Abstract

This paper proposes an analysis method of Petri nets(PNs) using the relational algebra(RA). More specifically, we represent PNs in relations of the relational model. Based on such representation, we first develop an algorithm for generating reachability trees of PNs. We then develop algorithms for analyzing properties of PNs, such as boundedness, conservation, coverability, reachability, and liveness.

The advantage of this approach is as follows: First, the algorithms represented by RA can be easily converted to a query language such as SQL of the widely used, commercial relational database management systems(DBMSs). Second, we can alleviate the problem of state space explosion because relational DBMSs can handle large amounts of data efficiently. Finally, we can use the DBMS's query language to interpret the Petri nets and make simulation.

1. Introduction

Petri nets are a powerful tool for describing and studying systems that are characterized as being concurrent, asynchronous, distributed, parallel, and/or nondeterministic. A strength of Petri nets is their support for analysis of many properties and problems associated with concurrent systems. But a major weakness is that modeled Petri nets tend to become too large for analysis even for a modest-size system. For example, in recent years considerable effort has been given to develop models for specification and validation of protocols. Petri nets, which have been designed for the purpose of communications between finite-state machines, seem a quite natural tool for modeling the protocols.\[1\] [2]
2. The Relational Model and Algebra

In this section we briefly introduce the relational model and algebra.

2.1 The Relational Model

The mathematical concept underlying the relational model is the set-theoretical relation (s-relation), which is a subset of the Cartesian product of a list of sets. Given sets $S_1, S_2, \ldots, S_n$, a $s$-relation $R$ is a subset of the Cartesian product $S_1 \times S_2 \times \cdots \times S_n$. A relation $R$ is said to be of degree $n$. Each of the sets $S_1, S_2, \ldots, S_n$ on which one or more $s$-relations are defined is called a domain.

A relation $R$ in the relational model\cite{[6]} \cite{[7]} is very similar to its counterpart in mathematics. Relations can be perceived as tables, in which each row is a tuple and each column has a distinct name called an attribute. A relation $R$ on $A = \{A_1, A_2, \ldots, A_n\}$ will be denoted by $R[A]$. Let $t$ be a tuple in $R[A]$. The components of $t$ corresponding to the set of attributes $X \subseteq A$ is denoted by $t[X]$. If $a_1$ is a constant from the domain of $A_1$, then $\langle a_1, a_2, \ldots, a_n \rangle$ is a constant tuple over $A_1, A_2, \ldots, A_n$.

2.2 Operations of Relational Algebra

There are five fundamental operations that serve to define relational algebra. These operations are: union, difference, Cartesian product, project, and select.

1. Union $\cup$ : The union of relations $R$ and $S$, denoted $R \cup S$, is smallest set containing all tuples of $R$ and all tuples of $S$.
   
   $R \cup S = \{ t \mid t \in R \lor t \in S \}$

2. Difference $-$ : The difference of relations $R$ and $S$, denoted $R - S$, is the set containing the tuples of $R$ that are not in $S$.
   
   $R - S = \{ t \mid t \in R \land t \notin S \}$

3. Cartesian Product $\times$ : The Cartesian product of $R$ and $S$ is the set of all ordered pairs $\langle r, s \rangle$ such that the first element of ordered pair, $r$, is from $R$ and second element of the ordered
pair, $s$, is from $S$.
\[ R \times S = \{(r, s) | r \in R \land s \in S\} \]

(4) **Projection $\pi$** : Projection chooses a subset of the columns. Let $R$ be a relation on a set of attributes $A = \{A_1, A_2, \ldots, A_n\}$ and $X$ be subset of $A$. The projection $\pi_X(R)$ is obtained by dropping columns with attributes not in the set $X$ and removing duplicate tuples in what remains.
\[ \pi_X(R) = \{t[X] | t \in R\} \]
In general, we can extend $X$ as a set of the arithmetic expression of elements of $A$.

(5) **Selection $\sigma$** : Let $F$ be a formula which is one of the following types:
1. $F = \phi$.
2. $F = a \theta b$ where,
   (a) $a(b)$ are constants, attribute names, or component numbers; component $i$ is represented by $Si$.
   (b) $\theta \in \{\langle, =, \rangle, \leq, \neq, \geq\}$.
3. $F_1$ and $F_2$ are two formulas and $F = F_1 \land F_2$ or $F = F_1 \lor F_2$ or $F = \neg F_1$.

Then $\sigma_F(R)$ is the set of tuples $t$ in $R$ such that when we substitute the $i$th component of $t$ for any occurrences of $Si$ in the formula $F$ for all $t$ and substitute the corresponding components of $t$ for attribute names in the formula $F$, the formula $F$ becomes true.
\[ \sigma_F(R) = \{t | F(t) \land t \in R\} \]

In addition to the five fundamental operations, there are some other useful operations on relations that can be defined in terms of the five fundamental operations above, namely, **intersection**, **theta join**, and **natural join**.

(6) **Intersection $\cap$** : The intersection of relations $R$ and $S$, denoted $R \cap S$, is the smallest set containing all tuples that are members of both $R$ and $S$.
\[ R \cap S = R \setminus (R - S) \]

(7) **Theta join $R \bowtie \theta$** : The theta join of $R$ and $S$ on columns $i$ and $j$, written $R \bowtie_{ij} S$, is shorthand for $\sigma_{A_i \theta A_j}(R \times S)$, if $R$ is of degree $r$. $\theta$ is an arithmetic comparison operator (\(\equiv\), (\(\prec\), and so on).
\[ R \bowtie_{ij} S = \sigma_{A_i \theta A_j}(R \times S), \text{ if } R \text{ is of degree } r \]

(8) **Natural Join $\bowtie$** : Let $R$ and $S$ be relations on a set of attributes $A = \{A_1, A_2, \ldots, A_n\}$ and $X$, $Y$, and $Z$ be a subset of $A$. The natural join combines two relations on all their common attributes. The natural join, written $R[XY] \bowtie S[YZ]$, is a relation $T[XYZ]$ of all tuples $t$ over $XYZ$ such that there are tuples $t[XY] \in R$ and $t[YZ] \in S$.
\[ R[XY] \bowtie S[YZ] = \{t[XY] \in R \land t[YZ] \in S\} \]

### 3. Modeling of Petri Nets using RA

Petri nets are a promising tool for describing and studying systems that are characterized as being concurrent, asynchronous, distributed, parallel, and/or nondeterministic. A strength of Petri nets is their support for analysis of many properties and problems associated with concurrent systems. But a major weakness is that modeled Petri nets tend to become too large for analysis, even for a modest-size system.

This problem can be alleviated by the relational approach, because relational DBMSs can handle large amounts of data efficiently. That is, if we represent Petri nets as relations, their reachability tree can be obtained as relations by using RA operators. Furthermore, we can draw several properties of Petri nets from the reachability tree in forms of relations.

#### 3.1 Definition and Properties of Petri Nets

In this section, we present basic definitions of Petri nets and explain behavioral properties of Petri nets. A formal definition of Petri net follows.
Definition 3.1 A Petri net, $PN$, is a five-tuple structure, where $PN = (P, T, I, O, M_0)$

1. $P$ is finite set of Place
2. $T$ is a finite set of transitions. The set of places and the set of transitions are disjoint, $P \cap T = \emptyset$
3. $I:T \rightarrow \mathbb{P}^\infty$ is the input function, a mapping from transition to bags of places.
4. $O:T \rightarrow \mathbb{P}^\infty$ is the output function, a mapping from transition to bags of places.
5. $M_0 : P \rightarrow \mathbb{N}$ is the initial marking function, a mapping from set of places $P$ to the nonnegative integers $\mathbb{N}$.

Note that the inputs and outputs of a transition are bags of places. The multiplicity of an input place $p$ for a transition $t$ is the number of occurrences of the place in input bag of a transition, $\#(p, T(t))$. Similarly, the multiplicity of an output place $p$ for transition $t$ is $\#(p, O(t))$.

definition 3.2 Let $PN = (P, T, I, O, M_0)$ be a Petri net.

1. A function $M_k : P \rightarrow \mathbb{N}$, where $k \in \mathbb{N}$ is called a marking of $PN$. $M_k(p)$ represents the number of tokens in the place $p$.

2. A transition $t \in T$ is enabled at marking $M_k$ iff $\#(p, T(t)) \leq M_k(p)$, $\forall p \in P$. An enabled transition may or may not fire.

3. If $t \in T$ is a transition which is enabled at $M_k$ then $t$ may fire, yielding a new marking $M_k'$ given by the equation:

$$M_k'(p) = M_k(p) - \#(p, I(t)) + \#(p, O(t)), \forall p \in P$$

4. Firing $t$ changes the marking $M_k$ into the new marking $M_k';$ we denote this fact by $M_k \xrightarrow{t} M_k'$.

5. The set of all possible markings reachable from $M_k$ in $PN$, denoted $R(M_k)$, is the smallest set of markings of $PN$ such that:

(a) $M_k \in R(M_k)$
(b) if $M_k' \in R(M_k)$ and $M_k \xrightarrow{t} M_k''$ for some $t \in T$ then $M_k'' \in R(M_k)$.

There are two types of properties studied with a Petri-net model: behavioral and structural properties. In this paper, we discuss only behavioral properties because structural properties depend on the topological structures of Petri nets and can be well characterized in terms of the incidence matrix and its associated homogeneous equations or inequalities.

Many behavioral properties have been studied in Petri nets[8][9][10] but we consider only boundedness, conservation, liveness, reachability, and coverability. Boundedness can be interpreted as a stable factor in the system. For example, if the modeled Petri net is unbounded, then this may indicate the occurrence of overflow of some buffers in the system. Conservation is an important property in that if a Petri net models resource allocation systems, then tokens, which represent resources, are neither created nor destroyed. Liveness has an important meaning for many systems and is closely related to the complete absence of deadlocks in concurrent systems. Reachability is a fundamental basis for studying the dynamic properties of any system. Coverability is closely related to $L1$-liveness.

[8] Let $M_k$ be the minimum marking needed a transition $t$. Then $t$ is dead if and only if $M_k$ is not coverable. That is, $t$ is $L1$-live if and only if $M_k$ is coverable.

Definition 3.3

1. Boundedness:

(a) $p \in P$ is n-bounded iff $\forall M_k \in R(M_0)$, $M_k(p) \leq n$;

(b) $PN$ is n-Bounded iff $\forall p \in P$, $p$ is n-bounded;

(c) $PN$ is safe iff $PN$ is 1-bounded.

(d) $PN$ is bounded iff $\exists n \in \mathbb{N}$, $PN$ is n-bounded.

2. Conservation:

(a) $PN$ is strictly conservative iff $\forall M_k \in R(M_0)$,
\[ \Sigma_{r \in R} M_k(p) = \Sigma_{r \in R} M_o(p) \]

(b) PN is **conservative** with respect to a weighting function \( W : P \to \mathbb{N}, \) iff \( \forall M_k \in R(M_o), \Sigma_{r \in R} W(p) \cdot M_k(p) = \Sigma_{r \in R} W(p) \cdot M_o(p) \)

3. Liveness:
   (a) \( t \in T \) is live iff \( \forall M_k \in R(M_o), \exists M_k' \in R(M_k), t \) is enabled at \( M_k' \).
   (b) PN is live iff \( \forall t \in T, t \) is live.
   (c) PN is **deadlock free** iff \( \forall M_k \in R(M_o), \exists t \in T, t \) is enabled \( M_k \).

4. Reachability Problem: Given a Petri net PN and a marking \( M_k \), is \( M_k \in R(M_o) \)?

5. Coverability Problem: Given a Petri net PN and a marking \( M_k \), is there a reachable marking \( M_k' \in R(M_o) \) such that \( M_k' \geq M_k \)?

1. \( P[pid] \) is a table to store place \( P \) of PN.
2. \( T[tid] \) is a set to store transitions \( T \) of PN.
3. \( I[tid, pid, cnt] \) is a table where each tuple \( i \) corresponds to an input arc of \( i[tid] \in T \).
   \[ I[tid, pid, cnt] = \{i|i \in I \land i[tid] \in T \land i[pid] \in P \land cnt = \#(pid, I[tid]) \} \]
4. \( O[tid, pid, cnt] \) is a table where each tuple \( o \) corresponds to an output arc of \( o[tid] \in T \).
5. \( M[mid, pid, cnt] \) is a table to store all marking \( M_k \in R(M_o) \). A marking \( M_k \) can be retrieved by \( \sigma_{pid} = k(M) \).
6. \( R[mid, pid, cnt'] \) is a table to store the reachability tree of PN.

The table defined above for Figure 1 are shown in Figure 2.

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### Figure 1

A simple Petri net \( PN_i \)

### Figure 2

The relation representation of the simple Petri net \( PN_i \)

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3.2 Tabular Representation of Petri nets

In this section we define relations to represent Petri nets.

**Schema definition:**
3.3 Algorithm for Generating the Reachability Tree

The reachability tree represents the reachability set of a Petri net. It is a useful tool for solving behavioral properties. For the reachability tree to be finite, the special symbol, \( \omega \), are necessary for the construction of the reachability tree[8]. The rules for \( \omega \) are:

- \( n \omega \) for all \( n \in \mathbb{N} \):
- \( \omega + \omega = \omega + n = \omega - n = \omega \) for all \( n \in \mathbb{N} \):
- \( \omega \leq \omega \).

Now, we develop the algorithm for generating the reachability tree of a Petri net based on RA.

1. Enabled-transition operation ET\((M_k)\): Given \( PN \) and \( M_k \), this operation, denoted ET\((M_k)\), yields a set of enabled transitions in the marking \( M_k \). It is defined by the following procedure:

   - **step1.** \( MInput := \pi_{\text{st}} \cdot \pi_{\text{in}} \cdot \pi_{\text{cm}}(I = (I_{\text{st}} = M_k \cap \neg I_{\text{cm}} \leq M_k_{\text{cm}}) \cdot M_k) \);
   - **step2.** \( UInput := I - MInput; \)
   - **step3.** \( NTrans := \pi_{\text{st}} \cdot \pi_{\text{in}} \cdot \pi_{\text{cm}}(UInput); \)
   - **step4.** \( ET(M_k) := T - NTrans; \)

2. Next-state operation NS\((M_k, t)\): This operation NS\((M_k, t)\) yields the new marking\((state)\) which results from firing the transition \( t \) in the marking \( M_k \). Since \( t \) can fire only if it is enabled, NS\((M_k, t)\) is not enabled in marking \( M_k \). If \( t \) is enabled, then NS\((M_k, t)\) is \( M_k' \), where \( M_k' \) is the marking which results from removing tokens from the inputs of \( t \) and adding tokens to the outputs of \( t \). It is defined by the following procedure:

   - **step1.** \( Pre := \pi_{\text{st}} \cdot \pi_{\text{in}} \cdot \pi_{\text{cm}}(I = (I_{\text{st}} = I(I)); \)
   - **step2.** \( Pos := \pi_{\text{st}} \cdot \pi_{\text{in}} \cdot \pi_{\text{cm}}(I = I(0)); \)
   - **step3.** \( Tmp1 := \pi_{\text{st}} \cdot \pi_{\text{in}} \cdot \pi_{\text{cm}}(M_k' = M_k \cdot \neg M_k_{\text{cm}} = \neg M_k \cdot \neg (\text{Pos} \cdot \text{Pre}); \)
   - **step4.** \( Tmp2 := \pi_{\text{st}} \cdot \pi_{\text{in}} \cdot \pi_{\text{cm}}(M_k' = M_k \cdot \neg \text{Pos} \cdot \text{Pre}; \)

3. Parent-marking operation PM\((R, M_k)\): Given the reachability tree \( R \) and any marking \( M_k \) which is a node of \( R \), this operation yields all the parent markings of \( M_k \). It is defined by the following procedure:

   - **step1.** \( Parent := \{k\}; New := \{k\}; \)
   - **step2.** while \( New \neq \emptyset \) do
     1. \( *\)Without loss of generality we assume \( New \) is queue *
     \( i := \text{DEQUEUE}(New); (*\)Extract front element from queue *
     if \( i \neq 0 \) then
       for each \( j \in \pi_{\text{mid}}(\pi_{\text{mid}}' = \pi(R)) \) do
         if \( j \notin Parent \) then
           Parent := Parent \cup j;
           New := New \cup j;
       end if
     end if
   end while
   - **step3.** \( PM(R, M_k) := Parent; \)

4. Update-marking operation UM\((M_k, M_k')\): Given \( M_k' \), which is immediately reachable from \( M_k \), if there exists a path from the root to \( M_k \) containing a marking \( M_j \) such that \( \pi_{\text{cnt}}(\sigma_{\text{pid}} = p(M_j)) \leq \pi_{\text{cnt}}(\sigma_{\text{pid}} = p(M_k')) \) for all \( p \in P \),

then replace \( m_k'[\text{cnt}] \) (where \( m_k' \subseteq M_k' \) and \( m_k' \) \[\{pid\} = p \) by \( \omega \) wherever \( \pi_{\text{cnt}}(\sigma_{\text{pid}} = p(M_j)) \lt \pi_{\text{cnt}}(\sigma_{\text{pid}} = p(M_k')) \).

   - **step1.** \( Tmp := \emptyset; \)
   - **step2.** \( Pmid := \text{PM}(R, M_k); \)
step3. for each $i \in P_{mid}$ do
  if $\pi_{cnt}(\sigma_{pid} = \nu(M_i)) \leq \pi_{cnt}(\sigma_{pid} = \nu(M_k'))$ for all $p \in P$ then
    for each $p \in P$ do
      if $\pi_{cnt}(\sigma_{pid} = \nu(M_i)) \leq \pi_{cnt}(\sigma_{pid} = \nu(M_k'))$ then
        $Tmp := Tmp \cup \pi_{pid \cdot cnt} = \omega(M_k')$;
      else
        $Tmp := Tmp \cup \pi_{pid \cdot cnt}(M_k')$;
      end if
    end for
    $M_k' := Tmp$
  end if
end for

5. Reachability-tree operation RT: Using the above defined operations, we could construct the reachability tree of a Petri net as follows. Note that the relation $R$ is initially empty.

step1. $New := \{o\}; j := o$;

step2. while $New \neq \phi$ do
  (* Without loss of generality we assume New is queue *)
  $i := \text{DEQUEUE}(New)$; (* Extract front element from queue *)
  if $\sigma_{mla} = i(\pi_{mla}(R)) = \phi$ then
    for each $t \in ET(M_i)$ do
      $M' := NS(M_i, t)$;
      $UM(M_k, M')$;
      if $M' = M_k$. where $k \in \pi_{mla}(M)$ then
        $R := R \cup \{(i, t, k)\}$;
      else
        $j := j + 1$;
        $M' := M \cup \{\langle j, m' \rangle | m' \in M'\}$;
        $R := R \cup \{(i, t, j)\}$;
    end for
  end if
end while

(Figure 3) The reachability tree of $PN_1$
\[
M = \begin{array}{|c|c|c|}
\hline
mid & ptid & cnt \\
\hline
0 & p_1 & 3 \\
0 & p_2 & 0 \\
0 & p_3 & 0 \\
1 & p_1 & 2 \\
1 & p_2 & 1 \\
1 & p_3 & 0 \\
2 & p_1 & 1 \\
2 & p_2 & 2 \\
2 & p_3 & 0 \\
3 & p_1 & 2 \\
3 & p_2 & 0 \\
3 & p_3 & 1 \\
4 & p_1 & 0 \\
4 & p_2 & 3 \\
4 & p_3 & 0 \\
5 & p_1 & 1 \\
5 & p_2 & 1 \\
5 & p_3 & 1 \\
6 & p_1 & 0 \\
6 & p_2 & 2 \\
6 & p_3 & 1 \\
7 & p_1 & 1 \\
7 & p_2 & 0 \\
7 & p_3 & 2 \\
8 & p_1 & 0 \\
8 & p_2 & 1 \\
8 & p_3 & 2 \\
9 & p_1 & 0 \\
9 & p_2 & 0 \\
9 & p_3 & 3 \\
\hline
\end{array}
\]

\[
R = \begin{array}{|c|c|c|}
\hline
mid & tid & mid' \\
\hline
0 & t_1 & 1 \\
1 & t_2 & 2 \\
1 & t_2 & 3 \\
2 & t_1 & 4 \\
2 & t_2 & 5 \\
3 & t_1 & 5 \\
4 & t_2 & 6 \\
5 & t_1 & 6 \\
5 & t_2 & 7 \\
6 & t_1 & 8 \\
7 & t_1 & 8 \\
7 & t_2 & 9 \\
8 & t_2 & 0 \\
\hline
\end{array}
\]

(Figure 4) The marking table \( M \) and reachability table \( R \) of \( PN \).

\[\text{New} = \text{New} \cup \{j\};\]
end if
end for
end if
end while

For example, we can obtain the reachability tree of Figure 3 by applying RT operation to the Petri net of Figure 1. The contents of \( M \) and \( R \) is shown Figure 4.
3.4 Algorithm for Analysis of Petri Nets

The reachability tree is a very powerful tool for analysis of behavioral properties. But, reachability and liveness, in general, cannot be solved by using the reachability tree because of the existence of the \( \omega \) symbol. The reachability tree does not necessarily contain enough information to solve reachability or liveness. However, if the modeled system satisfies boundedness then all of behavioral properties can be determined by using the reachability tree because it contains all possible markings. Fortunately, most realistic problems should be bounded. Thus, we assume that the reachability tree is bounded for properties such as reachability and liveness.

1. Safeness: Safeness is a special case of the more general boundedness property. This property requires that each place \( p \in P \) should not have more than one token for each marking \( M_k \in M \). To show safeness we first obtain the set of unsafe places. It is easy to show that the set of unsafe places is given by

\[
\text{Unsafe-Place} := \pi_{\text{vol}}(\pi_{\text{ctl}}(M)).
\]

Next, we justify the safeness with Unsafe-Place. That is, if Unsafe-Place is empty, then the modeled system is safe.

\begin{itemize}
  \item \textbf{step1.} Unsafe-Place: \( = \pi_{\text{vol}}(\pi_{\text{ctl}})1(M) \);
  \item \textbf{step2.} if Unsafe-Place \( \neq \emptyset \) then
    \text{return unsafe};
  \item \textbf{step3.} return safe;
\end{itemize}

In the reachability tree of Figure 3, there are three unsafe places given below:

\[
\text{Unsafe-Place} = \begin{bmatrix}
  \text{pd} \\
  p_1 \\
  p_2 \\
  p_3
\end{bmatrix}
\]

2. Boundedness: This property requires that each place \( p \in P \) should not have more than \( k \) tokens for each marking \( M_k \in M \). In other words, if the modeled system is bounded, then \( k \) is the maximum number among the number of tokens of all places for all markings.

\begin{itemize}
  \item \textbf{step1.} Unbounded-Place: \( = \pi_{\text{vol}}(\pi_{\text{ctl}} = \omega(M)) \);
  \item \textbf{step2.} if Unbounded-Place \( \neq \emptyset \) then
    \text{return unbounded};
  \item \textbf{step3.} \( k := \max(\pi_{\text{vol}}(M)) \);
  \item \textbf{step4.} return \( k \)-bounded;
\end{itemize}

In the reachability tree of Figure 3, \( PN_1 \) is 3-bounded.

3. Conservation: A petri net is strictly conservative if it does not lose or gain tokens but merely moves them around. This property can easily be tested by the following procedure. First, boundedness is checked, because the necessary condition of conservation is boundedness. Next, if the modeled system is bounded, then we check whether the sum of the number of tokens of each marking \( M_k \in M \) is equal or not.

\begin{itemize}
  \item \textbf{step1.} Unbounded-Place: \( = \pi_{\text{vol}}(\pi_{\text{ctl}} = \omega(M)) \);
  \item \textbf{step2.} if Unbounded-Place \( \neq \emptyset \) then
    \text{return unconservative};
  \item \textbf{step3.} \( Rmid := \pi_{\text{mid}}(M) \);
  \item \textbf{step4.} \( \text{sum}_0 := \sum(\pi_{\text{vol}}(\pi_{\text{ctl}} = i(M))) \);
  \item \textbf{step5.} for each \( i \in Rmid \) do
    \( \text{sum}_i := \sum(\pi_{\text{vol}}(\pi_{\text{ctl}} = i(M))) \);
    if \( \text{sum}_0 \neq \text{sum}_i \) then
     \text{return unconservative};
  \item \textbf{step4.} return conservative;
\end{itemize}

We can also test conservation by considering the weighting factor given to each place. Because this is very similar to the above procedure, the
procedure is not enumerated here. In the reachability tree of Figure 3, $PN_1$ is strictly conservative.

4. Coverability: The coverability problem is that given a Petri net $PN$ with initial marking $M_0$ and a marking $M_k$, is there a reachable marking $M'$ such that $\pi_{cm}(\sigma_{sta}=p(M')) \geq \pi_{cm}(\sigma_{sta}=p(M_k))$, for all $p \in P$?

step1. $Rmid := \pi_{md}(M)$;

step2. for each $i \in Rmid$

if $\pi_{cm}(\sigma_{vid}=p(M')) \leq \pi_{cm}(\sigma_{vid}=p(M_i))$ for all $p \in P$ then

return coverable;

end if

end for

step4. return uncoverable;

We can know that a marking $M=(1, 1, 0)$ is coverable in the reachability tree Figure 1 because $M_1$ is covered by $M_1$ or $M_2$.

5. Reachability: The reachability problem is that given a Petri net $PN$ with initial marking $M_0$ and a marking $M_k$, is $M_k \in M$?

step1. $Rmid := \pi_{md}(M)$;

step2. for each $i \in Rmid$

if $M_i = M_k$ then

return reachable;

end if

end for

step4. return unreachable;

We can know that a marking $M=(0, 1, 2)$ is reachable in the reachability tree of Figure 1 because $M_1$ is $M_0$.

6. Liveness: We say that a Petri net is in deadlock if no transition in the net is enabled. These deadlock markings correspond to the terminal nodes of the reachability tree.

$Deadlock := \pi_{md}(R) - \pi_{md}(R)$

We assume that a Petri net is live if it doesn't have any deadlock states. In the reachability tree of Figure 3, there is one deadlock marking (state) given below:

$Deadlock = \begin{array}{c}
mid \\
9
\end{array}$

4. Conclusions

We have shown the relational methodology for the analysis of Petri nets based on RA representation and manipulation. Specifically, we have described an implementation of reachability analysis for systems modeled by Petri nets. In this approach, systems are represented as a set of relations. Using these relations, the reachable global states are determined by an iterative sequence of operations of RA that eventually generate a reachability tree for the system. This final reachability tree can be examined by specific queries, again, described in terms of RA, to verify properties of Petri nets. Because the several properties of Petri nets have been formulated in terms of RA's operators, all procedure of analysis of Petri nets can easily transfer to commercial relational DBMSs. In conclusion, we believe that RA is a useful tool for manipulating data.

4. References


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