bar can finally be calculated by

\[
\sigma_{py}^* = \min \left\{ \sigma_{py} - 6.3 \frac{f_{c1}}{\rho E_c} \sqrt{\frac{d_p}{d_t}} \frac{1}{\rho_1} \right\} \text{ for pre-tensioning}
\]

\[
\sigma_{py}^* = \min \left\{ \sigma_{py}^* - 6.3 \frac{f_{c1}}{\rho E_c} \sqrt{\frac{d_p}{d_t}} \frac{1}{\rho_1} \right\} \text{ for post-tensioning}
\]

To trace the average stress–strain relation of a tendon generally used in a prestressed concrete structure, the yield stress differences defined in Eq. (9) are calculated and compared in Table 1. The tensile strength and Young’s modulus of concrete are assumed to be 2.75 MPa and 32,800 MPa, respectively, and the minimum steel ratio of \( \rho = 0.5\% \) is assumed to maximize the yield stress difference. The values in parentheses in Table 1 are determined from the upper boundary values calculated by Eq. (6). As shown in this table, the yield stress differences are gradually governed by Eq. (6) as the nominal diameter of the tendon increases, because this increase accompanies an extension of the bond–slip region at the cracked member.

The yield stress difference in Table 1 has a maximum value of 81.1 MPa corresponding to about 5% of the yield strength of the tendon. However, this appears to be relatively small in comparison with that of mild steel, which shows an approximate 25% difference between the yield strength \( \sigma_y \) and apparent yield stress \( \sigma_{py} \) (Belarbi and Hsu, 1994). The yield stress difference will also be smaller in real PSC structures where a tendon ratio of more than 0.5% is generally applied. Thus, no remarkable difference in the stress–strain relations of bare tendon and embedded tendon is expected. Accordingly, only slight modification of the stress–strain relation for a bare tendon, rather than deriving a new relation, is sufficient in defining the stress–strain relation of an embedded tendon. Fig. 5 shows the modification of the stress–strain relation. Since the linear elastic range is the same as that of the bare tendon, point A in Fig. 5, which represents the upper limit for the linear elastic behavior, can be determined easily from the stress–strain relation of the bare tendon. Point B, representing the apparent yield stress, is then calculated by Eq. (9), followed by determination of point C from the experimental data. In addition, each range bounded by two points (range A–B or B–C in Fig. 5) in a curved line connecting three points A–B–C can be simplified with a linear or multi-linear relation.

3.2. Unbonded tendon

In contrast with the bonded tendon in which the tension stiffening effect is emphasized, the most dominant effect in the structural response of PSC structures with unbonded tendons is the slip behavior along the tendon sheath. Since the stress increase in the tendons due to external loading is not section-dependent but member-dependent, it cannot be determined from the analysis of a beam cross-section. Rather, it must be determined from the total deformations of the entire structure in the elastic as well as the ultimate limit state. In order to consider the slip in the numerical analysis of a PSC structure with unbonded tendons, accordingly, many analysis methods have been introduced. Naaman and Alkhairi (1991) proposed a simplified analytical method to determine the strain in a simple beam with a symmetrical tendon profile and subject to symmetrical load. Wu et al. (2001) introduced a friction model that considers friction at the interface of the tendon and concrete using the spring element, and more general analyses of the slip behavior using the finite element method have also been proposed. However, most slip models are based on the spring element that connects the concrete node and the tendon node. As mentioned above, since the slip element in the finite element analysis imposes many restrictions in use, direct application of this element to a large complex structure reinforced with unbonded tendons may be impossible. Accordingly, to take into account the slip effect, an iterative approach is introduced in this paper, instead of taking double nodes as in the case of the spring element, and the cal-
Strain $\varepsilon$ is maintained bonding with the concrete. If the strain due to external load is calculated slip effect is implemented into the stress–strain relation of the tendon can be defined by Eq. (11). Occurs, and the corresponding point in the stress–strain relation to be the strain at a critical section in which the maximum stress value of tendon can be calculated by Eq. (10). In addition, the strain are expected to be produced in the same structure with unbonded tendons. When the results do not reach convergence, re-analyses are repeated on the basis of the revised stress–strain relation, as was the case for the first iteration (see Eq. (13)), until the convergence check is satisfied. These steps are presented in greater detail in Eq. (14).

First, a PSC structure with unbonded tendons subjected to an applied external load $p$ is analyzed under the assumption that the tendons maintain bonding with the concrete. If the strain $\varepsilon(x, p)$ in Fig. 6 represents the obtained strain distribution along the tendon length in a structure with bonded tendons, then the average strain $\varepsilon_{\text{ave}}(p)$ and the corresponding average stress $\sigma_{\text{ave}}(p)$, which are expected to be produced in the same structure with unbonded tendons, can be calculated by Eq. (10). In addition, the strain value $\varepsilon_{\text{max}}(p)$ in the strain distribution $\varepsilon(x, p)$ may be assumed to be the strain at a critical section in which the maximum stress occurs, and the corresponding point in the stress–strain relation of tendon can be defined by Eq. (11).

$$\varepsilon_{\text{ave}}(p) = \frac{1}{L} \int_0^L \varepsilon(x, p) \, dx, \quad \sigma_{\text{ave}}(p) = f(\varepsilon_{\text{ave}}(p))$$  \hspace{1cm} (10)$$

$$\varepsilon_{\text{max}}(p) = \max(\varepsilon(x, p)), \quad \sigma_{\text{max}}(p) = f(\varepsilon_{\text{max}}(p))$$  \hspace{1cm} (11)$$

where $f$ represents the stress–strain relation of a bare tendon and $L$ is the total length of a tendon between both anchorages.

As shown in Eqs. (10) and (11), both values of the average strain $\varepsilon_{\text{ave}}(p)$ and the maximum strain $\varepsilon_{\text{max}}(p)$ change with the magnitude of the applied load $p$, which directly affects the tendon force in the prestressing steel. This means that the modified stress–strain relation of embedded tendons, as shown in Fig. 7, is not uniquely defined, but rather it changes in accordance with the applied external load $p$ and the tendon layout along the length in a PSC structure. Accordingly, it is reasonable to construct the modified stress–strain relation of the tendon on the basis of the stress and strain at the critical section, where these represent the maximum values, because all the structural responses from the uncracked elastic to the ultimate limit states are dominantly affected by the structural behavior at this section. Unlike the bonded tendon, which presents the same strain distribution along the length as that of the concrete, the unbonded tendon presents a uniform strain distribution (see Fig. 6). Therefore, the tendon stress $\sigma_{\text{ave}}(p)$ corresponding to the maximum strain of $\varepsilon_{\text{max}}(p)$ needs to be revised to the average stress $\sigma_{\text{ave}}(p)$, because the non-uniform strain distribution with the maximum value of $\varepsilon_{\text{max}}(p)$ is averaged in the case of an unbonded tendon. That is, as shown in Fig. 7, point A in a bare tendon is moved to point B in an unbonded internal tendon to implement the slip behavior of the tendon. The same modification procedures are repeated for a few different load levels to obtain a completely modified stress–strain relation of the tendon defined for the entire stress range.

On the other hand, when a PSC structure with unbonded tendons experiences the crushing failure of concrete before yielding of the tendon, the modified stress–strain relation of the unbonded tendon is only defined up to the strain of the tendon corresponding to the failure of the structure. The following relation is then assumed to be extended to the ultimate state with the same modulus of elasticity determined at the modified stress–strain relation just before the failure of the structure.

Finally, the modified stress–strain relation of $g_1$ (continuous line in Fig. 7) can be redefined as $g_1^{-1}(\sigma_{\text{ave}}(p_i)) = f^{-1}(\sigma_{\text{max}}(p_i))$ (12) where $f$ is the stress–strain relation of a bare tendon (discontinuous line in Fig. 7) and $p_i$, where $f_{\text{ave}} \leq p_i \leq f_{\text{ave}}$, is the tendon force between the two boundary values of the effective tendon force and the ultimate tendon force.

On the basis of the modified stress–strain relation $g_1$, a second finite element analysis of the same structure is conducted. Even though the slip effect in the unbonded tendon is indirectly taken into account in the results obtained from the re-analysis, the obtained results must be checked in terms of whether they effectively represent the unbonded characteristics of the tendons. When the results do not reach convergence, re-analyses are repeated on the basis of the revised stress–strain relation, as was the case for the first iteration (see Eq. (13)), until the convergence check is satisfied. These steps are presented in greater detail in Eq. (14).
Since the maximum tendon stress \( \sigma_{j,\text{max}}(p_j) \) experienced at the critical section at each iteration \( 1 \leq j \leq n \) is adjusted to the average value reflecting the characteristics of the unbonded tendon, the convergence can be checked by comparing the two stress components of the maximum tendon stress \( \sigma_{j,\text{max}}(p_j) \) obtained from the finite element analysis and the average tendon stress \( \sigma_{j,\text{ave}}(p_j) \) corresponding to the average strain from Eq. (10). In this paper, the convergence criterion employed is

\[
\frac{\sqrt{\sum (\sigma_{j,\text{ave}}(p_j) - \sigma_{j,\text{ave}}(p_j))^2}}{\sqrt{\sum (\sigma_{j,\text{ave}}(p_j))^2}} \leq \text{TOLER} \tag{14}
\]

where TOLER is the specified tolerance and a tolerance of 1–5% gives a satisfactory convergence.

In addition to the slip phenomenon in an unbonded tendon, additional prestressing losses such as friction losses, anchorage slip and relaxation with time are developed even though the short-term losses occurred at the jacking stage represent relatively small values in the case of unbonded internal tendons and are assumed to be constant after anchorage. Moreover, the tendon force variation by these prestressing losses seems to be very small in comparison with the tendon force change along the length due to the slip behavior in an unbonded tendon and becomes negligibly small as the applied lateral load increases. Nevertheless, when required, additional consideration for these losses is possible by Naaman and Alkhairi (1991), which defines the ratio of the average concrete strain increment in the unbonded tendon to the strain increment in the equivalent bonded tendon at the section of maximum moment in a PSC beam, definition of a modified stress–strain relation for an unbonded tendon becomes possible. In particular, the derivation procedure on the basis of the strain compatibility condition makes it possible to consider the slip behavior indirectly even in the case of modeling an unbonded tendon with embedded and/or distributed steel models. In advance, since any limitation in numerical modeling of a structure to consider the relative slip behavior is not required, the introduced numerical model can effectively be used in the analysis of large complex PSC structures with unbonded tendons using commercialized large programs such as ADINA (2002), ABAQUS (2004) or DIANA (TNO, 2002), regardless of the structural type and loading history.
prestressing losses can be achieved according to the method adopted in classical approaches after determining the average stress along the length with the introduced numerical approach. More details related to the numerical implementation of prestressing losses can be found elsewhere (Collins and Mitchell, 1991).

4. Solution algorithm

Every non-linear analysis algorithm consists of four basic steps: the formation of the current stiffness matrix, the solution of the equilibrium equations for the displacement increments, the state determination of all elements in the model and the convergence check. Ultimately, since construction of the global stiffness matrix and determination of the deformation state of the structure are initiated from the definition of the stress–strain relation of each material, introduction of an accurate stress–strain relation that takes into account many influencing factors is important in simulating the non-linear behavior of PSC structures. In this regard, the tendon models introduced in this paper can be effectively used. To analyze PSC structures, the DIANA 8.1 general purpose finite element program (TNO, 2002) is used, and the other material models including that for concrete are defined according to the CEB-FIP MC90 (1990). In advance, the non-linear solution scheme selected in this paper uses the tangent stiffness matrix at the beginning of the load step in combination with a constant stiffness matrix during the subsequent correction phase, that is, the incremental–iterative method.

The criterion for measuring the convergence of the iterative solution is based on the accuracy of satisfying the global equilibrium equations or on the accuracy of determining the total displacements. The accuracy of satisfying the global equilibrium equations is controlled by the magnitude of the unbalanced nodal forces. The accuracy of the node displacements depends on the magnitude of the additional displacement increment after each iteration. The latter convergence criterion is used in this study (Kwak and Filippou, 1990). This can be expressed as

\[
\sqrt{\sum \left(\frac{\Delta d_i}{d_i}\right)^2} \leq \text{TOLER}
\]

where the summation extends over all degrees of freedom \( j \), \( d_i \) the displacement of degree of freedom \( j \), \( \Delta d_i \) the corresponding increment after iteration \( i \) and TOLER is the specified tolerance.

In the non-linear analysis of a RC structure the load step size must be small enough so that unrealistic “numerical cracking” does not take place. These spurious cracks can artificially alter the load transfer path within the structure and result in incorrect modes of failure. Crisfield (1982) has shown that such numerical disturbance of the load transfer path after initiation of cracking can give rise to alternative equilibrium states and thereby lead to false ultimate strength predictions. In order to avoid such problems after crack initiation the load is increased in steps of 2.5–5.0% of the ultimate load of the member.

The failure load is assumed to occur at a load level for which a large number of iterations is required for convergence. This means that very large strain increments take place during this step and that equilibrium cannot be satisfied under the applied loads. Obviously, the maximum number of iterations depends on the problem and the specified tolerance, but a maximum of 30 iterations seems adequate for a tolerance of 1%. This is the limit in the number of iterations selected in this study.

5. Applications

5.1. PSC beam

To verify the proposed analytical model, two two-span continuous PSC beams with bonded internal tendons are investigated. These beams, A and B, are tested by Lin (1955) to determine the cracking behavior and ultimate strength of PSC beams. The geometry and cross-section dimensions of the adopted beams are presented in Fig. 9, and the material properties of concrete reinforcing bar and tendon are summarized in Table 2. The other material properties not mentioned in this paper are determined in accordance with the CEB-FIP MC90 (1990). The prestressing tendon has a concordant profile in this specimen and consists of a straight part, which extends from the end of the beam to the point where the concentrated load is applied, and a curved part over the center support. Moreover, beam B

![Fig. 9. Configuration of two-span continuous PSC beam (unit: mm).](image-url)
is reinforced with mild reinforcing bars while beam A is not reinforced.

The concrete was modeled by eight-node serendipity plane stress elements with a $3 \times 3$ Gauss integration and the tendon and reinforcements were modeled by an embedded two-node truss element. The number of elements used through the depth and the length of the member are 4 and 150, respectively. Since the plastic hinge length $l_p$ calculated by the simple equation proposed by Sawyer (1964) is determined as 20 cm, the specimen is modeled along the entire span with an element of $l = 10$ cm to obtain an analytical result which is free from the mesh-dependency.

The correlation between the measured load to the mid-span deflection curves of the two beams and the analytical results is shown in Fig. 10. As shown in this figure, the numerical results obtained by using the modified stress–strain relation of tendon according to the introduced numerical approach and also by considering the tension stiffening effect in concrete give very good agreements with experimental results throughout the entire loading history. On the other hand, the inclusion of only tension stiffening effect produces a slightly overestimation of the ultimate resisting capacity, but the exclusion of tension stiffening effect underestimate the ultimate resisting capacity and gives a soft cracking behavior. It is clear from the comparison of these numerical results with the experimental data that the consideration of tension stiffening effect together with the modification of stress–strain relation of tendon yields a very satisfactory agreement for the structural stiffness and ultimate capacity.

Unlike a beam where the plastic deformation is widely distributed, the plastic deformation in beams A and B subjected to two concentrated load (see Fig. 9), is concentrated at the center support with narrow width, where the occurrence of plastic rotation is initiated and concentrated. This range is called the plastic hinge length. Various empirical expressions have been proposed by investigators for the equivalent length of the plastic hinge $l_p$ (Park and Paulay, 1975). Since the structure is modeled with finite elements whose displacement field is defined by the average deformation of nodes, the ultimate capacity can be overestimated if the plastic hinge length is not precisely taken into consideration.

In order to study the effect of finite element mesh size on the analytical results, accordingly, three different meshes with $l = 10$ cm, 43 cm and 130 cm for the region between the loading point and the center support are investigated. As shown in Fig. 11, the exclusion of plastic hinge length when the element size is greater than the expected plastic hinge length ($l_p$ is about 20 cm in this example structure) may yield an overestimated ultimate load. On the other hand, the numerical results when the element size was 5 cm was exactly the same with those of $l = 10$ cm. Accordingly, three effects of the tension stiffening, modification of stress–strain relation of tendon and plastic hinge length must be considered to reach to a very satisfactory agreement of the model with reality.

5.2. PSC slabs

Tension stiffening has a significant effect in the analysis of RC slabs. In order to investigate the validity of the proposed unbonded tendon model together with an assessment of the necessity for consideration of the tension stiffening effect, two three-span continuous slabs tested by Burns et al. (1978) are used in the correlation studies. The geometry and cross-section dimensions of the adopted slabs are presented in Fig. 12, and the material properties are summarized in Table 3. The unbonded internal tendons are placed along the total span and mild steels are reinforced at the maximum positive and negative moment

<table>
<thead>
<tr>
<th>Table 3 Material properties used in slabs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
</tr>
<tr>
<td>$f'_c$ [MPa] 32.4</td>
</tr>
<tr>
<td>Reinforced steel</td>
</tr>
<tr>
<td>$A_s$ [mm$^2$] 28.3</td>
</tr>
<tr>
<td>$f_y$ [MPa] 448</td>
</tr>
<tr>
<td>Tendon</td>
</tr>
<tr>
<td>$A_p$ [mm$^2$] 31.68</td>
</tr>
<tr>
<td>$f_{pe}$ [MPa] 976</td>
</tr>
<tr>
<td>$f_{pu}$ [MPa] 1655</td>
</tr>
</tbody>
</table>
regions to prevent abrupt failure and to reserve additional ductility of the structure.

The two slabs are identical except for the loading condition. The first slab, A108, is subjected to a gradually increased live load on the first and third spans under application of a self-weight of \( w_D = 3304 \text{ Pa} \) and an additional live load of \( w_L = 1317 \text{ Pa} \) on the center span, while the second slab, A109, is subjected to a gradually increased live load on the first and second spans under application of a self-weight of \( w_D = 3304 \text{ Pa} \) and an additional live load of \( w_L = 1317 \text{ Pa} \) on the third span. A commercialized program, DIANA 8.1 (2002), is used, after defining the stress–strain relations of tendon and concrete on the basis of the introduced tendon model and the strain softening branch of the concrete. Eight-node laminate shell elements are used for the finite element idealization of the slab and 60 elements with 8 imaginary concrete layers through the thickness of the slab are used in the analysis.

Figs. 13a and 14a, comparing the analytical load–deflection relation at point B with the measured experimental data and the other results by Allouche et al. (1999), show that the inclusion of both effects (tension stiffening and slip) yields satisfactory agreement between the model and reality. As shown in these figures, the specimen cracking behavior may be significantly affected by the loading history. Further, the numerical analyses may have some limitations in tracing the cracking behavior depending on the loading history in shallow bending members, which are dominantly affected by the tension stiffening effect. On the other hand, the live load of \( w_L = 7 \text{ kPa} \) closely approximates the ultimate load of the specimens.

In advance, Figs. 13b and 14b show that tension stiffening affects the non-linear behavior of the slab much more than slip, and ignoring the tension stiffening effect clearly leads to underestimation of the ultimate resisting capacity of PSC slabs. Conversely, ignoring the slip effect leads to overestimation of the ultimate resisting capacity of the slabs. The present finding that slip is an important factor in multi-span continuous slabs also agrees with the results of simply supported beams, where the slip behavior along the single curvature is clearly explained. The difference in numerical results between considering and not considering the tension stiffening or slip effect is enlarged with an increase in the applied load.

5.3. CANDU containment

The proposed analytical model is also applied to a 1/14 prestressed concrete containment model (PCCM) with internal bonded tendons, tested by Rizkalla et al. (1984). As shown in Fig. 15a, the containment is composed of a circular base slab, an upright cylinder with four vertical buttresses to accommodate anchorages located symmetrically around the wall, and a rela-
The containment is composed using 1380 eight-node layered shell elements for the wall and lower part of the dome, 125 six-node triangular layered shell elements for the upper part of the dome and 2024 twenty-node solid elements for the foundation and buttresses on the wall, respectively (see Fig. 15c and d). Since the non-linear finite element program DIANA 8.1 is used to investigate the ultimate pressure capacity and the failure mode of the containment, more details related to the characteristics for the finite elements used can be found elsewhere (TNO, 2002).

Fig. 16, representing typical sections across the depth in the wall and dome, shows the placement of reinforcing steels and internal tendons. Reinforcing steel bars placed in the circumferential direction and in the vertical direction are taken into account as an equivalent steel layer, while the internal tendons are described by the truss elements maintaining the placing space (see Fig. 15b). The material properties of concrete, reinforcing steel, and tendon are given in Table 4, and other assumed material properties not mentioned are determined on the basis of the CEB-FIP MC90 (1990).

To simulate the semi-rigid connectivity at the joint between the base slab and the wall in the 1/14 PCCM, where rotational deformation is allowed, the hinge connection is adopted for the containment, and the numerical modeling of the tendon is extended beyond the connection to provide an additional partial restraint to the rotation. In advance, to describe the tensioning sequence in the tendon, sequential loading steps of self-weight, post-tensioning and internal pressure are considered. In addition, the prestressing losses caused by the friction $\Delta f_{fr}$ and anchorage slip $\Delta f_{anc}$ are taken into consideration according to the relations of $\Delta f_{fr} = f_j(1 - e^{-\frac{(L - \Delta L)}{L_{not}}})$ and $\Delta f_{anc} = E_p \Delta L\Delta L_{not}$ noted in the ACI 318 (2004). However, the stiffness of the tendon is not added until the containment is subjected to internal pressure loading because the tendon is hereafter integrated with the concrete containment. Fig. 17 compares the analytical results with the measured internal pressure–displacement response of

![Fig. 13. Numerical results of slab A108: (a) load–displacement relationship and (b) tension stiffening and slip effect. Solution A: modified with tension stiffening; solution B: bare with tension stiffening; solution C: modified without tension stiffening; solution D: bare without tension stiffening.](image1)

![Fig. 14. Numerical results of slab A109: (a) load–displacement relationship and (b) tension stiffening and slip effect. Solution A: modified with tension stiffening; solution B: bare with tension stiffening; solution C: modified without tension stiffening; solution D: bare without tension stiffening.](image2)
Very satisfactory agreement between analysis and experiment is observed. To identify the relative contribution of the tension stiffening effect, another analysis ignoring the tension stiffening effect is performed for this example structure. From Fig. 17a and b, it is clear that inclusion of the strain softening branch in the tension part of concrete (see Fig. 2) with the modification of the stress-strain relation of reinforcing steel (see Fig. 3) and tendon (see Fig. 5) yields very satisfactory agreement between the model and reality. However, on the other hand, exclusion of the strain softening branch in the tension part of concrete causes underestimation of the ultimate resisting capacity of the structure in spite of using the original stress-strain relation.

Table 4

Material properties used in 1/14 PCCM

<table>
<thead>
<tr>
<th>Properties</th>
<th>Dome</th>
<th>Wall</th>
<th>Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>100 mm shell</td>
<td>127 mm shell</td>
<td>Solid</td>
</tr>
<tr>
<td></td>
<td>$\rho_{\text{c,in}} = 1.25%$, $\rho_{\text{c,out}} = 1.25%$</td>
<td>$\rho_{\text{c,in}} = 0.44%$, $\rho_{\text{c,out}} = 0.44%$</td>
<td></td>
</tr>
<tr>
<td>Concrete</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Re-bar</td>
<td></td>
<td>$f_y = 483$ MPa</td>
<td></td>
</tr>
<tr>
<td>Tendon</td>
<td>One 1/2 in. seven-wire strand ($A_p = 98.7 \text{mm}^2$) with $f_{pu} = 1862$ MPa</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 16. Typical section in 1/14 PCCM: (a) across the dome and (b) across the wall.

Fig. 17. Internal pressure–displacement relation of 1/14 PCCM: (a) radial displacement at wall, (b) vertical displacement at dome-apex, (c) hoop strain at wall and (d) meridional strain at dome-apex. Solution A: embedded tendon with tension stiffening; solution B: bare tendon without tension stiffening.
relations of bare reinforcing steel and bare tendon. However, no remarkable difference in the numerical results according to the modification of the stress–strain relation of tendon is found in this example structure. Notably, this structure shows brittle behavior, because of relatively large steel ratios of $\rho_s = 2.5\%$ at the dome and $\rho_s = 0.88\%$ at the wall.

In advance, Fig. 18, which represents the deformation shapes of the containment under a few typical internal pressure conditions, shows that remarkable deformation of the containment initiates at the wall to the foundation joint because these two parts are hinge-connected, while the four vertical buttresses restrict outward movement of the wall. As shown in Fig. 18b, the ring beam installed along the joint between the dome and the wall shows outward movement when the internal pressure reaches $P = 0.620$ MPa (equivalent to about two times the design internal pressure, $P = 0.3$ MPa). This means that the ring beam and four vertical buttresses anchor the prestressing pressure and finally leads to membrane failure of the structure.

At loads of $P = 0.248$ MPa, which is somewhat higher than the first cracking of the dome, circumferential interior cracking at the ring-beam region and exterior cracking at the mid-dome region are developed. An additional increase of the internal pressure causes the cracking of the wall. When $P = 0.372$ MPa, horizontal interior cracking at the buttresses is initiated together with vertical exterior cracking, and is uniformly distributed along the face between the two adjacent buttresses because of uniform

Table 5
Material properties used in 1/4 PCCV

<table>
<thead>
<tr>
<th>Property</th>
<th>Dome</th>
<th>Wall</th>
<th>Base</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td>325 mm + 1.6 mm shell</td>
<td>275 mm + 1.6 mm shell</td>
<td>Solid</td>
</tr>
<tr>
<td>$\rho_{sk}$</td>
<td>0.29%</td>
<td>0.80%</td>
<td>0.64%</td>
</tr>
<tr>
<td>Concrete</td>
<td>$f'<em>{c} = 53.4$ MPa, $f</em>{c} = 2.21$ MPa</td>
<td>$f'<em>{c} = 45.6$ MPa, $f</em>{c} = 2.21$ MPa</td>
<td></td>
</tr>
<tr>
<td>Re-bar</td>
<td>$f_{y} = 480$ MPa, $f_{u} = 629$ MPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liner</td>
<td>$f_{y1} = 375$ MPa, $\epsilon_{y1} = 0.18%$, $f_{y2} = 447$ MPa, $\epsilon_{y2} = 5.08%$, $f_{u} = 488$ MPa, $\epsilon_{u} = 33.2%$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tendon</td>
<td>Three 13.7 mm seven-wire strand ($A_p = 339$ mm$^2$) with $f_{pu} = 1886$ MPa, $\epsilon_{pu} = 3.37%$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 19. Finite element idealization of 1/4 PCCV: (a) configuration, (b) tendon layout, (c) layered shells and (d) solid elements.

Fig. 20. Typical section in 1/4 PCCV: (a) across the dome and (b) across the wall.
Fig. 21. Modified stress–strain relationships of unbonded tendons: (a) meridional tendon and (b) hoop tendon.

Pressure along the wall. Moreover, after maintaining the membrane behavior up to the ultimate loading condition, the example structure subsequently fails at $P = 0.930$ MPa because of yielding of the tendons in the wall.

5.4. PWR containment

The proposed analytical model for the unbonded internal tendon is also applied to a 1/4 PCCV tested by Hessheimer et al. (2003). As shown in Fig. 19a, the containment is composed of a circular base slab, an upright cylinder with two vertical buttresses to accommodate the anchorages being located symmetrically around the wall and a hemispherical dome. To simplify the analysis, equipment hatches and penetrations on the containment are not considered and a three-dimensional layered shell is based on the numerical modeling of the containment. Details of the geometry and dimensions of the containment are provided in reference Hessheimer et al. (2003).

Fig. 22. Internal pressure–displacement relation of 1/4 PCCV: (a) radial displacement at wall, (b) vertical displacement at dome-apex, (c) hoop strain at wall and (d) meridional strain at dome-apex. Solution A: modified tendon considering slip effects, solution B: bare tendon with no consideration of slip.
The containment is composed with 1144 eight-node layered shell elements for the wall and lower part of the dome, 180 six-node triangular layered shell elements for the upper part of the dome and 1928 twenty-node solid elements for the foundation and buttresses on the wall, respectively (see Fig. 19c and d). Since the non-linear finite element program DIANA 8.1 used to investigate the ultimate pressure capacity and the failure mode of the containment, more details related to the characteristics for the finite elements used can be found elsewhere (TNO, 2002). On the other hand, Fig. 20 representing typical sections across the depth in the wall and dome shows the placement of reinforcing steels and internal tendons. The reinforcing steel bars placed in the circumferential direction and in the vertical direction are taken into account as an equivalent steel layer, while the internal tendons are described by the truss elements maintaining the placing space (see Fig. 19b). The material properties of concrete, reinforcing steel and tendon are given in Table 5, and other assumed material properties not mentioned are determined on the basis of the CEB-FIP MC90 (1990).

In advance, to describe the tensioning sequence in the tendon, sequential loading steps of self-weight, post-tensioning and internal pressure are considered, and the prestressing losses caused by the friction \( \Delta f_0 \) and anchorage slip \( \Delta f_{anc} \) are taken into consideration according to the relations of \( \Delta f_0 = f_j (1 - e^{-k(\mu\alpha)}) \) and \( \Delta f_{anc} = E_p \Delta l/L \) mentioned in the ACI 318 (2004). However, the stiffness of the tendon is not added until the containment is subjected to the internal pressure loading because the tendon is hereafter integrated with the concrete containment. Fig. 21 shows the average stress–strain relations of unbonded internal tendons constructed according to the iteration procedure adopted to take into account the slip effect along the length. Unlike the tendons placed in the meridional direction, which shows a little different stress–strain relation from that of bare tendon, there is no difference for the tendons placed in the hoop direction. It means that no structural response between using bonded tendons and using unbonded tendons is expected in the hoop direction.

Fig. 22 compares the analytical results with the measured internal pressure–displacement responses of 1/4 PCCV. Very good agreement between analysis and experiment is observed for the ultimate resisting internal pressure in spite of a little stiff cracking response. From Fig. 22a and b, it is clear that the consid-
eration of the slip effect with the modification of the stress–strain relation of tendon yields a very satisfactory agreement between the model and reality. However, on the other hand, exclusion of the slip effect in tendon causes an overestimation of the ultimate resisting pressure of the structure.

In advance, Fig. 23, which represents the deformation shapes of the containment under a few typical internal pressure conditions, shows that the two vertical buttresses restrict outward movement of the wall. As shown in Fig. 23d, the result of the numerical model have the good agreement with the experiments, and finally lead to the membrane failure of the structure at \( P = 1.29 \) MPa (equivalent to about 3.3 times of the design internal pressure \( P = 0.39 \) MPa).

6. Conclusions

Modified stress–strain relationships of tendons are proposed for the non-linear finite element analysis of PSC containments with bonded and/or unbonded tendons. The proposed tendon models make it possible to analyze prestressed concrete contain- vessel structures using commercialized software such as ADINA, ABAQUS and DIANA, which are based on a perfect bond assumption. The proposed models do not require a double node to simulate tension stiffening or the slip effect developed at the interface of two adjacent materials of concrete and tendon, and as such they can effectively be used in modeling a large three-dimensional PSC structure. The introduced tendon models have been verified through a comparison of experimental data and numerical results.

Representative PSC beams were analyzed with the purpose of investigating the relative effects of bond-slip and tension stiffen- ing, and finally numerical analysis of a CANDU containment structure was conducted. On the other hand, representative PSC slabs were analyzed with the purpose of investigating the relative effects of slip and tension stiffening, and finally numerical analysis of a PWR containment structure was conducted. From the numerical analyses, in advance, the following conclusions were obtained: (1) the tension stiffening and slip effects are more dominant in a PSC structure with unbonded tendons; (2) ignoring the tension stiffening effect clearly leads to underestima- tion of the stiffness and ultimate resisting capacity of PSC structures, in contrast with the case of slip effect, and accord- ingly consistent numerical results for PSC structures can only be obtained when both effects are included in the numerical model; (3) the plastic hinge length must be considered to exactly predict the ultimate resisting capacity of concrete structures where the plastic deformation is concentrated at any location with narrow range; (4) for under-reinforced concrete structures, the use of modified stress–strain relations of reinforcing steel and tendon has the dominant influences in the cracking behavior and ultimate resisting capacity of structure, while its effect has been decreased with an increase of the steel ratio; (5) however, modi- fication of the stress–strain relation of an unbonded tendon must be preceded if no additional slip model is taken into account in the numerical modeling of a PSC structure, regardless of the steel ratio, because the slip effect, representing member-dependent behavior, is dominant.

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References


