Series-Series Compensated Wireless Power Transfer at Two Different Resonant Frequencies

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Abstract— Recently, resonance phenomenon in wireless power transfer (WPT) is widely used to deliver power to a load over relatively large air gap via magnetic coupling. Since many reactive components in WPT system are contributed to the resonance phenomenon, there are many resonant frequencies. These resonant frequencies have different and distinguishing features. For this reason, the analysis of the resonant tank at various resonant frequencies is required to understand WPT system more easily. Therefore, in this paper, three resonant frequencies in series-series compensated WPT (SS-WPT) system are defined. And then, this paper is aiming to present the features of SS-WPT system in the major two of the three resonant frequencies. The analysis is supported with experimental results.

Keywords—Coupled energy transfer, inductive power transfer, series-series compensation, resonant converter, wireless power transfer.

I. INTRODUCTION

When windings share a magnetic core and are close to each other, magnetic coupling is good and tight. This tightly magnetic coupling enables most of magnetic field generated by the one winding to pass through the other winding. For this reason, tightly magnetic coupling was the fundamental principle of transferring power effectively. On the contrary, when windings do not share a magnetic core and are instead separated over large air gap, magnetic coupling is bad and loose. As a result, most of magnetic field generated on one winding does not link with the other winding and return to the generated winding. Therefore, loosely magnetic coupling was considered as non-effective way to transfer power. However, loosely magnetic coupling has huge advantages of low maintenance cost, high reliability and the ability to operate in extreme environment from the absence of conductive contact. Furthermore, recently, resonance phenomenon is widely used to compensate the returned magnetic field in wireless power transfer (WPT) system [1]-[18] and thus begins to create new possibilities of effective power transmission.

Generally, capacitors are inserted to resonate with windings and overcome the returned magnetic field and low efficiency in WPT system. It is called as compensation. Since many reactive components are contributed to compensation, there are many resonant frequencies in compensated WPT system. Unlike the

other frequencies, the resonant frequencies have different and distinguishing features. For this reason, the analysis of the resonant tank at various resonant frequencies is required to understand WPT system more easily.

Furthermore, in various research areas, the preferred resonant frequency is divided into two: one resonant frequency by self-inductance and resonant capacitor [1]-[18] and the other resonant frequency by leakage inductance and resonant capacitor [19], [20]. Therefore, the difference between two resonant frequencies is needed to be clarified.

In this paper, series-series compensated WPT (SS-WPT) system is analyzed at the major two resonant frequencies. First of all, the configuration and equivalent circuit of the SS-WPT system are explained and resonant frequencies are defined from the well-known open-circuit and short-circuit tests. And then, the features of the SS-WPT in the major two of the three resonant frequencies are presented. To analyze the SS-WPT system effectively, input impedance analysis and frequency domain analysis based on Fundamental Harmonic Approximation (FHA) are used. And the result of analysis is verified by experiment.

II. STRUCTURE OF SS-WPT SYSTEM

SS-WPT system consists of an input voltage source, a full-bridge inverter, a resonant tank, a full-bridge rectifier, a capacitive filter, and a load, as shown in Fig. 1. The full-bridge inverter connected to the input voltage source V_{IN} applies a square wave of voltage to the resonant tank. The square wave of voltage $V_{AB}(t)$ can be described by

$$V_{AB}(t) = \begin{cases} V_{IN} & (0 < \omega t \leq \pi) \\ -V_{IN} & (\pi < \omega t \leq 2\pi) \end{cases} \quad (1)$$

As the resonant tank with series-series (SS) external capacitors acts as a band pass filter, the effect of any harmonic components of $V_{AB}(t)$ can be neglected, except for the fundamental harmonic component. Therefore, the fundamental harmonic of the square wave voltage $V_{AB}^F(t)$ is only considered [21]. From Fourier expansion of the square wave voltage $V_{AB}(t)$, the fundamental voltage $V_{AB}^F(t)$ is given by

$$V_{AB}^F(t) = \frac{4}{\pi} V_{IN} \sin \omega t = V_m \sin \omega t \quad (2)$$

Fig. 1 Configuration of series-series compensated wireless power transfer system

The loosely coupled primary winding and secondary winding is usually modeled by a coupled-inductor model, as shown in Fig. 1. Assuming that the diameter of the primary winding is the same diameter as the secondary winding, the coupled-inductor model is transformed into the two-leakage inductance transformer model, as shown in Fig. 2(a).

For resonance and compensation, external series resonant capacitors C_1 and C_2 are added to the two-leakage inductance transformer model. Therefore, the resonant tank of the SS-WPT system is formed from two-leakage inductance transformer model and series resonant capacitors. Since the two-leakage inductance transformer model tank with series resonant capacitors has a current-source characteristic, it should be connected to a capacitive filter having a voltage-source characteristic. Therefore, the full-bridge rectifier with the capacitive filter provides DC voltage at the output, which can be represented as a load resistance R_L . Since the fundamental harmonic of the square wave $V_{AB}^F(t)$ at the input is considered, the full-bridge rectifier, capacitive filter, and load can be modeled by ac load resistance $\frac{8}{\pi^2} R_L$ [21], as shown in Fig. 2(a).

To analyze the resonant tank effectively, the components in the secondary side of the ideal transformer are transformed to their equivalents in the primary side in Fig. 2(b). The ac load resistance is transformed to the equivalent load resistance R_{eq} .

$$R_{eq} = \frac{8}{\pi^2} n^2 R_L \quad (3)$$

The primary leakage inductance L_{lkp} is equal to the secondary leakage inductance reflected to the primary side $n^2 L_{lks}$ in Fig. 2(b).

$$L_{lkp} = (1 - \kappa) L_P = (1 - \kappa) n^2 L_S = n^2 L_{lks} \quad (4)$$

where κ is coupling coefficient.

Also, since the resonant frequency in the primary side is the same as the resonant frequency in the secondary side, the primary resonant capacitance C_1 is equal to the secondary resonant capacitance reflected to the primary side C_2/n^2 , and the reactance X_1 is equal to the reactance X_2 in Fig. 2(b).

$$C_1 = \frac{C_2}{n^2} \quad \left(\frac{1}{\sqrt{L_P C_1}} = \frac{1}{\sqrt{n^2 L_S C_1}} = \frac{1}{\sqrt{L_S C_2}} \right) \quad (5)$$

$$X_1 = \omega L_{lkp} - \frac{1}{\omega C_1} = \omega n^2 L_{lks} - \frac{n^2}{\omega C_2} = X_2 \quad (6)$$

And then, the reactance in the SS compensated resonant tank is represented as the primary reactance. As a result, the frequency-domain equivalent circuit is shown in Fig. 2(c).

Fig. 2 Equivalent circuit of series-series compensated wireless power transfer system and its frequency-domain equivalent circuits

III. RESONANCE FREQUENCIES

From Fig. 2(c), resonant frequencies are defined from the well-known open-circuit and short-circuit tests. The input impedance under the secondary open-circuit condition Z_{IN-oc} is given by

$$Z_{IN-oc} = s(1 - \kappa) L_P + \frac{1}{s C_1} + s \kappa L_P = \frac{s^2 L_P C_1 + 1}{s C_1} \quad (7)$$

In (7), the input impedance under the secondary open-circuit condition Z_{IN-oc} has two complex conjugate zeros at the first resonant frequency ω_{O1} and one pole at zero. And then, the input impedance under the secondary short-circuit condition Z_{IN-sc} is given by

$$\begin{aligned} Z_{IN-sc} &= s(1 - \kappa) L_P + \frac{1}{s C_1} + s \kappa L_P \parallel (s(1 - \kappa) L_P + \frac{1}{s C_1}) \\ &= \frac{[s^2(1 - \kappa) L_P C_1 + 1][s^2(1 + \kappa) L_P C_1 + 1]}{s C_1 [s^2 L_P C_1 + 1]} \end{aligned} \quad (8)$$

In (8), the input impedance has two pair of complex conjugate zeroes at the second and third resonant frequencies ω_{O2} and ω_{O3} , one pole at zero and two complex conjugate zero at the

Fig. 3 Input current and equivalent circuits by input impedance analysis: (a) at first resonant frequency ω_{O1} (b) at second resonant frequency ω_{O2}

first resonant frequency ω_{O1} . Therefore, three resonant frequencies in SS-WPT system are as follows:

$$\omega_{O1} = \frac{1}{\sqrt{L_P C_1}} = \frac{1}{\sqrt{L_S C_2}} \quad (9)$$

$$\omega_{O2} = \frac{1}{\sqrt{(1-\kappa)L_P C_1}} = \frac{1}{\sqrt{L_{lkp} C_1}} = \frac{\omega_{O1}}{\sqrt{1-\kappa}} \quad (10)$$

$$\omega_{O3} = \frac{1}{\sqrt{(1+\kappa)L_P C_1}} = \frac{1}{\sqrt{(L_P + L_m) C_1}} = \frac{\omega_{O1}}{\sqrt{1+\kappa}} \quad (11)$$

The first resonant frequency ω_{O1} is fixed. On the other hand, the second and third resonant frequencies ω_{O2} and ω_{O3} are determined by the coupling coefficient κ .

The second resonant frequency ω_{O2} is preferred in Power Electronics, whereas the first resonant frequency ω_{O1} is preferred in other research areas. Therefore, the first and second resonant frequency ω_{O1} and ω_{O2} are mainly analyzed in this paper.

IV. INPUT IMPEDANCE ANALYSIS

With the known input voltage $V_{AB}^F(t)$, the input impedance Z_{IN} provides the information of input current $I_{IN}(t)$, which determines the operation of inverter from the phase difference between voltage and current at the input. The input impedance Z_{IN} of the SS-WPT system in Fig. 2(c) can be obtained as:

$$Z_{IN} = jX_1 + jX_m \parallel (jX_2 + R_{eq}) = jX_1 + \frac{jX_m(jX_1 + R_{eq})}{jX_m + jX_1 + R_{eq}} \quad (12)$$

The input impedance Z_{IN} is investigated at the two major resonant frequencies ω_{O1} and ω_{O2} to analyze the SS-WPT system at the resonant frequencies.

At the first resonant frequency ω_{O1} , the reactance X_l and the reactance X_m can be written as

$$X_1 = \omega L_{lkp} - \frac{1}{\omega C_1} = \omega L_P \left(1 - \frac{\omega_{O1}^2}{\omega^2} - \kappa\right) = -\omega_{O1} \kappa L_P \quad (13)$$

$$X_m = \omega_{O1} \kappa L_P \quad (14)$$

At the first resonant frequency ω_{O1} , the input impedance Z_{IN} and the phase difference θ_{O1} can be written by

$$\begin{aligned} Z_{IN}(\omega=\omega_{O1}) &= -j\omega_{O1} \kappa L_P + \frac{j\omega_{O1} \kappa L_P (-j\omega_{O1} \kappa L_P + R_{eq})}{R_{eq}} \\ &= \frac{\omega_{O1}^2 \kappa^2 L_P^2}{R_{eq}} = \frac{\omega_{O1}^2 L_m^2}{R_{eq}} = \kappa^2 Q_L^2 R_{eq} \end{aligned} \quad (15)$$

$$\theta_{O1} = \tan^{-1}(0) = 0 \quad (16)$$

$$Q_L = \frac{\omega_{O1} L_P}{R_{eq}} \quad (17)$$

where Q_L is load quality factor

Since the input impedance Z_{IN} contains no imaginary term in (15), the input impedance is resistive and the equivalent circuit at the first resonant frequency ω_{O1} consists of only a resistor, as shown in Fig. 3(a). Thus, the input current $I_{IN}(t)$ is in phase with the input voltage $V_{AB}^F(t)$, as shown in Fig. 3(a). As a result, all switches in the inverter are turned on and off at zero current. Also, the equivalent load resistance R_{eq} is in inverse proportion to the input impedance Z_{IN} . Thus, the input current $I_{IN}(t)$ increases at the lighter load due to the low input impedance. That is, SS-WPT system has poor output voltage regulation at the first resonant frequency ω_{O1} .

At the second resonant frequency ω_{O2} , the reactance X_l and X_m can also be written as

$$X_1 = \omega L_{lkp} - \frac{1}{\omega C_1} = \omega L_P \left(1 - \frac{\omega_{O2}^2}{\omega^2}\right) = 0 \quad (18)$$

$$X_m = \omega_{O2} \kappa L_P \quad (19)$$

The input impedance Z_{IN} and the phase difference θ_{O2} at the second resonant frequency ω_{O2} can be written by

$$Z_{IN}(\omega=\omega_{O2}) = \frac{j\omega_{O2} \kappa L_P R_{eq}}{j\omega_{O2} \kappa L_P + R_{eq}} = \frac{j\omega_{O2} L_m R_{eq}}{j\omega_{O2} L_m + R_{eq}} = sL_m \parallel R_{eq} \quad (20)$$

$$\theta_{O2} = \tan^{-1}\left(\frac{R_{eq}}{\omega_{O2} \kappa L_P}\right) = \tan^{-1}\left(\frac{R_{eq}}{\omega_{O2} L_m}\right) \quad (21)$$

The equivalent circuit at the second resonant frequency ω_{O2} consists of the equivalent load resistance R_{eq} and the magnetizing inductance L_m from (20), as shown in Fig. 3(b). As a result, the input impedance is inductive and the input current $I_{IN}(t)$ lags the input voltage $V_{AB}^F(t)$ by the phase difference θ_{O2} in (21), as shown in Fig. 3(b). Thus, all switches in the inverter can be turned on with zero-voltage-switching (ZVS). Also, unlike the input impedance at the first resonant frequencies ω_{O1} , the input current $I_{IN}(t)$ decreases at the lighter load.

Note that the input impedances Z_{IN} at the second resonant frequencies ω_{O2} in (20) are about the same with the equivalent load resistance R_{eq} and the phase differences θ_{O2} in (21) become almost zero when the terms $\omega_{O2} \kappa L_P$ is much more than the equivalent load resistance R_{eq} (the load quality factor Q_L is very high).

$$Z_{IN}(\omega=\omega_{O2}) \approx R_{eq} \quad (22)$$

$$\theta_{O2} \approx 0 \quad (23)$$

V. FHA-BASED FREQUENCY ANALYSIS

The FHA-based frequency analysis is carried out to help more understand the operation of the SS-WPT system graphically. The FHA-based frequency analysis consists of voltage gain G_V , current gain G_I , and power gain G_P .

Fig. 4 Equivalent circuit for fundamental harmonic approximation-based frequency analysis

A. Voltage Gain G_V

From Fig. 4, the voltage gain G_V is as follows:

$$G_V = \left\{ \left[\frac{1}{\kappa} \left(1 - \frac{1}{\omega_n^2} \right) \right]^2 + \left[Q_L \omega_n \left(\frac{1}{\kappa} \left(1 - \frac{1}{\omega_n^2} \right) - \kappa \right) \right]^2 \right\}^{-0.5} \quad (24)$$

where the normalized frequency ω_n is defined by

$$\omega_n = \omega / \omega_{O1}$$

The voltage gains G_V at the first and second resonant frequency ω_{O1} and ω_{O2} can be obtained as:

$$G_V(\omega = \omega_{O1}) = \left| \frac{R_{eq}}{-j\kappa\omega_{O1}L_P} \right| = \frac{R_{eq}}{\omega_{O1}L_m} = \frac{R_{eq}}{\kappa Z_{O1}} = \frac{1}{\kappa Q_L} \quad (24)$$

$$G_V(\omega = \omega_{O2}) = \left| \frac{j\kappa\omega_{O2}L_P R_{eq}}{j\kappa\omega_{O2}L_P R_{eq}} \right| = \left| \frac{j\omega_{O2}L_m R_{eq}}{j\omega_{O2}L_m R_{eq}} \right| = 1 \quad (25)$$

Fig. 5 shows the voltage gain G_V of the SS-WPT system according to three key parameters of loaded quality factor Q_L : first resonant frequency f_{O1} , self-inductance L_P , and load resistance R_L . The voltage gain G_V at the first resonant frequency ω_{O1} decreases as the load quality factor Q_L increases, as shown in Fig. 5. Since the equivalent load resistance R_{eq} is in inverse proportion to the input impedance Z_{IN} , SS-WPT system has very poor output voltage regulation at the first resonant frequency ω_{O1} . On the other hand, as long as the inverter operates at the second resonant frequencies ω_{O2} (or at the third resonant frequencies ω_{O3}), the voltage gain G_V is always one since the reactance X_L is zero at the second resonant frequencies ω_{O2} .

When the voltage gain G_V at the first resonant frequency is much higher than one due to the low load quality factor Q_L , the voltage gain G_V has a single peak. And then, as the load quality factor Q_L increases, the voltage gain G_V decreases and becomes almost one. At that time, a single peak is being divided into a double peak. At the moment it is called critical coupling in the view point of Coupled Mode Theory. After that, as the voltage gain G_V at the first resonant frequency ω_{O1} becomes much lower, the distance between two peaks increases.

Fig. 6 shows voltage gain G_V of SS-WPT system according to coupling coefficient κ . The first resonant frequency ω_{O1} is fixed and the voltage gain G_V at the first resonant frequency ω_{O1} decreases, as the coupling coefficient κ increases. On the other hand, the second resonant frequencies ω_{O2} is variable although the voltage gain G_V at the second resonant frequencies ω_{O2} is always one.

B. Current Gain G_I and Power Gain G_P

In a similar way to the voltage gain G_V , the current gain G_I is as follows:

Fig. 5 Voltage gain G_V according to loaded quality factor (a) first resonant frequency f_{O1} (b) self-inductance L_P (c) load resistance R_L ($f_{O1} = 50\text{kHz}$, $n = 1:1$)

Fig. 6 Voltage gain G_V according to coupling coefficient κ ($R_L = 15\Omega$, $L_P = 100\mu H$, $f_{O1} = 50\text{kHz}$, $n:1 = 1:1$)

$$G_I = \left\{ \left[\frac{1}{\kappa} \left(1 - \frac{1}{\omega_n^2} \right) \right]^2 + \left[\frac{1}{\kappa Q_L \omega_n} \right]^2 \right\}^{-0.5} \quad (26)$$

The current gains G_I at the first and second resonant frequency ω_{O1} and ω_{O2} can be obtained as:

$$G_{I(\omega=\omega_{O1})} = \left| \frac{j\kappa\omega_{O1}L_P}{R_{eq}} \right| = \kappa Q_L \quad (27)$$

$$G_{I(\omega=\omega_{O2})} = \left| \frac{j\kappa\omega_{O2}L_P}{R_{eq} + j\kappa\omega_{O2}L_P} \right| = \left| \frac{j\omega_{O2}L_m}{R_{eq} + j\omega_{O2}L_m} \right| \quad (28)$$

From the above-mentioned the voltage gain G_V and the current gain G_I , the power gain G_P can be obtained as:

$$G_P = G_V G_I \quad (29)$$

The power gains G_P at the first and second resonant frequency ω_{O1} and ω_{O2} can be obtained as:

$$G_{P(\omega=\omega_{O1})} = \frac{1}{\kappa Q_L} \kappa Q_L = 1 \quad (30)$$

$$G_{P(\omega=\omega_{O2})} = \left| \frac{j\kappa\omega_{O2}L_P}{R_{eq} + j\kappa\omega_{O2}L_P} \right| = \left| \frac{j\frac{\kappa\omega_{O2}L_P}{R_{eq}}}{1 + j\frac{\kappa\omega_{O2}L_P}{R_{eq}}} \right| = \left| \frac{j\frac{\kappa}{\sqrt{1-\kappa}}Q_L}{1 + j\frac{\kappa}{\sqrt{1-\kappa}}Q_L} \right| \quad (31)$$

Fig. 7 shows the characteristics of the power gain G_P versus frequency according to the load quality factor Q_L : first resonant frequency f_{O1} , self-inductance L_P , and load resistance R_L . As long as the inverter operates at the first resonant frequency ω_{O1} , the power gain G_P keeps unity. On the other hand, the power gains G_P at the second resonant frequency ω_{O2} is less than one. However, when load quality factor Q_L is very high, the power gains G_P at the second resonant frequency ω_{O2} increases to almost unity.

At the very low load quality factor Q_L , the power gain has a single sharp peak. And then, as the load quality factor Q_L increases, the single sharp peak is changed into a single blunt peak. After that, a single peak is divided into a trident peak as the load quality factor Q_L becomes much higher. At the very high load quality factor Q_L , the trident peak becomes much sharper.

Also, Fig. 9 shows the characteristics of the power gain G_P versus frequency according to the coupling coefficient κ .

Fig. 7 Power gain G_P according to loaded quality factor (a) first resonant frequency f_{O1} (b) self-inductance L_P (c) load resistance R_L ($f_{O1} = 50\text{kHz}$, $n:1 = 1:1$)

Fig. 8 Power gain G_P according to coupling coefficient κ ($R_L = 15\Omega$, $L_P = 100\mu\text{H}$, $f_{O1} = 50\text{kHz}$, $n:1 = 1:1$)

Fig. 9 Resonator (size: 20cm x 20cm, separated distance: 8cm and equivalent AWG: 14)

Fig. 10 Experimental waveforms (a) at the first resonant frequency ω_{O1} (b) at the second resonant frequency ω_{O2} (Time: $5\mu\text{s/div}$, $V_{IN} = 50\text{V}$ and $R_L = 15.5\Omega$)

Fig. 11 Output voltage according to load resistance R_L at the first resonant frequency ω_{O1} and at the second resonant frequency ω_{O2} ($V_{IN} = 20\text{V}$)

Fig. 12 Experimental waveform at the same output power (a) at the first resonant frequency ω_{O1} (b) at the second resonant frequency ω_{O2} (Time: $5\mu\text{s/div}$ and $V_{IN} = 20\text{V}$)

Regardless of the variation of the coupling coefficient κ , the power gain G_P keeps unity. Theoretically, at the first resonant frequency ω_{O1} , the input power P_{IN} is totally transferred to the output since the input impedance Z_{IN} contains no imaginary term and the impedance matching is perfectly accomplished at the resonant frequency ω_{O1} . Therefore, low coupling coefficient κ by long separated distance does not matter at the first resonant frequency ω_{O1} .

On the other hand, at the second resonant frequency ω_{O2} , the power gain G_P is less than one from (31) when the load quality factor Q_L and the coupling coefficient κ are low. That is, a part of input power P_{IN} is transferred to the output because of the low current gain G_I . However, when the load quality factor Q_L and the coupling coefficient κ are very high, the low current gain G_I and the power gain G_P is almost one at the second resonant frequency ω_{O2} . In other words, when the magnetic coupling is loose and the coupling coefficient κ is low, the very high load quality factor Q_L is required for high efficiency. Thus, when the load quality factor Q_L is not high, the separated distance is limited at the second resonant frequency ω_{O2} .

VI. EXPERIMENTAL VERIFICATION

To support the analysis, experimental results are performed with specifications of the self-inductance $L_P = L_S = 55.9\mu\text{H}$ (Fig. 9), the resonant capacitor $C_1 = C_2 = 15\text{nF}$, the separation distance 8cm, and coupling coefficient $\kappa = 0.19$.

Fig. 10 shows the phase difference between the voltage and current at the input. Since the input impedance at the first resonant frequency ω_{O1} is resistive, the voltage waveform and current waveform are in phase, as shown in Fig. 10(a). On the other hand, since the input impedance at the second resonant frequency ω_{O2} is inductive, the current waveform lags the voltage waveform, as shown in Fig. 10(b). At that time, all the MOSFET are turned on with ZVS.

Fig. 11 shows the output voltage V_O according to load resistance R_L at the first resonant frequency ω_{O1} and at the second resonant frequency ω_{O2} . At the first resonant frequency ω_{O1} , the input impedance Z_{IN} is in reverse proportion to the equivalent load resistance R_{eq} . For this reason, as the load resistance R_L increases, the input impedance Z_{IN} decreases. And then, since the input current and input power increase, the output voltage V_O also increases. On the other hand, since the output voltage at the second resonant frequency ω_{O2} is insensitive to load resistance R_L , the output voltage V_O is almost constant.

Also, when the coupling coefficient κ are very low, the power gain G_P at the first resonant frequency ω_{O1} is higher than that at the second resonant frequency ω_{O2} . Therefore, the higher input current (primary current) at the second resonant frequency ω_{O2} is required to transfer the same output voltage to a load, as shown in Fig. 12.

VII. CONCLUSION

To understand WPT system more easily, this paper presents the features of the SS-WPT system in two major resonant frequencies: one resonant frequency by self-inductance and resonant capacitor and the other resonant frequency by leakage inductance and resonant capacitor.

From the input impedance analysis, this paper shows that the phase difference at two resonant frequencies determines the operation of the inverter. Also, from the voltage gain G_V in the FHA-based frequency analysis, this paper describes that the operation at the second resonant frequencies ω_{O2} is suitable for a wide range of load variation due to the characteristic of good load regulation. Furthermore, from the power gain G_P in the FHA-based frequency analysis, this paper explains that the operation at the first resonant frequency ω_{O1} is preferred for transferring power in large separation distance.

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