bandwidth, the capability to achieve the performance is obvious.

We made a comparison of the results obtained with the structure maintained by the U shape without any soldering and the one obtained with soldering without support. Except in the upper frequency band, there is no different between the results, showing again that this concept is not especially sensitive to the supporting structure provided that the material is chosen adequately.

In Figure 5, we present the radiation pattern of such a disk at 2.4 GHz. This disk shows the same behavior as the classical monopole disk described in [4]. However, it presents many ripples because the dimensions of the supporting substrate which were only 8 cm × 10 cm.

In Figure 6, we present the matching performances of a double ellipse of ratio 0.5, the major axis of which is 10 mm. The structure is set on the same line as that described previously, and should be matched to 8 GHz. Figure 6 shows a matching better than 7 dB over the 1.5 GHz band. Again, it is possible by sliding the monopole to optimize the matching according to the frequency band desired.

This structure could be used in higher frequency ranges with only the following limitations taken into account:

- the manufacturing of the radiating structure,
- the coupling effect in between the matching line and the radiating structure.

However, up to 20 GHz, the structure has dimensions which are acceptable for the purposes of incorporating it into a complete system.

CONCLUSION

The realization of a new surface-mounted antenna type has been shown. A comparison between a conventional structure and this new way of attaching the monopole shows the consistency of the approach. The major advantages and limitations of the concept were described; it is shown that this approach avoids the use of a connector through the ground plane, and that it is mainly suitable for broadband monopole application. We were able to achieve 7 dB of matching through a 3 GHz bandwidth. The only limitations were shown to be the manufacturing limitations if one intends to use it for a frequency higher than 20 GHz.

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COMPUTATION OF RESONANT MODES OF OPEN RESONATORS USING THE FEM AND THE ANISOTROPIC PERFECTLY MATCHED LAYER BOUNDARY CONDITION

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ABSTRACT: A finite-element approach for the eigenmode analysis of open resonators is presented using a perfectly matched layer (PML) absorbing boundary condition. The PML is a nonphysical anisotropic lossy material absorbing outgoing waves from the computational domain. The validity of the approach is examined via the application to a Fabry–Perot resonator problem. Numerical error analyses for the quality factors and resonant frequencies demonstrate the accuracy and reliability of the anisotropic PML method applied to the eigenvalue problem.


Key words: perfectly matched layer; finite-element method; open resonators; eigenvalue problems; Fabry–Perot resonator

1. INTRODUCTION

In recent years, the exact numerical analysis of inhomogeneous waveguides and resonators without metal enclosures has gained increasing interest because of the rapid development of devices such as optical fibers, dielectric resonators [1], and photonic bandgap devices [2]. Although the finite-element method (FEM) is known to be suitable for the eigenmode analysis of various waveguiding structures, many difficulties arise in dealing with the open-boundary or unbounded field problems [3]. Several methods have been proposed to deal with these unbounded domains in eigenvalue problems. These include the fictitious boundary method [4], the infinite-element method [5], and the boundary integral method [6]. Although the boundary integral method is exact among them, it destroys the sparsity of matrixes generated in this formulation. The infinite-elements approach has been applied to a wide range of waveguiding problems such as optical fibers and dielectric waveguides. The method, however, is not rigorous in the sense that it requires a knowledge of the asymptotic behavior of solutions in the far-field region. Thus, a more rigorous and local boundary condition is required to estimate the boundary-sensitive parameters such as the quality factors.

In solving deterministic problems such as scattering and antenna problems, on the other hand, new concepts have recently been introduced to deal with unbounded infinite regions. The perfectly matched layer (PML) introduced by Berenger [7] provides many advantages in finite-difference time-domain (FDTD) computations. Pakel and Mittra [8] have modified Berenger’s equation to make it suitable for the frequency-domain FEM applications. Sacks et al. [9] have proposed an anisotropic PML, which is a nonphysical anisotropic lossy material backed with a perfect electric conductor (PEC). Although these PMLs have been applied to some deterministic problems, to our knowledge, the
anisotropic PML concept has not been employed yet to eigenvalue problems of open resonators.

In this letter, we apply the PML concept to the formulation of eigenvalue equations for the open resonators in order to accurately calculate the resonant frequencies, the quality factors, and field distributions of two-dimensional open resonators. A finite-element formulation for the two-dimensional anisotropic media with diagonal permittivity and permeability tensors is derived from Maxwell’s equations. We restrict our attention to the resonant modes that have no variation of fields along the axial or z-direction (\(k_z = 0\)). With this assumption, we can take the axial electromagnetic-field (\(E_z - H_z\)) formulation because spurious solutions do not appear in this formulation [10]. By imposing the PML boundary condition on the formulation, a matrix equation for complex eigenmodes is obtained. The resonator characteristics such as resonant frequencies, quality factors, and modal field patterns are obtained by solving the final matrix equation. The main advantage of this approach is that no assumption or knowledge of a far-field solution is required since the PML can absorb all outgoing electromagnetic waves from the computational domain. To check the validity of the present approach, we studied an isolated Fabry–Perot resonator (FPR) which can also be regarded as a one-dimensional photonic bandgap crystal for light emission control [2]. The numerical errors associated with the finite-element discretization and the PML reflection are also presented to serve as a guide in selecting the optimal parameters of the PML.

2. FORMULATION

We consider a two-dimensional, infinite, linear, anisotropic, and source-free region that is divided into an interior finite-element domain \(\Omega\) and an exterior infinite region \(\Omega^{ext}\) by an artificial boundary \(\Gamma\) as shown in Figure 1. The boundary \(\Gamma\) should enclose all inhomogeneous materials. With the time dependence of the form \(\exp(\imath \omega t)\), the electric field equation is given by

\[
\nabla \times (\{\mu_r\}^{-1} \cdot \nabla \times E) - k_0^2 \{\varepsilon_r\} \cdot E = 0
\]

\(\nabla \times (\{\mu_r\}^{-1} \cdot \nabla \times E) - k_0^2 \{\varepsilon_r\} \cdot E = 0
\)

where \(\{\varepsilon_r\}\) and \(\{\mu_r\}\) are the relative permittivity and permeability of the medium, respectively, and \(k_0\) is the free-space wavenumber. According to Sacks et al. [9], \(\{\varepsilon_r\}\) and \(\{\mu_r\}\) are complex diagonal tensors of the form

\[
\{\varepsilon_r\} = \begin{bmatrix}
\varepsilon_{xx} & 0 & 0 \\
0 & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{bmatrix}, \quad \{\mu_r\} = \begin{bmatrix}
\mu_{xx} & 0 & 0 \\
0 & \mu_{yy} & 0 \\
0 & 0 & \mu_{zz}
\end{bmatrix}
\]

\[
\{\varepsilon_r\} = \begin{bmatrix}
\varepsilon_{xx} & 0 & 0 \\
0 & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{bmatrix}, \quad \{\mu_r\} = \begin{bmatrix}
\mu_{xx} & 0 & 0 \\
0 & \mu_{yy} & 0 \\
0 & 0 & \mu_{zz}
\end{bmatrix}
\]

In the PML region, where the air–PML interface planes are assumed to be normal to the x-direction as shown in Figure 2, these tensors satisfy the conditions

\[
\varepsilon_{xx} = \frac{1}{1 - \beta(x)},
\varepsilon_{yy} = \varepsilon_{zz} = \mu_{yy} = \mu_{zz} = 1 - \beta(x).
\]

Here, the absorption function \(\beta(x)\) determines the rate of decay of transmitted waves traveling in the x-direction, and it is given by

\[
\beta(x) = \frac{(m + 1) \ln R}{2 k_0 d} \left( \frac{x}{d} \right)^m
\]

where \(x\) is the distance to the interface of the PML, \(d\) is the depth of the PML, \(m\) is the order of the polynomial function, and \(R\) is the desired overall reflection coefficient of the waves traveling in the x-direction.

Restricting our attention to the modes that have no variation of fields along the z-direction, which is a valid assumption for the two-dimensional resonators, it is adequate to use the simple \(E_z - H_z\) formulation because spurious solutions do not appear in this case [10]. In the \(E_z - H_z\) formulation, applying the Ritz variational principle to Eq. (1) results in the...
following functional $F$:

$$F(E, H) = \frac{1}{2} \int_{\Omega} \left[ \frac{1}{\omega \mu_{yy}} \left( \frac{\partial E_x}{\partial x} \right)^2 + \frac{1}{\omega \mu_{xx}} \left( \frac{\partial E_z}{\partial y} \right)^2 - e_{zz} E_z \right] d\Omega$$

$$+ \frac{1}{2} \int_{\Omega} \left[ \frac{1}{\omega^2 \varepsilon_{yy}} \left( \frac{\partial H_z}{\partial x} \right)^2 + \frac{1}{\omega^2 \varepsilon_{xx}} \left( \frac{\partial H_z}{\partial y} \right)^2 - \mu_{zz} H_z \right] d\Omega$$

$$- \frac{j}{2 \omega \hat{n}} \int_{\Gamma} (E_x(H_y\hat{x} - H_z\hat{y}) + H_z(E_y\hat{x} - E_z\hat{y})) \cdot \hat{n} d\Gamma \quad (5)$$

where $\hat{n}$ denotes the unit normal vector on the boundary $\Gamma$ as in Figure 1 and the unit $e_0 = \mu_0 = 1$ is employed. The first two terms in the functional can be readily discretized in a standard finite-element procedure without looking at the boundary. The real issue is to consider the last line integral (to be denoted as $F_{\phi}$) along the boundary $\Gamma$. The simple truncation method [4], which imposes a perfect electric conductor or a perfect magnetic conductor condition on the boundary to eliminate the term $F_{\phi}$, gives serious errors in quality factors because the boundary prevents the energy from traveling away from the open resonator. In the infinite-elements approach, the boundary $\Gamma$ can be regarded as a far-field boundary at which infinite elements are connected to ordinary elements in $\Omega$. This approach does not ensure that the reflection error introduced due to terminating the finite-element mesh with the infinite-elements region is negligible. It implies that a small amount of traveling waves can be reflected at the boundary. For high-$Q$ resonators, however, this minute feedback could significantly affect the computed values of quality factors or energy losses. For the computation of quality factors, therefore, it is especially important to treat the boundary more rigorously. On the other hand, it is interesting to note that the resonant frequency would hardly be changed since the small reflected waves are not expected to modify the shape of modal fields appreciably.

Our approach is to eliminate the term $F_{\phi}$ rigorously by introducing the PML concept. Since the boundary $\Gamma$ truncating the PML region is a perfectly conducting backing, the tangential components of electric fields on the boundary must vanish. In other words, since $E_z = (E_x\hat{x} - E_z\hat{y}) \cdot \hat{n} = 0$ on the boundary $\Gamma$, the boundary-dependent term $F_{\phi}$ is removed out of the functional $F$. Therefore, it is possible to decouple the electric fields and the magnetic fields. Consequently, the solution reduces to a transverse magnetic (TM) or transverse electric (TE) mode. In this letter, we focus on the TM mode solutions. The TE mode solutions are also obtained by the same procedure.

After following the standard finite-element discretization procedure with the functional $F$, we obtain a generalized eigenvalue equation given by

$$[A][E_z] - \omega^2[B][E_z] = 0 \quad (6)$$

where $[0]$ is a null vector, $(E_z)$ is a column vector whose components are the values of $E_z$ at nodal points in the problem domain $\Omega$, and $[A]$ and $[B]$ are global system matrices assembled from their corresponding elemental matrices given by

$$A_{ij} = \int_{\Omega} \left( \frac{1}{\mu_{yy}} \frac{\partial N_i^e}{\partial x} \frac{\partial N_j^e}{\partial x} + \frac{1}{\mu_{xx}} \frac{\partial N_i^e}{\partial y} \frac{\partial N_j^e}{\partial y} \right) d\Omega$$

$$B_{ij} = \int_{\Omega'} e_{zz} N_i^e N_j^e d\Omega \quad (7)$$

Here, the superscript $e$ denotes an element number, and $N_i^e$ is the interpolation function corresponding to node $i$ of the element.

Finally, the resonant characteristics such as resonant frequencies, quality factors, and modal field distributions are obtained by solving Eq. (6). The quality factor $Q$ that describes electromagnetic energy losses in open resonator structures, and the resonant frequency $f_r$ are obtained by

$$Q = \frac{\omega_p}{2 \omega_0}, \quad f_r = \frac{\omega_0}{2 \pi} \quad (9)$$

where $\omega_p$ and $\omega_0$ are the real and imaginary parts of the eigenfrequency $\omega$ in Eq. (6), respectively. Note that the quality factor is only defined when Eq. (6) gives a complex number of eigenvalue. To describe the numerical error introduced due to the finite-element discretization and the numerical reflections at the PML interface, we define the percentage normalized error as

$$\text{error} = \left| \frac{f_r, \text{numerical} - f_r, \text{exact}}{f_r, \text{exact}} \right| \times 100(\%) \quad (10)$$

for the resonant frequency $f_r$, and similarly for the $Q$-factor.

3. NUMERICAL RESULTS

To check the validity of the present approach, we considered an isolated FPR as shown in Figure 2 with a refractive index $n_H = 3.4$, a thickness of high-index $\lambda/4$ layer $t_H = a/(n_H + 1)$, and a spacing in the center $t_s = 2an_H/(n_H + 1)$, where $a$ denotes the thickness of a pair of layers. The $a$ can be taken as a scale factor. In this study, we set $a = 1$ for convenience. The exact values of the resonant frequency and quality factor of the fundamental mode of the FPR are obtained by the thin-film calculation method [11] and they are $(n_H + 1)/(4a \cdot n_H)$ and 1717.387, respectively. For this one-dimensional FPR structure, our two-dimensional FE code is used by imposing periodic boundary conditions at the top ($\Gamma_1$) and bottom ($\Gamma_2$) boundaries. In order to reduce the computer memory needed for the large non-Hermitian matrices $[A]$ and $[B]$ with a size of over 10,000, the iterative Arnoldi method was employed instead of the conventional packages such as EISPACK.

The normalized numerical error in Eq. (10) was computed while varying several parameters such as $d$, $R$, $m$ as defined in Eq. (4), the distance $r$ between the edge of the resonator and the interface of PML, and the nodal density normalized by the wavelength $\lambda/\delta x$ [12]. $\delta x$ denotes the nodal separation. Since the error is a function of many parameters, only major characteristic results are presented in this letter. Figure 3 shows the normalized error of the resonant frequency and the quality factor of the fundamental mode versus the nodal density. In quadratic eight-node rectangular elements, the nodal density is twice the elemental density that is the number of elements per wavelength. The PML parameters
Since the numerical errors of both $f_L$ and $f_R$ are governed by the same dispersion relation as Eq. (11), the numerical value of $Q$ can be written as follows:

$$Q_{\text{numerical}} = \frac{f_r D(\delta x)}{f_L D(\delta x) - f_R D(\delta x)} = Q_0.$$  

Here, the numerical dispersion function $D(\delta x) = 1/(1 - \gamma \cdot (\delta x)^2)$ is derived from Eq. (11). Note that the numerical dispersion functions are canceled out in Eq. (13). Consequently, the numerical value of $Q$ hardly depends on the value of the nodal density.

It is worth pointing out that the error of resonant frequency is nearly independent of PML parameters $(d, r, R, m)$, as well as negligible (less than 0.01%) with 24 nodes per wavelength using quadratic elements. The normalized errors of the $Q$-factor are plotted in Figure 4 as a function of the depth of the PML for the case $\lambda / \delta x = 24$, $r = H_0/2$, and $m = 2$. The error is not negligible when the depth is smaller than $H_0/4$. The optimal depth is about $H_0/2$, which corresponds to six elements. With the $H_0/2$-thick parabolic grading PML, the normalized error is about 0.1%. The influence of the distance $r$ on the normalized error of the $Q$-factor is shown in Figure 5 when $\lambda / \delta x = 24$, $d = H_0/2$, and $m = 2$.

In general, a sufficiently accurate solution is obtained when $r$ is larger than $H_0/2$. This is advantageous in the sense that
nodes per wavelength, and the PML parameters are quadratic eight-node rectangulars, the nodal density is 40 with those of the exact method 11. The elements employed by this approach is presented in Figure 7, which agrees well finally, the field pattern for the fundamental mode computed the error. These results are similar to those reported in 13.

We plot the error in the quality factor as a function of linear rectangular, Figure 6 when the nodal density is 80. Here, the elements are the same as those shown in Figure 3.

The effect of changing \( R \) and \( m \) in Eq. (4) is also studied. We plot the error in the quality factor as a function of \( R \) in Figure 6 when the nodal density is 80. Here, the elements are linear rectangular, \( d = \lambda_0/2 \), and \( r = \lambda_0/2 \). This figure demonstrates that the error is reduced when only employs the parabolic grading of \( \beta(x) \) instead of the constant \( \beta \) which produces errors up to a few percent. The higher polynomial order in Eq. (4) \( (m = 4) \) does not further reduce the error. These results are similar to those reported in [13]. Finally, the field pattern for the fundamental mode computed by this approach is presented in Figure 7, which agrees well with those of the exact method [11]. The elements employed are quadratic eight-node rectangulars, the nodal density is 40 nodes per wavelength, and the PML parameters \((d, r, m, R)\) are the same as those shown in Figure 3.

4. CONCLUSIONS

A finite-element approach for the eigenmode analysis of two-dimensional unbounded resonator problems is presented using the anisotropic PML concept. To our knowledge, the anisotropic PML is successfully applied to an eigenvalue problem of open resonators for the first time. The validity of our approach is tested via application to a one-dimensional Fabry–Perot resonator. From the numerical results, we have also obtained the conditions of the PML for sufficiently accurate and reliable computation of the resonant frequencies, the quality factors, and the patterns of modal fields of two-dimensional open resonators.

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NUMERICAL SOLUTION OF SCATTERING OF WAVES BY LOSSY DIELECTRIC SURFACES USING A PHYSICS-BASED TWO-GRID METHOD

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ABSTRACT: The numerical solution of the scattering of waves by lossy dielectric surfaces often requires a dense surface discretization of many points per wavelength. To reduce the CPU requirements, we have used