Derivation of simplified analytic formulae for magneto-optical Kerr effects

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We have derived simplified analytic expressions for various magneto-optical Kerr effects of an optically thick medium having an arbitrary magnetization direction. We have found that the simplified formulae for the Kerr effects of p and s waves consisted of a product of two factors: the prefactor dependent pure optical parameters of the system and the main factor of the polar Kerr effect for a normal incidence case. The measured magneto-optic polar Kerr rotation angles of Co/Pd multilayers and the longitudinal Kerr rotation angles of Cu/Co multilayers with varying incident angles could be well fitted using the simplified analytical formulae. © *1996 American Institute of Physics.* [S0003-6951(96)01135-7]

Recently, the magneto-optical Kerr effect (MOKE) has been attracting much attention due to applications of the effect as a tool in probing magnetism as well as a readout mechanism in high-density magneto-optical recording. The MOKE is manifested itself by change of polarization and/or intensity of incident polarized light when reflected from the surface of a magnetized medium and is fundamentally related to the spin polarized electronic band structure.

Based on the fact that the MOKE is proportional to the projection of the magnetization to the incident beam direction, Haijar *et al.*[1] and Purcell *et al.*[2] have discussed the scope of employing MOKE to study the magnetic anisotropy energy of a sample having perpendicular magnetic anisotropy. Weller *et al.*[3] applied this method to obtain the magnetic anisotropy energy of Co/X multilayers (X=Pd, Pt, and Ni). However, only a few studies have been reported for the general case where the direction of the magnetization is arbitrary and the direction of the incident beam is not normal; the interpretation is not so simple because of the complicated relations.[4–6] In this letter, using Snell's law we have derived the simplified analytic expressions for the various magneto-optical Kerr effects of an optically thick medium having an arbitrary magnetization direction.

As depicted in Fig. 1, there are various kinds of MOKEs depending on the relative direction of the magnetization to the plane of incidence. Depending on the direction of the magnetization, whether it is parallel to the surface normal, parallel to the surface and in the plane of incidence, or parallel to the surface and perpendicular to the plane of incidence, the MOKE is called the polar Kerr effect, the longitudinal Kerr effect, and the transverse (or equatorial) Kerr effect, respectively.[7] When a beam of light is incident from a nonmagnetic medium 0 to a magnetic medium 1, having an arbitrary direction of the magnetization as shown in Fig. 2, the dielectric tensor ϵ can be generalized using Euler's angle as follows:[4,6]

$$\epsilon=\epsilon_{xx}\begin{pmatrix} 1 & -iQm_z & iQm_y \\ iQm_z & 1 & -iQm_x \\ -iQm_y & iQm_x & 1 \end{pmatrix}. \tag{1}$$

For generality, we treat all physical quantities as complex numbers. We assume $\epsilon_{zz}=\epsilon_{xx}$ for simplicity. The magneto-optical constant Q in Eq. (1) is defined as

$$Q=i\frac{\epsilon_{xy}}{\epsilon_{xx}}. \tag{2}$$

Here, we follow the same sign convention proposed by Atkinson and Lissberger[8] for Q. It is same as that proposed by Hunt[4] and Yang and Scheinfein,[6] but opposite to that of Zak *et al.*[5] We consider the magnetic permeability is equal to 1 in our treatment since we are interested in the optical wavelength region.[9] In Eq. (1), m_x, m_y, and m_z are the direction cosines of the magnetization vector $\mathbf{M}_s$. Solving Maxwell equations for the above dielectric tensor, the magneto-optical Fresnel reflection matrix can be given as follows:

$$\hat{\mathfrak{R}}=\begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix}, \tag{3}$$

where r_{ij} is the ratio of the incident j polarized electric field and the reflected i polarized electric field, and expressed by[4–6]

$$r_{pp}=\frac{n_1\cos\theta_0-n_0\cos\theta_1}{n_1\cos\theta_0+n_0\cos\theta_1}-\frac{i2n_0n_1\cos\theta_0\sin\theta_1 m_xQ}{n_1\cos\theta_0+n_0\cos\theta_1}, \tag{4}$$

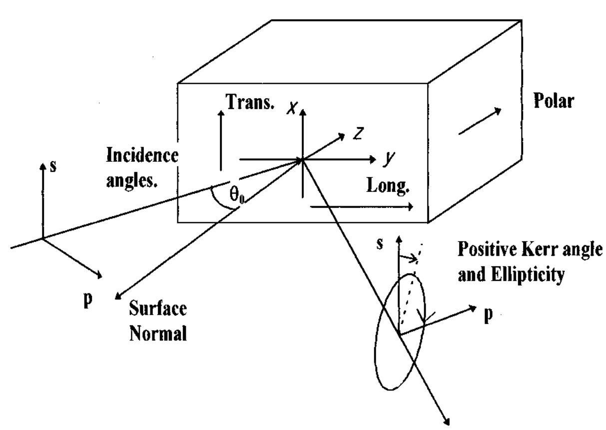

FIG. 1. Schematic configurations for the polar, longitudinal, and transverse (or equatorial) magneto-optical Kerr effects. The definitions of the coordinate system are also shown.

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$$r_{sp}=\frac{in_0n_1\cos\theta_0(m_y\sin\theta_1+m_z\cos\theta_1)Q}{(n_1\cos\theta_0+n_0\cos\theta_1)(n_0\cos\theta_0+n_1\cos\theta_1)\cos\theta_1}, \tag{5}$$

$$r_{ss}=\frac{n_0\cos\theta_0-n_1\cos\theta_1}{n_0\cos\theta_0+n_1\cos\theta_1}, \tag{6}$$

$$r_{ps}=-\frac{in_0n_1\cos\theta_0(m_y\sin\theta_1-m_z\cos\theta_1)Q}{(n_1\cos\theta_0+n_0\cos\theta_1)(n_0\cos\theta_0+n_1\cos\theta_1)\cos\theta_1}. \tag{7}$$

In the above expressions, θ_0, n_0, and n_1 are the angle of incidence, the refractive index of the nonmagnetic medium 0, and that of the magnetic medium 1, respectively. They are valid within the first-order approximation of the magneto-optical constant Q. In the above equations, the complex refractive angle θ_1 in the magnetic medium 1 is determined by Snell's law. The differences in the signs of r_{sp} and r_{ps} compared to those of Yang and Scheinfein[6] are ascribed to the sign conventions as mentioned earlier.

To simplify the above expressions, we define the complex Kerr effects as follows:

$$\theta_K^p\equiv r_{sp}/r_{pp}, \tag{8}$$

$$\theta_K^s\equiv r_{ps}/r_{ss}. \tag{9}$$

Using the above definitions, we first consider the simple cases of the polar and longitudinal configurations and then extend them to the general case.

In the polar configuration, $m_z=1$ and $m_y=m_x=0$. For the p-polarized wave in this configuration, by substituting Eqs. (4) and (5) in Eq. (8) the following relation could be obtained:

$$\left(\frac{r_{sp}}{r_{pp}}\right)^{\text{pol}}=\frac{in_0n_1\cos\theta_0Q}{(n_0\cos\theta_0+n_1\cos\theta_1)(n_1\cos\theta_0-n_0\cos\theta_1)}. \tag{10}$$

The denominator can be expanded as follows:

$$(n_1^2-n_0^2)\cos\theta_0\cos\theta_1+n_0n_1(\cos^2\theta_0-\cos^2\theta_1). \tag{11}$$

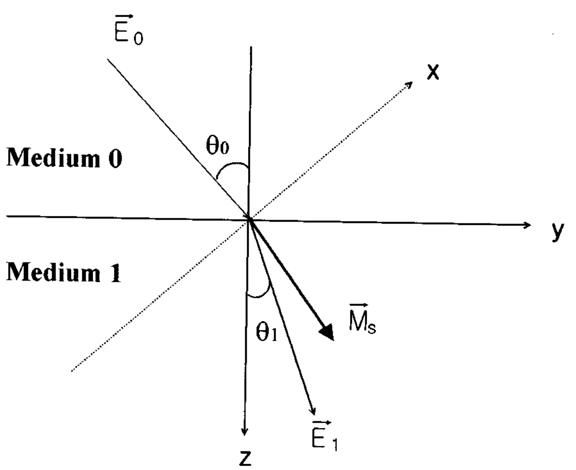

FIG. 2. The coordinate system of the nonmagnetic medium 0 and the magnetic medium 1. The direction of the magnetization of medium 1 is arbitrary.

Using the relation $\cos^2\theta_0-\cos^2\theta_1=\sin^2\theta_1-\sin^2\theta_0$ and Snell's law, $n_0\sin\theta_0=n_1\sin\theta_1$, the second term of the denominator can be simplified as follows:

$$\begin{aligned}n_0n_1(\cos^2\theta_0-\cos^2\theta_1)&=n_0n_1(\sin^2\theta_1-\sin^2\theta_0)\\&=n_0n_1\sin^2\theta_1\left(1-\frac{n_1^2}{n_0^2}\right)\\&=\sin\theta_0\sin\theta_1(n_0^2-n_1^2).\end{aligned} \tag{12}$$

By substituting this result in Eq. (11), the denominator can be simplified as follows:

$$\begin{aligned}&(n_1^2-n_0^2)(\cos\theta_0\cos\theta_1-\sin\theta_0\sin\theta_1)\\&\quad=(n_1^2-n_0^2)\cos(\theta_0+\theta_1).\end{aligned} \tag{13}$$

Substituting Eq. (13) in Eq. (10), $(\theta_K^p)^{\text{pol}}$ can be expressed as

$$(\theta_K^p)^{\text{pol}}\equiv\left(\frac{r_{sp}}{r_{pp}}\right)^{\text{pol}}=\frac{\cos\theta_0}{\cos(\theta_0+\theta_1)}\cdot\frac{in_0n_1Q}{(n_1^2-n_0^2)}. \tag{14}$$

In this expression, the second factor, $in_0n_1Q/(n_1^2-n_0^2)$, is just the well-known polar Kerr effect for normal incidence.[7] For the oblique incident p-polarized wave, the Kerr effect therefore can be described by a product of two factors. The prefactor, $\cos\theta_0/[\cos(\theta_0+\theta_1)]$, is a simple function of the incident angle and the refractive angle determined by the refractive indices of the media. And the main factor contains information about the magneto-optical properties of the medium 1.

We get the following similar expression for the s-polarized wave from Eqs. (6) and (7) using Snell's law:

$$(\theta_K^s)^{\text{pol}}\equiv\left(\frac{r_{ps}}{r_{ss}}\right)^{\text{pol}}=\frac{\cos\theta_0}{\cos(\theta_0-\theta_1)}\cdot\frac{in_0n_1Q}{(n_1^2-n_0^2)}. \tag{15}$$

The only difference between Eqs. (14) and (15) is the sign of the argument of the cosine function in the denominator of the prefactor.

In the longitudinal configuration, $m_y=1$ and $m_z=m_x=0$. By similar mathematical treatment of Eqs. (4)–(7) as the polar configurations, the complex Kerr effects for the longitudinal configuration can be expressed by

$$(\theta_K^p)^{\text{long}}\equiv\left(\frac{r_{sp}}{r_{pp}}\right)^{\text{long}}=\frac{\cos\theta_0\tan\theta_1}{\cos(\theta_0+\theta_1)}\cdot\frac{in_0n_1Q}{(n_1^2-n_0^2)}, \tag{16}$$

$$(\theta_K^s)^{\text{long}}\equiv\left(\frac{r_{ps}}{r_{ss}}\right)^{\text{long}}=-\frac{\cos\theta_0\tan\theta_1}{\cos(\theta_0-\theta_1)}\cdot\frac{in_0n_1Q}{(n_1^2-n_0^2)}. \tag{17}$$

The expressions for the longitudinal Kerr effects are also similar to those of the polar Kerr effects and can be split into

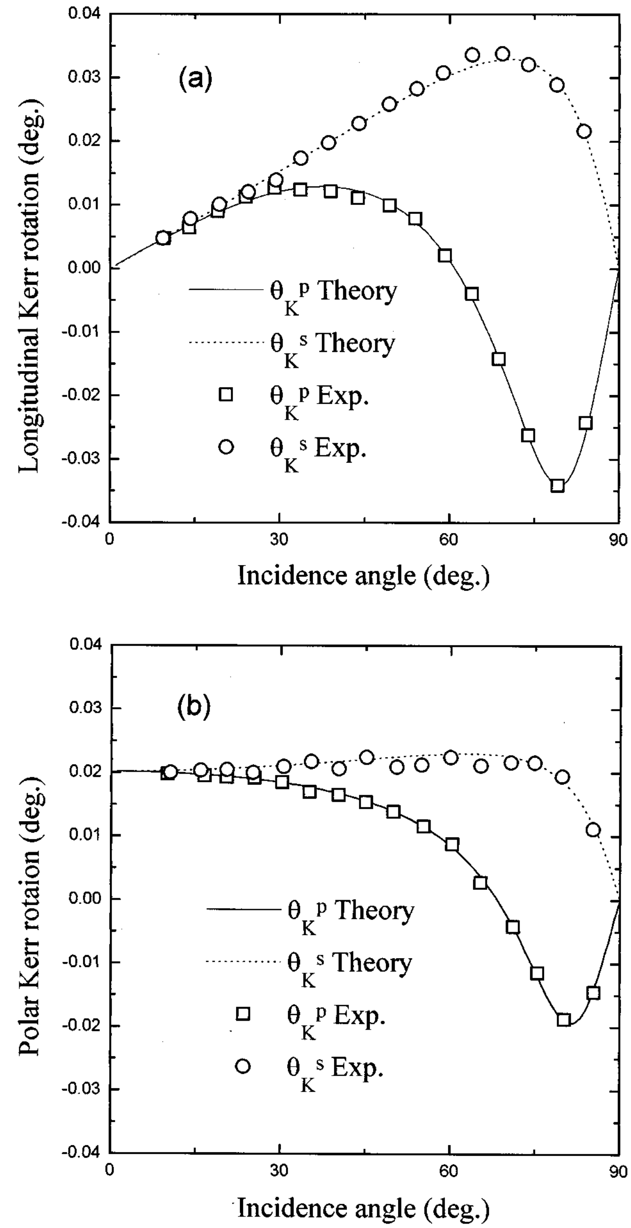

FIG. 3. Experimental and theoretical p- and s- Kerr rotation curves of (a) the Cu/Co multilayer in a longitudinal configuration and (b) the Co/Pd multilayer in a polar configuration.

two factors. The prefactors, $\cos\theta_0 \tan\theta_1/[\cos(\theta_0 \pm \theta_1)]$, are also simple functions of the optical parameters. The main factor, $in_0 n_1 Q/(n_1^2 - n_0^2)$, is the polar Kerr effect for normal incidence. In this case the only difference between p- and s-polarized waves is also the sign of the argument of the cosine function in the denominator of the prefactor, as it was in the polar configuration.

Since there is no contribution by the perpendicular component of the magnetization to the plane of incidence within the first-order of Q, the contribution by the arbitrary magnetization comes from the component parallel to the plane of incidence. Therefore, one can ignore m_x and consider only m_y and m_z. Then the Kerr effects in the general case of arbitrary magnetization direction and oblique incidence can be expressed from Eqs. (14)–(17) as follows:

$$\theta_K^p \equiv \frac{r_{sp}}{r_{pp}} = \frac{\cos\theta_0(m_z + m_y \tan\theta_1)}{\cos(\theta_0 + \theta_1)} \cdot \frac{in_0 n_1 Q}{(n_1^2 - n_0^2)}, \tag{18}$$

$$\theta_K^s \equiv \frac{r_{ps}}{r_{ss}} = \frac{\cos\theta_0(m_z - m_y \tan\theta_1)}{\cos(\theta_0 - \theta_1)} \cdot \frac{in_0 n_1 Q}{(n_1^2 - n_0^2)}. \tag{19}$$

Again, the Kerr effect is expressed as the product of two simple factors: the prefactor is a simple function of the optical parameters of the media and the direction of the magnetization, and the main factor is the well-known polar Kerr effect for normal incidence.

To verify the validity of the simplified expressions, we have applied these results to the published experimental data on Co/Pd and Cu/Co multilayers by Deeter and Strid,[10] where they measured the polar Kerr rotation angles of a (1.8 Å Co/9 Å Pd)$_{200}$ multilayer having a perpendicular magnetic anisotropy and the longitudinal Kerr rotation angles of (50 Å Cu/55.8 Å Co)$_{10}$ multilayer having in-plane magnetic anisotropy. The polar and longitudinal Kerr rotation angles of the p- and s-polarized waves were reported at wide incident angles ranging from 5° to 85° with an increment of 5°. The reflectivities for the p- and s-polarized waves were also reported at various incident angles. The complex refractive indices n_1 and the magneto-optical constants Q of the samples at a wavelength of 6328 Å, determined using the least-square fitting method, were $n_1 = 1.58 + 3.58i$ and $Q = 0.0177 - 0.0063i$ for the Cu/Co multilayer, and $n_1 = 2.04 + 4.06i$ and $Q = 0.00038 - 0.00314i$ for the Co/Pd multilayer. These values were used in Eqs. (14)–(17) to calculate various Kerr rotation angles. The calculated results, together with the experimental data, are shown in Fig. 3. The open circles and rectangles represent the experimental results of Deeter and Strid, and the solid and dashed lines represent the theoretical results obtained using Eqs. (14)–(17). As seen in Fig. 3, the experimental data are well explained by the present simplified formulae.

In conclusion, we have found that the expression for any kind of the MOKE can be expressed as a simple formula, where it can be described by a products of two factors. The prefactor is a simple function of the optical parameters of the system and the main factor is just the well-known polar Kerr rotation for normal incidence. The validity of the simple formulae was verified by fitting the experimental data of Cu/Co and Co/Pd multilayers with the present formulae. The derived simple formulae will be useful to study the MOKE or the magnetic properties of the sample in the general case.

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