Graphene-assisted Si-InSb thermophotovoltaic system for low temperature applications

Mikyung Lim, Seokmin Jin, Seung S. Lee, and Bong Jae Lee

Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology, Daejeon 305-701, South Korea

*bongjae.lee@kaist.ac.kr

Abstract: The present work theoretically analyzes the performance of the near-field thermophotovoltaic (TPV) energy conversion device for low temperature applications (Tsource ~ 500 K). In the proposed TPV system, doped Si is employed as the source because its optical property can be readily tuned by changing the doping concentration, and InSb is selected as a TPV cell because of its low bandgap energy (0.17 eV). In order to enhance the near-field thermal radiation between the source and the TPV cell, monolayer of graphene is coated on the cell side so that surface plasmon can play a critical role in heat transfer. It is found that monolayer of graphene can significantly enhance the power throughput by 30 times and the conversion efficiency by 6.1 times compared to the case without graphene layer. The resulting maximum conversion efficiency is 19.4% at 10-nm vacuum gap width.

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References and links
1. Introduction

Global warming and energy crisis are long-term issues resulted from prevalent use of fossil fuels. In developing the carbon-free, high-efficiency and low-cost renewable energy harvesting and recycling technology, thermophotovoltaic (TPV) energy conversion system is considered as a promising alternative to mitigate the current crisis [1]. A TPV system that generates electric power directly from absorbed thermal radiation can reap benefits because it can reuse the wasted heat in industry. In addition, a TPV system has no moving parts, leading to low noise and high portability [2].

Major challenges in practical use of a TPV system are relatively low power throughput and energy conversion efficiency because the source temperature in many industrial processes is considerably lower than that of the sun. In order to resolve these issues, the near-field thermal radiation has been employed by several groups [2–7]. Considering that the radiative heat trans-

fer between two objects can exceed the blackbody limit through tunnelling of evanescent waves when distance between them is smaller than the characteristic wavelength of thermal radiation [8, 9], the source can transfer more radiative energy to the TPV cell in certain temperature difference, leading to increased power throughput. Whale and Cravalho [2] considered a Drude-model-based fictitious source and InGaAs cell to analyze the performance of TPV cell, while Pan et al. [3] calculated the near-field radiation between two dielectric materials for a near-field TPV system. Narayanaswamy and Chen [4] studied the effect of surface phonon-polaritons on the near-field radiative transfer for increasing the conversion efficiency of a TPV system. Later, the performance of TPV system consisting of tungsten and GaSb TPV cell was analyzed by Laroche et al. [5] by assuming the quantum efficiency to be 100% (i.e., all electron-hole pairs generated from TPV system can contribute to the photogeneration current). However, because generated electron-hole pairs can experience the recombination as they are transported in the TPV cell, the assumption of 100% quantum efficiency can lead to overestimation of the performance of a TPV system. Park et al. [6] considered this minority carrier transport and conducted more rigorous analysis on the performance of TPV system. Later, Francaur et al. [7] developed a theoretical model for coupled near-field thermal radiation, charge and heat transport problems to ensure required cooling for maintaining near-field TPV performance.

The conversion efficiency of a TPV system can be defined as ratio of electric power generated from a TPV cell to the absorbed radiative power. Given that low-energy photons (i.e., energy lower than bandgap energy of a TPV cell) cannot generate the electron-hole pair and the excessive energy absorbed from high-energy photons is eventually dissipated in the form of heat, the spectral radiative heat transfer in frequency slightly higher than bandgap energy is preferred for achieving high conversion efficiency. Recently, spectral tuning of the thermal radiative heat flux using gold reflector [10] or hyperbolic metamaterial [11] has been investigated to increase the conversion efficiency.

In order to achieve both enhanced power throughput and conversion efficiency, recent studies focused on the use of graphene for TPV application [12–14]. Because graphene, a two-dimensional lattice of carbon atoms with exceptionally high crystal and electronic quality [15, 16], can support surface plasmon in the near infrared (IR) spectral region [17, 18], it can enhance the near-field thermal radiation by matching the surface plasmon frequencies of the source and the receiver [19–23]. For example, Ilic et al. [12] analyzed the performance of InSb TPV cell with suspended graphene source, and Messina et al. [13] investigated the effect of graphene on matching frequencies of hBN source and InSb TPV cell. However, all these analyses on the performance of graphene-assisted TPV system are based on the assumption of 100% quantum efficiency, resulting in inevitable overestimation of the performance.

In this work, we solve the near-field thermal radiation together with the minority carrier transport model used in Park et al. [6] in a semi-analytic manner to conduct more realistic performance analysis of the graphene-assisted near-field TPV system. In the proposed TPV system, doped Si is employed as the source because its optical property can be readily tuned by changing the doping concentration, and InSb is selected as a TPV cell for low temperature applications (∼500 K) because of its low bandgap energy (0.17 eV). In order to enhance the near-field thermal radiation between the source and the TPV cell, monolayer of graphene is employed to the cell side so that surface plasmon can play a critical role in heat transfer.

2. Theoretical modeling
2.1. Near-field thermal radiation

Figure 1 depicts the proposed near-field TPV system consisting of a doped-Si source at $T_1 = 500$ K and an InSb TPV cell coated with monolayer of graphene at $T_2 = 300$ K, separated by vacuum gap width $d$. The doping concentration of phosphorous-doped Si, $N_{Si}$, varies from...
where coefficients from the source to the receiver for a given polarization state:

\[ \Theta(\omega, T_1) = \left( n + i \frac{\alpha(\omega)}{2\varepsilon_0} \right)^2 \]

[12, 13, 25], where \( n \) is the refractive index and \( \alpha(\omega) = 0 \) for \( \omega < \omega_g \) and \( \alpha(\omega) = (0.7 \times 10^6 \text{ m}^{-1})\sqrt{(\omega - \omega_g)/\omega_g} \) for \( \omega > \omega_g \). Here, \( k_0 = \omega/c_0 \) with \( c_0 \) being the speed of light in vacuum, and \( \omega_g \) is the angular frequency corresponding to the bandgap energy (i.e., \( \omega_g = 2.583 \times 10^{14} \text{ rad/s} \)). In the present study, \( \varepsilon_2(\omega) \) is assumed to be independent of the doping concentration of InSb, and the value of refractive index is taken from [26] near \( n_g = 7.29 \mu \text{m} \) (i.e., the wavelength corresponding to the bandgap energy). Therefore, the TPV cell consisting of two quasi-neutral \( p \)- and \( n \)-doped InSb can be treated as a homogeneous isotropic medium, which has commonly done in several previous works [6, 7, 10].

The net spectral heat flux from the semi-infinite doped-Si source (i.e., medium 1) to the semi-infinite TPV cell (i.e., medium 2) can be calculated from [9, 27]:

\[
q_{12, \omega, \text{net}} = \int_0^{\omega_g} S_{\beta, \omega}(\beta, \omega) d\beta = \left[ \frac{\Theta(\omega, T_1) - \Theta(\omega, T_2)}{4\pi^2} \right] \int_0^{\omega_g} \beta d\beta \Re(k_{1z}) \\
\times \left[ \Re\left( \frac{k_{1z}^2 + k_{2z}^2}{k_{2z}} \right) \left( k_{1z} + k_{2z} \right)^2 \right] T_{12}^P T_{12}^{\pi*} + \Re(k_{2z}) \left( \frac{1}{k_{1z} k_{1z}^*} \right) T_{12}^P T_{12}^{\pi*} 
\]

(1)

where \( \Theta(\omega, T_i) = \frac{\hbar \omega}{e^{\hbar \omega/(k_B T_i)} - 1} \) is the mean energy of the Planck’s oscillator at \( T_i \) with \( \hbar \) being the Planck constant divided by \( 2\pi \) and \( k_B \) being the Boltzmann constant. In the above equation, \( \Re \) takes the real part of a complex quantity and * denotes the complex conjugate.

In addition, \( \beta \) represents the parallel wavevector component and \( k_{ic} = \sqrt{\omega^2/c_0^2 - \beta^2} \) is the normal wavevector component in the \( i \)th medium. Here, \( T_{12}^P \) and \( T_{12}^{\pi*} \) represent the transmission coefficients from the source to the receiver for a given polarization state:

\[
T_{12}^{\pi, P} = \frac{r_{12}^{P, P} r_{12}^{P, P} e^{ik_{0c}d}}{1 - r_{12}^{P, P} r_{12}^{P, P} e^{2ik_{0c}d}} 
\]

(2)

where \( r_{12}^{P, P} \) and \( r_{12}^{P, P} \) are the Fresnel reflection and transmission coefficients, respectively, at the \( i-j \) interface for a given polarization state, and the subscript 0 represents the vacuum gap.
Because the TPV system generates the electric power from the absorbed radiation inside the TPV cell only, the absorption by the graphene layer should be excluded in the TPV performance analysis. It is important to understand that Eq. (1) contains the factor of $T_{12}T′_{12}$, where $T_{12}$ is the transmission coefficient for the electromagnetic wave from the doped-Si source (medium 1) to the InSb cell (medium 2) passing through the graphene layer. Thus, the absorption by the graphene layer is not included. However, the effect of graphene is fully accounted for when calculating the Fresnel reflection and transmission coefficients at the vacuum- graphene/InSb interface in Eq. (2) [12, 13, 23, 28, 29]. Consequently, Eq. (1) represents the net spectral radiative heat flux absorbed only by the TPV cell, whose optical absorption is tuned by the graphene layer. Finally, the total net heat flux $q''_{net}$ from the Si source to the InSb cell can be calculated as $q''_{net} = \int_{0}^{\infty} d\omega q''_{\omega,net} = \int_{0}^{\infty} d\omega \int_{0}^{\infty} S_{\beta,\omega}(\beta, \omega) d\beta$ or $q''_{net} = \int_{0}^{\infty} d\lambda q''_{\lambda,net} = \int_{0}^{\infty} d\lambda \int_{0}^{\infty} S_{\beta,\lambda}(\beta, \lambda) d\beta$.

2.2. TPV performance

As mentioned earlier, the TPV cell consisting of two quasi-neutral $p$- and $n$-doped InSb is treated as a homogeneous isotropic medium. Therefore, the net radiative heat flux $Q(z)$ inside the TPV cell is simply [30]

$$Q(z) = q''_{net} e^{-2\Im(\kappa_{z})} = \int_{0}^{\infty} d\lambda Q_{\lambda}(z, \lambda) = \int_{0}^{\infty} d\lambda \int_{0}^{\infty} S_{\beta,\lambda}(\beta, \lambda) e^{-2\Im(\kappa_{z})} d\beta$$

(3)

where $\Im$ takes the imaginary part of a complex quantity. When the TPV cell absorbs the photons whose energy are greater than bandgap energy, it generate the electron-hole pairs that move toward the opposite region. The minority carrier (i.e., electron in the $p$-region and hole in the $n$-region) can recombine until it reaches the depletion region. At steady state, the minority carrier density by diffusion in the $p$- and $n$-regions is described by [6, 31]

$$D_{e,h} \frac{d^{2}\{n_{e,h}(z, \lambda) - n_{e,h}^{0}\}}{dz^{2}} - \frac{n_{e,h}(z, \lambda) - n_{e,h}^{0}}{\tau_{e,h}} + \dot{g}(z, \lambda) = 0$$

(4)

where $D_{e,h}$ is the diffusion coefficient, $n_{e,h}(z, \lambda)$ is the minority carrier concentration, $n_{e,h}^{0}$ is the equilibrium carrier concentration, and $\tau_{e,h}$ is the relaxation time of electron or hole. In the above equation, $\dot{g}(z, \lambda)$ represents the photogeneration rate of electron-hole pairs due to the absorption of spectral radiative heat flux and can be expressed as:

$$\dot{g}(z, \lambda) = -\frac{dQ_{\lambda}}{dz} \cdot \frac{\lambda}{hc_{0}} = \frac{\lambda}{hc_{0}} \int_{0}^{\infty} 2\Im(\kappa_{z}) S_{\beta,\lambda}(\beta, \lambda) e^{-2\Im(\kappa_{z})} d\beta$$

(5)

Because Eq. (4) is identical to the energy equation for the extended surface with internal heat generation [32], the homogenous solution to Eq. (4) is readily available. Regarding the particular solution, the source term $\dot{g}(z, \lambda)$ given in Eq. (5) is simply an integration of $\exp\{-2\Im(\kappa_{z})z\}$ multiplied by some function of $\beta$ and $\lambda$ with respect to $\beta$. Therefore, the particular solution associated with $\dot{g}(z, \lambda)$ can also be expressed as an integration of some exponential function of $z$ with respect to $\beta$. Finally, we can write the general solution to Eq. (4) as a semi-analytic form:

$$n_{e,h}(z, \lambda) - n_{e,h}^{0} = A_{e,h} \exp\left(\frac{z}{\sqrt{D_{e,h} \cdot \tau_{e,h}}}\right) + B_{e,h} \exp\left(\frac{-z}{\sqrt{D_{e,h} \cdot \tau_{e,h}}}\right) + \frac{\lambda}{hc_{0}} \int_{0}^{\infty} 2\tau_{e,h} \Im(\kappa_{z}) S_{\beta,\lambda}(\beta, \lambda) e^{-2\Im(\kappa_{z})} d\beta$$

(6)
where constants $A_{e,h}$ and $B_{e,h}$ can be obtained by applying boundary conditions for electron or hole. Boundary conditions for Eq. (4) at the edge of the depletion region (i.e., $z = a$ and $z = b$) is that $n_e$ and $n_h$ are equal to $n_0^e$ and $n_0^h$, respectively, leading to zero excess minority carrier. At the edge of the TPV cell (i.e., $z = 0$ and $z = b + t_n$ with $t_n$ being the thickness of $n$-region), the recombination processes is significant due to surface defects and metallic contact [33]. However, the surface recombination velocity $u_{sa}$ in the $n$-region can be set to be zero given that $n$-region is fabricated to be thicker than the hole diffusion length, $L_h$ [6]. With the assumption that the TPV cell consisting of two quasi-neutral $p$- and $n$-doped InSb can be treated as a homogeneous isotropic medium, neither multilayer formulation in the near-field thermal radiation nor finite difference method in solving the charge transport is necessary to conduct the performance analysis of near-field TPV system.

With the solution of Eq. (6), the photocurrent density flowing from the $n$-region to the $p$-region (i.e., $-z$ direction) can be obtained from the derivative of the minority carrier concentration at the edge of the depletion region [33]:

$$J_e(\lambda) = eD_e \frac{dn_e(z, \lambda)}{dz} |_{z=a} \quad \text{and} \quad J_h(\lambda) = -eD_h \frac{dn_h(z, \lambda)}{dz} |_{z=b} \quad (7)$$

The magnitude of drift current in the depletion region can be calculated by neglecting the recombination of generated electron-hole pairs as

$$|J_{dp}(\lambda)| = e \frac{Q_e(a, \lambda) - Q_e(b, \lambda)}{h\gamma_0/\lambda} \quad (8)$$

where $e = 1.602 \times 10^{-19}$ C is the electron charge. Because $J_e(\lambda)$, $J_h(\lambda)$, and $J_{dp}(\lambda)$ flows the same direction (i.e., from the $n$-region to the $p$-region), the magnitude of spectral photocurrent density of the TPV cell is simply the summation of the photocurrent in each region; that is, $|J(\lambda)| = |J_e(\lambda)| + |J_h(\lambda)| + |J_{dp}(\lambda)|$. Then, the total photocurrent is the integration of the spectral photocurrent over spectrum: $|J_{ph}| = \int_0^\infty |J(\lambda)| d\lambda$.

The total current density is required to calculate the maximal power output that can be derived by the ideal diode equation [34]: $|J| = |J_{ph}| - |J_s| \times \left\{ \exp \left( \frac{eV}{kT} \right) - 1 \right\}$, where $|J_s| = eN_i^2 \left( \frac{\sqrt{D_e/\tau_e}}{N_A} + \frac{\sqrt{D_h/\tau_h}}{N_p} \right)$ is the magnitude of saturation current density with $N_i$ being the intrinsic carrier concentration, and $V$ is the bias voltage of the TPV system. The maximum value of electric power can then be calculated by [5, 6]:

$$P_e = V_{max} \times |J_{max}| = V_{max} \times \left( |J_{ph}| - |J_s| \times \left\{ \exp \left( \frac{eV_{max}}{kT} \right) - 1 \right\} \right) \quad (9)$$

In the above equation, $0 < V_{max} < V_{oc}$, where $V_{oc} = (kT/e) \ln(|J_{ph}|/|J_s| + 1)$ is the open-circuit voltage occurred when net current density $J$ is set to be zero. The conversion efficiency $\eta$ is the ratio of generated maximal electric power to total radiative heat flux reached to the surface of cell and can be expressed as $\eta = P_e/q_{net}$.

In the calculation, the impurity concentrations of InSb TPV cell are set as $N_{A} = 10^{19}$ cm$^{-3}$ ($p$-region) and $N_{D} = 10^{19}$ cm$^{-3}$ ($n$-region). The properties of the InSb are adopted from the works of González-Cuevas et al. [35] and Francoeur [7]. The diffusion coefficients in the $p$- and $n$- region are $D_e = 186$ cm$^2$/s and $D_h = 5.21$ cm$^2$/s, respectively. The relaxation times at each region are $\tau_e = 1.45$ ns and $\tau_h = 1.81$ ns, respectively. The width of $p$-region is set to be $a = 400$ nm, and the $n$-region is thicker than the hole diffusion length $L_h = 970$ nm. The depletion region is estimated to be $L_{dp} = b - a = 10$ nm at the initial impurity condition. The surface recombination velocity is taken as $u_p = 10^4$ m/s [36].
Table 1. The enhancement factor compared to the case without graphene layer in terms of the conversion efficiency ($EF_\eta$) or the power throughput ($EF_P$).

<table>
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<tr>
<th>$d$ (nm)</th>
<th>$N_{Si}$ (cm$^{-3}$)</th>
<th>$\mu$ (eV)</th>
<th>$\eta$ (%)</th>
<th>$EF_\eta$</th>
<th>$EF_P$</th>
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3. Results and Discussion

Figure 2 shows the calculated spectral radiative heat flux between the doped-Si source and the graphene-coated InSb cell while varying the chemical potential ($\mu$) of graphene. It can be clearly seen that higher chemical potential yields higher spectral heat flux, regardless of $N_{Si}$ and $d$ values. Because the excessive energy absorbed from high-energy photons is eventually dissipated in the form of heat, the spectral radiative heat transfer in slightly lower wavelengths than $\lambda_g$ is preferred for achieving high conversion efficiency. The maximum enhancement achieved by applying monolayer graphene occurs at slightly lower wavelength than $\lambda_g$, reaping the benefits of increasing the conversion efficiency. For $N_{Si} = 1 \times 10^{20}$ cm$^{-3}$ in Fig. 2(a), we can obtain five times higher heat flux with graphene of $\mu = 1.0$ eV compared to the case without graphene layer at $d = 10$ nm. Because presence of the graphene layer greatly increases the net heat flux in the vicinity of $\lambda_g$, graphene of $\mu = 1.0$ eV can significantly enhance both the power throughput ($EF_P = 30.4$) and the conversion efficiency ($EF_\eta = 6.1$). On the other hand, for higher $N_{Si}$ values in Figs. 2(b) and 2(c), although graphene increases the spectral heat flux to some extent, the enhancement occurs rather in a broad wavelength range of $3.5 \mu m < \lambda < \lambda_g$. Consequently, the enhancement in the conversion efficiency is less than that of the case for $N_{Si} = 1 \times 10^{20}$ cm$^{-3}$, as can be read from Table 1. If the vacuum gap increases to 50 nm as shown in Fig. 2(d), graphene of $\mu = 1.0$ eV still enhances the conversion efficiency by 11%, but its effect is not as considerable as the case of $d = 10$ nm.

In order to elucidate the effect of graphene, Fig. 3 shows the contour of $S_{\beta,\lambda} (\beta, \lambda)$ together with the surface plasmon polariton (SPP) dispersion curves that are calculated by finding poles of the transmission coefficient for $p$ polarization in Eq. (2). As shown in Fig. 3(a), the SPP at the vacuum-InSb interface is not even close to the wavelength regions in which heat transfer dominantly occurs. When graphene of $\mu = 0.5$ eV is coated on the InSb cell, however, it shifts the SPP at the vacuum-graphene/InSb interface to the shorter wavelength region so that the SPP at the vacuum-Si interface can interface with that at the vacuum-graphene/InSb near 8 $\mu$m. Be-
Fig. 2. Spectral radiative heat flux between the doped-Si source and the InSb cell: (a) \(d = 10\ \text{nm}\) and \(N_{\text{Si}} = 1 \times 10^{20}\ \text{cm}^{-3}\); (b) \(d = 10\ \text{nm}\) and \(N_{\text{Si}} = 5 \times 10^{20}\ \text{cm}^{-3}\); (c) \(d = 10\ \text{nm}\) and \(N_{\text{Si}} = 1 \times 10^{21}\ \text{cm}^{-3}\); and (d) \(d = 50\ \text{nm}\) and \(N_{\text{Si}} = 1 \times 10^{20}\ \text{cm}^{-3}\). In the figure, \(\lambda_{g}\) indicates the wavelength corresponding to the bandgap energy of InSb.

cause the InSb cell rarely absorbs the radiation when \(\lambda > \lambda_{g}\), the spectral heat flux is mainly enhanced along the SPP dispersion curve at the vacuum-graphene/InSb interface due to evanescent waves with \(\beta > 200 \times \frac{2\pi}{\lambda_{g}}\) in \(6 \ \mu\text{m} < \lambda < \lambda_{g}\) (i.e., refer to Fig. 3(b)). If graphene’s chemical potential increases to 1 eV, then the SPP dispersion curve at the vacuum-graphene/InSb interface increases faster with respect to \(\beta\), as can be seen from Fig. 3(c). In other words, as graphene’s \(\mu\) increases, the SPP at the vacuum-graphene/InSb interface can be excited near \(\lambda_{g}\) by evanescent waves with relatively lower \(\beta\) values (i.e., \(\beta > 50 \times \frac{2\pi}{\lambda_{g}}\)). Therefore, the resulting heat transfer enhancement becomes larger than the case of \(\mu = 0.5\ \text{eV}\) because the evanescent waves with smaller \(\beta\) values are more favorable to the photon tunneling.

For \(N_{\text{Si}} = 5 \times 10^{20}\) in Figs. 3(d)-3(f), the SPP dispersion curve at the vacuum-Si interface appears when \(\lambda\) is much less than \(\lambda_{g}\), resulting in rather broadband absorption over \(3 < \lambda < 7\ \mu\text{m}\). However, similar to Fig. 3(c), the graphene of \(\mu = 1.0\ \text{eV}\) in Fig. 3(f) makes the SPP dispersion curves at the vacuum-Si interface and at the vacuum-graphene/InSb interface get closer, resulting in larger \(S_{\beta,\lambda}(\beta, \lambda)\) values along both dispersion curves.

If \(d\) increases to 50 nm as shown in Figs. 3(g)-3(i), although SPPs are excited at the vacuum-graphene/InSb interface, evanescent waves associated with SPPs do not actively participate in the photon tunneling. Graphene of \(\mu = 1.0\ \text{eV}\) has certain effects on enhancing the radiative heat flux at \(d = 50\ \text{nm}\) because it’s SPPs are associated with smaller \(\beta\) at \(\lambda_{g}\) than that of 0.5 eV;
Fig. 3. Contour of $S_{\beta, \lambda} (\beta, \lambda)$ in logarithmic scale: (a)-(c) $d = 10 \text{ nm}$ and $N_{Si} = 1 \times 10^{20} \text{ cm}^{-3}$; (d)-(f) $d = 10 \text{ nm}$ and $N_{Si} = 5 \times 10^{20} \text{ cm}^{-3}$; and (g)-(i) $d = 50 \text{ nm}$ and $N_{Si} = 1 \times 10^{20} \text{ cm}^{-3}$. For simplicity, the parallel wavevector component $\beta$ is normalized by bandgap wavelength ($\lambda_g = 7.29 \mu \text{m}$). Surface plasmon dispersion curves are also overlaid.

however, the heat transfer enhancement is not as significant as in Fig. 3(c).

The magnitude of spectral photocurrent density is plotted in Fig. 4 for selected $N_{Si}$ and $d$ values. The spectral photocurrent density exhibits similar wavelength-dependency with the net spectral heat flux shown in Fig. 2. In addition, the graphene changes the spectral photocurrent density according to its chemical potential, $\mu$, in the similar manner as in the spectral heat flux. The difference between the spectral photocurrent density and the spectral heat flux lies at wavelengths longer or far shorter than $\lambda_g$. Because InSb cell cannot generate electron-hole pairs by absorbing photons of $\lambda > \lambda_g$, there is no spectral photocurrent density for $\lambda > \lambda_g$. On the other hand, in the short wavelength region, the excessive energy of photons cannot generate extra electron-hole pairs and will eventually be dissipated as heat generation inside the TPV cell. Therefore, the spectral photocurrent is relatively small when $\lambda > \lambda_g$ or $2 < \lambda < 4 \mu \text{m}$, resulting in low conversion efficiency.

The power throughput of the graphene-assisted TPV system is plotted in Fig. 5(a) with respect to the vacuum gap width when $N_{Si} = 1 \times 10^{20} \text{ cm}^{-3}$. Monolayer of graphene enhances the...
power throughput about 30 times than the case without graphene for \( d = 10 \text{ nm} \), but its effect diminishes quickly as \( d \) increases to 50 nm. This is because SPPs at the vacuum-graphene/InSb interface do not contribute to the photon tunneling at larger vacuum gap widths as noted by Figs. 3(g)-3(i). Similarly, the effect of graphene of \( \mu = 0.5 \text{ eV} \) decreases faster than graphene of \( \mu = 1.0 \text{ eV} \) because graphene of lower \( \mu \) supports SPPs at larger \( \beta \) values with which the photon tunneling hardly occurs.

Figure 5(b) shows the conversion efficiency with respect to the vacuum gap width. In contrast to previous works [6, 7, 10] that show nearly constant conversion efficiencies in \( 10 \text{ nm} < d < 10 \text{ \mu m} \), the graphene-assisted TPV system exhibits monotonically decreasing conversion efficiency with respect to the vacuum gap width. Concerning the near-field thermal radiation, as the vacuum gap width decreases, evanescent waves with larger \( \beta \) values start to participate in heat transfer. As mentioned earlier, the radiative heat transfer inside the TPV cell is proportional to \( \exp\{-2\Im(k_2)z\} \), and the corresponding radiation penetration depth is \( \{2\Im(k_2)\}^{-1} \) [6]. When \( \beta \gg \omega/c_0 \), \( k_2 = \sqrt{k_2^2 - \beta^2} \) can be approximated as \( i\beta \). In such cases, the radiation penetration depth is simply \( 1/(2\beta) \); thus, the larger \( \beta \) is, the shorter penetration depth is, resulting in the smaller quantum efficiency because most of the incident radiation is absorbed near the surface (i.e., far away from the junction). In our case, however, large portion (\( \sim 90\% \)) of the radiative heat transfer is carried by evanescent waves with \( \beta < 5\omega/c_0 \) especially when \( d \geq 50 \text{ nm} \) because of the relative low \( T_1 \) as well as the mismatch between plasmon

Fig. 4. Spectral photocurrent density generated in the InSb cell: (a) \( d = 10 \text{ nm} \) and \( N_{Si} = 1 \times 10^{20} \text{ cm}^{-3} \); (b) \( d = 10 \text{ nm} \) and \( N_{Si} = 5 \times 10^{20} \text{ cm}^{-3} \); (c) \( d = 10 \text{ nm} \) and \( N_{Si} = 1 \times 10^{21} \text{ cm}^{-3} \); (d) \( d = 50 \text{ nm} \) and \( N_{Si} = 1 \times 10^{20} \text{ cm}^{-3} \).
Fig. 5. Effect of the vacuum gap width on the TPV performance: (a) power throughput; and (b) conversion efficiency.

frequencies of doped Si and InSb cell. In addition, the quantum efficiency of the proposed TPV system does not increase with the vacuum gap because of its negligible $\beta$-dependence, which will be further discussed later. As a result, the resulting conversion efficiency monotonically decreases with respect to the vacuum gap. Similar to the case of power throughput in Fig. 5(a), graphene rarely affects the conversion efficiency when $d \geq 50$ nm.

It becomes clear now that in order for a near-field TPV system to take benefit of monolayer graphene even for $d \geq 50$ nm, graphene’s chemical potential should be higher than 1.0 eV so that graphene can support SPPs at much smaller $\beta$ values in the vicinity of $\lambda_g$. This is because the radiative heat transfer is dominated by evanescent waves with $\beta < 5\omega/c_0$ when $d \geq 50$ nm. For example, if we consider doped Si with $N_{Si} = 1.2 \times 10^{20}$ cm$^{-3}$ and graphene with $\mu = 2.0$ eV, the resulting $EFP = 28.5$ and $EF_\eta = 4.8$ at $d = 10$ nm, and $EFP = 2.1$ and $EF_\eta = 2.0$ at $d = 50$ nm. Although monolayer graphene with $\mu > 1.0$ eV has rarely been fabricated so far, multilayer graphene with moderate chemical potential can be used instead [37].

Figure 6 depicts the quantum efficiency of the InSb cell for selected $N_{Si}$ and $d$ values. In general, the quantum efficiency becomes lower at shorter wavelengths and at smaller vacuum gap widths in which the corresponding radiation penetration depth is smaller. However, the InSb cell without the graphene layer has higher quantum efficiency at shorter wavelengths and
smaller vacuum gaps, due to the fact that contribution of evanescent waves with $\beta < 5\omega/c_0$ is substantial at shorter wavelengths and larger vacuum gap widths. Given that the radiation penetration depths of such evanescent waves are comparable to the total thickness of $p$- and $n$-regions, the smaller penetration depth is, the more photons can be absorbed, leading to higher quantum efficiency. In the similar manner, at larger vacuum gap widths, radiative heat transfer occurs dominantly by evanescent waves with $\beta < 5\omega/c_0$ so that most of photons are absorbed beyond the hole diffusion length, reducing the quantum efficiency.

When graphene is coated on the InSb cell, graphene can affect the contribution by evanescent waves whose penetration depth is smaller than the thickness of $p$-region. Considering that graphene with higher chemical potential enhances the heat transfer through evanescent waves with smaller $\beta$ values (i.e., with larger penetration depths; refer to Fig. 3), graphene can increase the quantum efficiency around $\lambda_g$, as shown in Figs. 6(a)-6(d). Recalling that the net spectral heat flux is also enhanced near $\lambda_g$, graphene can enhance both the net spectral heat flux and the quantum efficiency, which in turn leads to the increased power throughput with high conversion efficiency. As clearly seen from Fig. 6, quantum efficiency, $\eta_q < 80\%$ in most cases; thus, considering the minority carrier transport inside the TPV cell is very important to properly estimate the TPV system performance.

It was proposed that decreasing the thickness of the $p$-region can improve the quantum efficiency [6,10] by reducing the probability of recombination in the $p$-region. Consistently, when
we decrease the thickness of $p$-region, it certainly increases both the power throughput and the conversion efficiency in $10 < d < 40$ nm. For $40 < d < 100$ nm, however, $p$-region with thick-
ness of 1.6 ∼ 2.0 μm yields the maximum power throughput and the conversion efficiency, as can be seen from Figs. 7(a) and 7(b). This can be explained based on Fig. 7(c) in which |J_e| and |J_h| are plotted together. Because |J_{dp}| ≪ |J_h|, it is not shown in the figure. For 40 < d < 100 nm, evanescent waves with β < 5ω/c₀ are dominantly contribute to the heat transfer and the corresponding radiation penetration depths become comparable to the total thickness of p- and n-regions (i.e., few micrometers). In this d range, |J_e| has the higher value for thicker p-region as more photons are absorbed in the p-region, but |J_h| always decreases as the p-region becomes thicker because less photons are absorbed in the n-region. On the other hand, if the thickness of p-region becomes too small, then the whole p- and n-regions cannot absorb all the photons from the source. Therefore, there exists an optimal thickness of p-region for the maximum power throughput and the conversion efficiency. For the considered graphene-assisted InSb cell, the optimal p-region thickness is found to be 1.6 ∼ 2.0 μm. With these p-region thicknesses, we can obtain further enhancement in the power throughput and the conversion efficiency for large vacuum gap width, although the enhancement factor is not as significant as for the smaller vacuum gap cases. It can be readily seen that solving minority carrier transport is very important to get physical insight on the proper p-region thickness for a certain vacuum gap width.

4. Concluding remark

In the present work, we proposed a graphene-assisted near-field TPV system composed of a doped-Si source and InSb cell for low temperature applications. The performance of the proposed TPV system was theoretically analyzed by solving the near-field thermal radiation with the minority carrier transport in semi-analytic manner. In order to tune the surface plasmon frequency, monolayer graphene was applied to the TPV cell side and then SPPs at the vacuum-graphene/InSb interface can interact with those at the vacuum-Si interface. It was found that the graphene layer (μ = 1.0 eV) can enhance the power throughput by 30 times and the conversion efficiency by 6.1 times for the doped-Si source (N_{Si} = 1 × 10²⁰ cm⁻³) at d = 10 nm. However, such an enhancement diminishes quickly as the vacuum gap increases. We also showed that the InSb cell with thicker p-region is advantageous for the larger vacuum gaps. In the previous studies proposing the near-field TPV system for the tungsten source [6, 7, 10] at 2000 K, the conversion efficiency was around 20 ∼ 30% when the corresponding Carnot efficiency was 85%. It is worthwhile to mention that we achieved the maximum conversion efficiency of 19.4% under the Carnot efficiency of 40% at the source temperature of 500 K. Therefore, monolayer graphene played significant impact on further enhancing the efficiency of near-field TPV system.

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