The service curve service discipline for the rate-controlled EDF service discipline in variable-sized packet networks

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Abstract

Guaranteed service will provide high quality services to real-time applications, e.g., audio or video, over packet networks such as the Internet. To support guaranteed service, a service discipline must guarantee a delay bound to each session. In addition, a preferred service discipline should achieve high network utilization and good scalability. The service disciplines studied so far have problems in achieving these two objectives at the same time. Generalized processor sharing (GPS) service disciplines can have low network utilization. Rate-controlled (RC) service disciplines have difficulty in scalability because of regulators. For service curve (SC) service disciplines, both the network utilization and the scalability depend on the adopted SC. To date, there have been no studies on an SC which can make an SC discipline achieve these two objectives. We propose a new service discipline based on SC service disciplines. The proposed discipline achieves these two goals in a variable-sized packet environment. We show that the discipline can achieve the network utilization achievable by the RC service disciplines. We further show that our SC requires O(1) complexity for deadline calculation. Different from the RC service disciplines, the SC service discipline with our SC does not need regulators at all. Thus, it has better scalability than the RC service disciplines and is work-conserving. We also show that the proposed SC makes SC service disciplines have strictly higher network utilization than the GPS service disciplines including the multi-rate service discipline.

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1. Introduction

Guaranteed service will provide high quality services to real-time applications such as audio/video transmissions over packet networks such as the Internet [19]. To support guaranteed service, each router employs a service discipline. Here, a service discipline refers to an aggregation of a packet scheduler and other required elements to provide a guaranteed service. Such a service discipline should guarantee a delay bound as well as bandwidth to each session at all the output ports. An important aspect of a preferred service discipline is the level of network utilization it can achieve. In addition, a good service discipline should be scalable to support a high number of concurrent sessions.

Service disciplines studied so far have problems with either network utilization or scalability. Generally, a service discipline can achieve high network utilization when it can provide different delay bounds to a given traffic specification. (Here, network utilization indicates how many real-time sessions can be admitted.) A service discipline has good scalability if its implementation cost and scheduling complexity are not much dependent on the number of sessions. There have been three major trends for service disciplines that can support guaranteed service in the literature: generalized processor sharing (GPS) service disciplines.
disciplines [13,14,1,2,9,20,17], rate-controlled (RC) service disciplines [25,5,8], and service curve (SC) service disciplines [4,21,18]. (A more detailed survey is given in Section 1.1.) In the GPS service disciplines, delay bounds are decided by specified traffic rates. Thus, achievable network utilization can be very low [8]. Among the RC service disciplines, the RC-EDF service discipline achieves the highest network utilization. However, it has problems in scalability because regulators are required for each session [24,8]. In the case of the SC service disciplines, network utilization and scalability depend on the adopted service curve. In the literature, so far, there have been no studies on an SC that can make an SC service discipline achieve both high network utilization and good scalability.

In this paper, we propose a new service discipline that can achieve both high network utilization and scalability. The proposed discipline can be used in a variable-sized packet network environment, such as the Internet. It is a special case of an SC service discipline [18]. Devising an SC which can achieve both scalability and high network utilization is not easy. We can achieve high scalability using a simple (e.g., linear) service curve. However, this approach usually results in very low network utilization. On the contrary, if we adopt a complex service curve to achieve high network utilization, scalability becomes a problem since it requires high complexity in deadline calculation. We propose a new SC, called SC-EDF\(^1\) that allows the service discipline to achieve both objectives.

An SC-EDF for a session is constructed from the output function of the regulator of the session and the delay bound at the packet scheduler of the RC-EDF discipline. We show that SC-EDF can make the SC service discipline achieve the network utilization which the RC-EDF discipline can. We further show that SC-EDF becomes piecewise concave linear in a range of our interest. Therefore, the process for deadline calculation can be simplified requiring only O(1) complexity. Moreover, different from the RC-EDF service discipline, regulators are not required. Therefore, scalability is significantly improved.

Another advantage of the proposed scheme over the RC-EDF service discipline is that it is work-conserving. A service discipline is said to be non-work-conserving if some packets cannot be transmitted to the link although packets are in the service discipline. Generally, a non-work-conserving service discipline makes the average delay of packets larger than that in the work-conserving service disciplines. Regulators make the RC service disciplines non-work-conserving.

Most service curve disciplines such as SCED (service curve based earliest deadline first policy) are developed for fixed-sized packet networks such as ATM networks, not for variable-sized networks. These disciplines use a discrete time model and assume that packets arrive synchronously in a slot time. They update data structures every packet arrival time. However, in variable-sized networks, the number of packet arrival times can be extraordinarily large even within a short interval due to the bursty traffic nature. Although the authors in [3] have studied how to allocate a service curve in SCED under a variable-sized environment, they do not address the issue of heavy implementation cost imposed by asynchronous packet arrivals. On the contrary, the proposed service discipline updates some information whenever each session becomes newly backlogged, not every packet arrival time. Note that the number of backlogged times is significantly smaller than that of packet arrival times.

Lastly, compared to the GPS service disciplines including the multi-rate service discipline [17], the proposed discipline can achieve strictly higher network utilization.

1. Related work

1.1. GPS disciplines

The GPS service disciplines such as GPS [13,14], WF\(^2\)Q [2], H-PFQ [1], SCFQ [9], and rate-proportional servers [20] are rate-based service disciplines. These disciplines are different from one another in terms of fairness. They provide the same delay bound to the same traffic rate, and hence, achievable network utilization can be very low. It is theoretically possible to have different delay bounds to the same traffic rate if the admission test for each session considers all the sessions in all the routers [14]. However, such cases are not practical since they utilize the information of all sessions in all routers for high network utilization [14,21].

A traffic rate can be specified either by a constant rate over all intervals or by different rates over different intervals. The traffic rate specified by a constant rate is called a single-rate and that specified by multiple rates is called a multi-rate. The multi-rate service discipline is a GPS service discipline that guarantees a multi-rate to each session [17]. In the context of scheduling methodology, the multi-rate service discipline may look somewhat similar to service curve service disciplines. (We go through a detailed comparison with SC-EDF in Section 6.) A difference is that the multi-rate service discipline do not consider delay bounds for sessions when next service packet is selected. Due to this indifference on delays, the delay bounds to the same multi-rate are identical. Thus, achievable network utilization can still be low although higher network utilization than the single-rate GPS service disciplines can be achieved. In all GPS disciplines, including the multi-rate discipline, scalability is dependent on the scheduling complexity, which is O(\(\text{log}N\)) to maintain a sorted priority queue where \(N\) is the number of sessions. Also, the disciplines are work-conserving.

Compared to SC-EDF, all GPS service disciplines have similar scalability and lower network utilization.
1.1.2. RC disciplines

An RC service discipline [25,24,5,8] consists of regulators and a packet scheduler. It can adopt a different packet scheduler such as a static-priority (SP) scheduler or an earliest deadline first (EDF) scheduler. Network utilization and scalability are affected by the scheduler. The RC service discipline adopting the SP/EDF scheduler is called an RC-SP/RC-EDF service discipline.

Consider the case of the RC-EDF service discipline [5,8], One regulator is required for each session. Each regulator controls the output rate of the corresponding session and stamps a deadline on each incoming packet. The deadline becomes the departure time plus the associated delay bound at the EDF scheduler. The EDF scheduler transmits packets in an increasing order of the deadlines. The RC-EDF service discipline can achieve higher network utilization than GPS disciplines since, it can provide different delay bounds to the same traffic rate [8]. However, the RC-EDF service discipline has problems in scalability mainly due to the cost of implementing regulators. (It requires N regulators, one for each session. Implementing a regulator can require significant overhead for buffer space and a timer that can be expensive.) In addition, the scheduler requires O(\log N) scheduling complexity. In [24], a method using a calendar queue has been proposed to avoid implementing separate regulators for each session. However, although a calendar queue is used, more than N packets may have to be moved from the calendar queue to the EDF scheduler within a short duration. Note that the proposed service curve service discipline requires no regulators at all since it computes deadlines for packets smartly; both the traffic characterization function and the delay bound are considered together to compute deadlines.

In the case of the RC-SP discipline, the SP scheduler maintains a certain number of priority groups in advance. A session is assigned to a priority group during admission control. The SP scheduler transmits a packet in the highest priority group that has packets in the queue. Compared to RC-EDF, RC-SP has advantage in scalability if a calendar queue method is used [25]. Since, the number of priority groups is usually a small constant, the movements between the calendar queue and the SP scheduler require O(1) complexity. However, the RC-SP service discipline can suffer from low network utilization [12]. Both RC-EDF and RC-SP are non-work-conserving due to the regulators in the disciplines.

Compared to the proposed SC-EDF, RC-SP has better scalability and lower network utilization and RC-EDF has worse scalability and the same level of network utilization.

1.1.3. SC disciplines

The SC service disciplines such as SCED (service curve based earliest deadline first policy) [4,18] or H-FSC (hierarchical fair service curve) [21] are deadline-based disciplines and guarantee an SC for each session. Intuitively, a service curve indicates the minimum service amounts during (different) intervals. In the SC disciplines, different level of network utilization can be achieved, depending on the selected SC’s for sessions. If the SC’s are constructed by considering only traffic rates, the SC service disciplines can achieve the same level of network utilization as the GPS. If the SC’s are constructed by considering both delay bounds and traffic rates, the SC disciplines can achieve the same level of network utilization as the RC-EDF. When an SC of an arbitrary shape is assumed, the SC disciplines can achieve the highest network utilization ever known. The level of scalability is also dependent on the selection of an SC. An SC of an arbitrary shape makes the deadline calculation more complex. Apart from the deadline calculation, the disciplines also require O(\log N) for scheduling. The SC service disciplines are work-conserving. Most research on SC service disciplines has been performed in the context of a restricted, fixed-sized environment that is applicable to ATM networks.

A general framework for SC service disciplines was proposed in [18,3]. Our work differs from the work in [18] in the followings. First, our work considers a variable-sized packet environment whereas the work in [18] considered a fixed-sized packet environment. Thus, the results of our work are applicable to packet networks such as the Internet. Second, we focus on an SC-EDF whereas the work in [18] studied the SC service disciplines in a general fashion. We give a solid proof that the SC-EDF can make the SC service disciplines to achieve the network utilization achievable by the RC-EDF service discipline. We also prove that the SC-EDF requires little overhead (O(1)) for deadline calculation. The work in [3] extends the algorithm proposed in [18] to cover the variable-sized packet environment. However, as mentioned in Section 2.2, the extended algorithm can have the implementation cost problem, and the work in [3] does not touch how to resolve this.

The works in [21,16] studied a hierarchical link-sharing and priority service in the context of an SC service disciplines in a variable-sized packet environment. Although the focus of the work was different from ours, we base our work on it because of the common interest in the packet environment.

This paper is organized as follows. In Section 2, we present the background helpful to understand the service curve service discipline. In Section 3, we present a revised version of the service curve service discipline presented in [15]. In Section 4, we focus on the SC-EDF and discuss network utilization. In Section 5, we show that SC-EDF results in a constant time for computing each deadline. In Section 6, we compare the SC service discipline using SC-EDF with the multi-rate service discipline. Lastly, Section 7 concludes this paper.

2. Background

We review the definition of a service curve presented in [15] and some analysis results applicable to such service curves. Also, the network modeling and traffic characterizations are explained.
A network is modeled by a series of routers (or servers). The transmission links are ignored for the convenience of discussion. We denote the input amount from session $i$ at a router during the time interval $(s,t]$ by $R_{bi}^i(s,t)$ and the output amount by $R_{bi}^o(s,t)$. We define $R_{bi}^i(s,t) = R_{bi}^o(s,t) = 0$ if $s > t$. For notational convenience, we denote $R_{bi}^i(0,t)$ and $R_{bi}^o(0,t)$ by $r_{bi}^i(t)$ and $r_{bi}^o(t)$, respectively. It is said that session $i$ is in a backlogged period in a router if there exist any packets to serve from session $i$.

A traffic specification is expressed by a traffic rate function $b_i(t)$. Intuitively, $b_i(t)$ indicates the maximum amount of traffic that is allowed to be transmitted from session $i$ during a time interval of length $t$. Thus, $b_i(t)$ must be a non-decreasing function and we say that $b_i(u) = 0$ for all $u \leq 0$. We say that session $i$ is $b_i$-smooth or traffic envelope function of session $i$ is $b_i$ if the incoming traffic amount from session $i$ to the network during the interval $(s,t]$ is not greater than $b_i(t - s)$.

As a special case of $b_i(t)$, we say that session $i$ is $K(\sigma, \rho)$-smooth or session $i$'s traffic envelope function is $K(\sigma, \rho)$ if $b_i(t) = \min_{k=1, \ldots, K} \{ \sigma^k + \rho^k t \}$. Without loss of generality, we assume that $0 \leq \sigma^1 \leq \cdots \leq \sigma^K$ and $\rho^1 \geq \cdots \geq \rho^K \geq 0$. By using a $K(\sigma, \rho)$ as the traffic envelope function, we can accurately characterize the traffic and its variability of a VBR video [22,17]. Fig. 1 illustrates how the traffic is characterized when a session transmits an MPEG-coded video. An MPEG encoder produces three types of encoded frames: $I$ (intra-coded), $P$ (predicted), and $B$ (bidirectional). We can use the knowledge of the encoding pattern of frames, say $IBBPBB$, and the size of the largest frames of each type, i.e., $I$, $B$, and $P$ to characterize the video traffic. In the figure, the traffic is characterized by either a $(\sigma, \rho)$ function or a $3(\sigma, \rho)$ function. Using a $3(\sigma, \rho)$ function, the traffic can be characterized more accurately than using a $(\sigma, \rho)$ function.

In particular, two specializations of $K(\sigma, \rho)$, $(\sigma, \rho)$ and $\min\{\sigma + \rho t, P\}$, are adopted by the IETF Int-Serv working group [19,23]. $\sigma$ represents the burstiness allowed to session $i$. $\rho_i$ is the average traffic rate. $P_i$ is the peak rate allowed to session $i$.

### 2.2. Service curves and some analysis results

Let $S(t)$ be a non-decreasing function with $S(u) = 0$ for all $u \leq 0$. We say that a service curve $S(t)$ is guaranteed for session $i$ by a router if, for a packet departure time $t$ of session $i$, there exists a time $s, s < t$, such that $s$ is the beginning of one of the session’s backlogged periods and $r_i^o(t) \geq r_i^o(s) + S(t - s)$. (1)

Intuitively, $S(t)$ indicates the minimum amount of service, i.e., minimum amount of data transmitted, by the router for session $i$ during an interval $t$ starting from the beginning of a session $i$’s backlogged period. Eq. (1) can be equivalently expressed as follows:

$$r_i^o(t) \geq \min_{s \in B(t)} \{ r_i^o(s) + S(t - s) \}.$$  (2)

where $B(t)$ is the set of the start time points of the session $i$’s backlogged periods that is not greater than the packet departure time $t$.

Now, consider multiple routers which a session $i$ passes through. Given service curves which are guaranteed in each of those routers, we can derive a single service curve which is guaranteed by those multiple routers as a whole. In other words, the multiple routers can be reduced to a single composite router (or a network) which guarantees the derived service curve. The derivation is presented in Theorem 1. Intuitively, the theorem tells that network service curve $S_{net}(\epsilon)$ is constructed by concatenating, in an increasing order of slopes, the line segments of the guaranteed service curves. (Theorems similar to the following three can be found in [18]. As mentioned earlier, they deal with different packet environments, namely, a fixed-sized vs. variable-sized packet environment.)

2 Our definition is the same as in [21]. (In [21], the definition is expressed by $W(s,t) \geq S(t - s)$. Since, at the time $s$, the queue for session $i$ is empty, both expressions becomes identical if $W(s)$ is substituted for $A(s)$ in Eq. (1).) However, our definition is different from that in [18], which is applicable only to a fixed-sized packet environment. In [18], it is said that a service curve $S(t)$ is guaranteed for session $i$ by a router if, for any time $t$, there exists a time $s, s \leq t$, that satisfies Eq. (1). Although both definitions use the same equation, the respective scheduling service disciplines have different implementation implications due to different conditions for the variables $s$ and $t$. The scheduling service discipline which guarantees service curves using the definition in [18] should check Eq. (1) whenever a packet arrives. In a variable-sized packet environment such as the Internet, the number of packet arrival times can be extraordinarily large even within a short interval. (A packet may arrive at any time since we cannot put any restriction on packet arrival times.) On the contrary, the scheduling service discipline using our definition has only to check the equation whenever each session becomes newly backlogged, not every packet arrival time. Note that the number of backlogged times is significantly smaller than that of packet arrival times.
Theorem 1 (Network service curve theorem). Suppose that session $i$ passes through $M$ routers in tandem and the $m$-th router, $1 \leq m \leq M$, strictly guarantees $S^i_m(\cdot)$ to session $i$. Then, the $M$ routers guarantee $S^i_{net}(\cdot)$ to session $i$ where

$$S^i_{net}(t) = \min \left\{ \sum_{m=1}^{M} S^i_m(A_m) : A_m > 0 \text{ and } \sum_{m=1}^{M} A_m = t \right\}.$$  

(3)

Proof. See the appendix. □

Fig. 2 illustrates the derivation when two routers are given. In the figure, $S^i_1(\cdot)$ and $S^i_2(\cdot)$ are the strictly guaranteed service curves at the first and second routers, respectively. $S^i_1(\cdot)$ has slope $s_1$, that is less than $s_2$, during the interval $(0, t_1]$. $S^i_2(\cdot)$ has slope zero until time $t_2$. The two curves have the same slope $s_2$ from the times $t_1$ and $t_2$, respectively. By Theorem 1, the network service curve $S^i_{net}(\cdot)$ has slope zero until the time $t_2$, slope $s_1$ until the time $(t_1 + t_2)$, and slope $s_2$ afterwards.

When a router guarantees a service curve for session $i$, the delay and backlog bound for session $i$ can be derived.

Theorem 2 (Delay bound theorem). Let $b_i(\cdot)$ be the traffic envelope function of session $i$. When a router guarantees $S^i(\cdot)$ to session $i$, the delay at the router is not greater than

$$\max_{k \geq 0} \min \{A : A > 0 \text{ and } b_i(k) \leq S^i(k + A)\}.$$  

(4)

Proof. See the appendix. □

Theorem 3 (Backlog bound theorem). Let $b_i(\cdot)$ the traffic envelope function of session $i$. If a router guarantees $S^i(\cdot)$ to session $i$, at any moment, the backlog is not greater than

$$\max_{k \geq 0} \{b_i(k) - S^i(k)\} + l_{max}.$$  

(5)

where $l_{max}$ is the maximum packet size.

Proof. See the appendix. □

Fig. 3 graphically illustrates the delay and backlog bound. In the figure, $b_i(t)$ is the maximum amount of traffic from session $i$ for interval $(0, t]$ and $S_i(t)$, the minimum service amount. It can be easily understood that the maximum horizontal distance from $b_i(t)$ to $S_i(t)$, denoted by $D(b_i[S_i])$, becomes the delay bound as described in Theorem 2. Similarly, the backlog bound described in Theorem 3 comes from the maximum vertical distance between $b_i(t)$ and $S_i(t)$. In the figure, the vertical distance becomes the maximum at time $t_0$. In the theorem, the backlog bound includes the term $l_{max}$ since each packet is cleared in the backlog after the last bit of the packet is served.

3. A service curve service discipline

It is shown in Section 2.2 that the delay and backlog bound at a router can be derived if the router guarantees a service curve for a session. This section presents how to guarantee a service curve for a session in a router. We propose a deadline-based service discipline applicable to the variable-sized packet network, called an SC (service curve) service discipline to guarantee a service curve. In fact, the proposed SC service discipline is a revised version of [15] that computes exact deadline values. (A detailed discussion on such modification is deferred later.)

The SC service discipline allocates a service curve to each session for guaranteeing a service curve. Each incoming packet from a session is stamped a deadline from the allocated service curve. The SC service discipline transmits a packet with the smallest deadline non-preemptively when the link is idle. Ties are broken arbitrarily.

Throughout this section, we denote the $k$-th packet from session $i$ by $p^i_k$. The arrival time, the departure time at the router, and the length of the packet are denoted by $a^i_k$, $d^i_k$, and $l^i_k$, respectively. The start time of the $m$-th backlogged period of session $i$ is denoted by $b^i_m$.

Let's see how to stamp deadlines in the SC service discipline. A deadline is assigned to each packet in such a way that if all the packets are transmitted until their deadlines, the allocated service curve is guaranteed for each session. Suppose that a packet $p^i_k$ arrives during the $m$-th backlogged period of session $i$. Let $S_i(\cdot)$ be the allocated service
curve to the session and $D_{i}^{m,k}$ the deadline of the packet. The SC service discipline allocates the deadline $D_{i}^{m,k}$ as follows:

$$D_{i}^{m,k} = \min \left\{ d : \min_{s \in B_i(t)} \left\{ r_i^m(s) + S_i(d-s) \right\} \geq \sum_{j=1}^{k} l_j^i \right\}. \quad (6)$$

Note that $B_i(d_i^k) = B_i(b_m^i)$ since there exists no new backlogged period of session $i$ during the interval $[b_m^i, d_i^k]$. Thus, from Eq. (6), if $d_i^k \leq D_{i}^{m,k}$, then $\min_{s \in B_i(t)} \left\{ r_i^m(s) + S_i(d_i^k-s) \right\} \leq \sum_{j=1}^{k} l_j^i$. In this case, since $\sum_{j=1}^{k} l_j^i = r_i^{out}(d_i^k)$, $r_i^{out}(d_i^k) \geq \min_{s \in B_i(t)} \left\{ r_i^m(s) + S_i(d_i^k-s) \right\}$. Thus, if each packet is transmitted until its deadline, $S_i(t)$ is guaranteed for each session $i$ by Eq. (2).

Note that Eq. (6) uses “$\geq$” instead of “$=$” as in [21] or “$>$” as in [15]. If “$=$” is used, we have difficult in determining deadlines given step functions as service curves. We explain the reason by an example. Suppose that a service curve $S_i(t)$ shown in Fig. 4 is allocated for a session $i$. The function $S_i(t)$ has zero value before the time $d_i$ and, at the $d_i$ time, the function jumps to $r_i$, that is, such a value is greater than zero. Consider the first packet $p_i^1$ from the session $i$. Let $l_1^i < r_i$. In this case, Eq. (6) becomes $\min \left\{ d : S_i(d-a_i^1) \geq l_1^i \right\}$. Thus, the deadline of $p_i^1$ becomes $a_i^1 + d_i$. However, if “$=$” is used in Eq. (6), we cannot express the exact deadline value due to the discontinuity of $S_i(t)$ at the time $d_i$ although such a value exists in the context of mathematics. A similar problem occurs if “$>$” is used in Eq. (6).

For the computation of the deadlines in Eq. (6), the SC service discipline keeps a deadline curve $D_i(t)$ for each session $i$, which is defined as follows:

$$D_i(t) = \min \left\{ r_i^m(s) + S_i(t-s) \right\}. \quad (7)$$

$D_i(t)$ is initialized to $S_i(t)$ when session $i$ becomes backlogged for the first time. Subsequently, it is sufficient to update $D_i(t)$ only when session $i$ becomes newly backlogged. We denote the updated deadline curve at each backlogged point $b_m^i$ by $D_i(b_m^i)$. Then, $D_i(b_m^i)$ can be recursively computed as follows:

$$D_i(b_m^i) = \min \left\{ D_i(b_{m-1}^i), r_i^m(b_m^i) + S_i(t-b_m^i) \right\} \quad \text{for } t \geq b_m^i + d_i. \quad (8)$$

Note that if $D_i(b_m^i)$ in Eq. (8) is maintained for $t \geq b_m^i$ as in [21], the maintenance for this range can incur unnecessary complexity. This problem is discussed in Section 5 in detail. However, we show in the following theorem that only a part of a service curve is used to compute deadlines.

**Theorem 4.** Let a service curve $S_i(t)$ be allocated to a session $i$. Let $d_i$ be the first time point at which $S_i(t)$ has a non-zero value. Consider the packets from the session which arrive after the last backlogged time point $b_m^i$. Then, the deadlines of those packets are always greater than $b_m^i + d_i$.

**Proof of Theorem 4.** Consider a packet $p_i^k$ which arrives after the last backlogged time point $b_m^i$. The accumulated amount of data including the packet $p_i^k$ to be transmitted until the deadline $D_i^{m,k}$ becomes $\sum_{j=1}^{k} l_j^i$. Thus,

$$D_i^{m,k} = \min \left\{ t : D_i(b_m^i) \geq \sum_{j=1}^{k} l_j^i \right\} = \min \left\{ t : \min \left\{ D_i(b_{m-1}^i), r_i^m(b_m^i) + S_i(t-b_m^i) \right\} \geq \sum_{j=1}^{k} l_j^i \right\} \quad \text{max} \min \left\{ t : D_i(b_{m-1}^i) \geq \sum_{j=1}^{k} l_j^i \right\} \geq \min \left\{ t : S_i(t-b_m^i) > 0 \right\} \quad \text{since } \sum_{j=1}^{k} l_j^i > r_i^m(b_m^i) \geq b_m^i + d_i. \quad (9)$$

**Theorem 4** tells us that it is sufficient to maintain $D_i(b_m^i)$ only for $t \geq b_m^i + d_i$, where $d_i$ is the first time point at which $S_i(t)$ has a non-zero value. For instance, we have only to consider the linear function $\ell(t)$ for $t \geq d_i$ of the service curve $S_i(t)$ presented in Fig. 4, not the entire non-linear function.

Although we have mentioned that each packet needs to be transmitted until its deadline to guarantee an allocated service curve, some may be transmitted after their deadlines since packets are transmitted non-preemptively. However, the departure time of each packet is still bounded. **Theorem 5** tells us that if the sum of the allocated service curves does not exceed the capacity, the delay of each packet is bounded. Note that **Theorem 5** can also be used for admission control.

**Theorem 5.** Consider a router with the SC service discipline with a capacity $r$ that serves $N$ sessions. Let $S_i(t)$ be the allocated service curve for each session $i$. All the packets are served until their deadlines plus $p_{max}^i r$ if

$$\sum_{i=1}^{N} S_i(t) \leq r t. \quad (10)$$
where $l_{\text{max}}$ is the maximum packet size.

Proof. See the appendix.

Now we derive a guaranteed service curve when the SC service discipline assigns $S_i(\cdot)$ to each session $i$ and satisfy Eq. (10). Suppose that a packet $p_i$ arrives during the $m$-th backlogged period. From Theorem 5, the packet departs until the deadline plus $l_{\text{max}}/r$. That is,

$$d_i^k - l_{\text{max}}/r < D_i^k.$$  (11)

Observe that from Eq. (6), if a time $t$ is less than or equal to $D_i^k$, $\min_{\forall t \in [0, D_i^k]} \{r_i^m(s) + S_i(t - s)\} \leq \sum_{j=1}^k l_j^i$. From Eq. (11), $d_i^k - l_{\text{max}}/r$ is such a time that is not greater than $D_i^k$. Thus,

$$\min_{\forall t \in [0, D_i^k]} \{r_i^m(s) + S_i(d_i^k - l_{\text{max}}/r - s)\} \leq \sum_{j=1}^k l_j^i. \quad (12)$$

In addition, $r_i^\text{out}(d_i^k) = \sum_{j=1}^k l_j^i$. Therefore,

$$r_i^\text{out}(d_i^k) \geq \min_{\forall t \in [0, D_i^k]} \{r_i^m(s) + S_i(d_i^k - l_{\text{max}}/r - s)\}$$

$$= \min_{\forall t \in [0, D_i^k]} \{r_i^m(s) + S_i(d_i^k - s)\}, \quad (13)$$

where $S_i(\cdot)$ is equal to $S_i(\cdot)$ shifted right by $l_{\text{max}}/r$, i.e.,

$$S_i(t) = \begin{cases} 0, & t < l_{\text{max}}/r, \\ S_i(t - l_{\text{max}}/r), & \text{otherwise}. \end{cases} \quad (15)$$

From Eq. (14), we conclude that the SC service discipline guarantees $S_i(t)$ for session $i$ if the allocated service curve is $S_i(\cdot)$. Reversely, whenever session $i$ requires $S_i(\cdot)$ as a guaranteed service curve, the SC service discipline has only to assign $S_i(\cdot)$ shifted left by $l_{\text{max}}/r$.

4. SC-EDF

An SC-EDF is an SC with which an SC service discipline can achieve the same network utilization as the RC-EDF service disciplines. The SC-EDF for each session is constructed from an underlying RC-EDF service discipline. Suppose that, in the underlying RC-EDF service disciplines, the set is also admitted by the corresponding SC service disciplines with the SC-EDF. Then, we show that the end-to-end delay bound to each session is the same when a session passes through the network composed by either of the two cases.

Let us see the first step in detail. Since, each SC-EDF is constructed from a corresponding RC-EDF discipline, the comparison of admission test can be localized. If $N$ sessions are admitted by an underlying RC-EDF service discipline, the following must be satisfied by admission control [6]:

$$\sum_{i=1}^N b_i(t - d_i) + l_{\text{max}} \leq rt \quad \text{for} \quad t \geq \min_{i=1 \cdots N} \{d_i\}, \quad (18)$$

where $b_i(\cdot)$ is the regulator function of session $i$. We can reduce Eq. (18) as follows:

$$\sum_{i=1}^N b_i(t - d_i) \leq rt \quad \text{for} \quad t \geq \min_{i=1 \cdots N} \{d_i\}. \quad (19)$$

Let $S_i(\cdot)$ be the SC-EDF for session $i$. Since, $S_i(t) = b_i(t - d_i)$ for $t \geq d_i$ from Eq. (16), it follows that from Eq. (19),

$$\sum_{i=1}^N S_i(t) \leq rt \quad \text{for} \quad t \geq \min_{i=1 \cdots N} \{d_i\}. \quad (20)$$

Also, since $S_i(t) = 0$ for $t < d_i$,

$$\sum_{i=1}^N S_i(t) \leq rt \quad \text{for} \quad t \leq \min_{i=1 \cdots N} \{d_i\}. \quad (21)$$

From Eqs. (20) and (21), $\sum_{i=1}^N S_i(t) \leq rt$ for any interval $t$. Therefore, by Theorem 5, the $N$ sessions are also admitted by the SC service discipline with the SC-EDF's.

Now let us see the second phase. Consider the case that a session $i$ passes through multiple RC-EDF service disciplines as shown in Fig. 5. $b_i(\cdot)$ is the output function of the regulator. Since, there is no reason to use different output functions in the respective RC-EDF service disciplines [8], the same output function $b_i(\cdot)$ is used at all the stages.

<table>
<thead>
<tr>
<th>Source</th>
<th>$b_1$</th>
<th>$d_1^1$</th>
<th>$b_2$</th>
<th>$d_2^1$</th>
<th>...</th>
<th>$b_j$</th>
<th>$d_j^M$</th>
<th>Destination</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_i$ : output function of the regulator</td>
<td></td>
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<tr>
<td>$r_i^m$ : the $m$-th link capacity</td>
<td></td>
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<td></td>
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<tr>
<td>$d_i^m$ : associated delay bound at the $m$-th scheduler</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Fig. 5. Example – $M$ number of RC-EDF service disciplines in a sequence.
The scheduling complexity of the SC service discipline consists of two parts, computing deadlines for incoming packets and transmitting those packets. For packet transmission, a sorted-priority queue is maintained for deadlines. The maintenance cost for a sorted priority queue is \( O(\log N) \), where \( N \) is the number of sessions. This cost can be further reduced to \( O(1) \) with a special hardware such as a sequencer [10] or a systolic array [11]. Note that whatever service curves are used, this cost for packet transmission remains unchanged.

However, computing deadlines has different complexities depending on allocated service curves. To compute deadlines, each session maintains a deadline curve, which depend on the allocated service curve. The deadline of a packet is calculated from the inverse of the deadline curve.

We first try to give an intuition on how each deadline curve is updated by a simple example. Suppose that the following SC-EDF \( S_i(t) \) is allocated for a \((\sigma, \rho)\)-smooth session \( i \) with local delay requirement \( d_i \):

\[
S_i(t) = \begin{cases} 
0, & 0 \leq t < d_i \\
\sigma + \rho(t - d_i), & \text{otherwise}
\end{cases}
\]  

In this case, the deadline curve \( D_i() \) is initialized to \( S_i() \) in \( O(1) \) complexity. Consider the case that \( D_i() \) is secondly updated. \( D_i(b_i^2; t) \) becomes the following:

\[
D_i(b_i^2; t) = \min \left\{ S_i(t - b_i^2), S_i(t - b_i^2 + r_i^m(b_i^2)) \right\}
\]  

for \( t \geq b_i^2 + d_i \).

In Eq. (28), we maintain \( D_i(b_i^2; t) \) for the time \( t \) such that \( t \geq b_i^2 + d_i \). This is so since \( S_i() \) has the first non-zero value \( \sigma \) at the time \( d_i \). Fig. 7 illustrates how to update \( D_i(b_i^2; t) \). \( D_i(b_i^2; t) \) becomes linear from the time \( b_i^2 + d_i \). Thus, \( D_i(b_i^2; t) \) can be updated in \( O(1) \) complexity for this range. Inductively, it can be easily understood that \( D_i(b_i^2; t) \) can also be updated in \( O(1) \). Note that, however, \( D_i(b_i^2; t) \) is not linear for \( t \geq b_i^2 \). Thus, if we would maintain \( D_i(b_i^2; t) \) for \( t \geq b_i^2 \), updating \( D_i(b_i^2; t) \) could require extremely high complexity to remember the changing time points.

All the SC-EDF’s are \( K \)-piece concave linear curve from the non-zero area as shown in Fig. 8. We can represent the SC-EDF \( S_i() \) for each \( K(\sigma, \rho) \)-smooth session \( i \) as follows:

5. Implementation

If service curves are allocated without any restriction, computation of deadlines requires high complexity. However, we show in this section that if SC-EDF’s are allocated, just a constant time is required for computing deadlines. That is, compared to other service disciplines using a sorted-priority queue, such as GPS service disciplines, the SC service discipline adopting either scheme has better network utilization while having the same scheduling complexity in the single and multiple routers, respectively.
The following theorem tells that if such a concave service curve is allocated, the deadline curve $D_i(\cdot)$ becomes also $K$-piece concave linear.

**Theorem 6.** Let the SC-EDF $S_i(\cdot)$ for session $i$ be

$$S_i(t) = \begin{cases} 0, & 0 \leq t < d_i, \\ \min_{k=1}^{m} \{ \sigma^k_i + \rho^k_i(t - d_i) \}, & \text{otherwise.} \end{cases}$$  

Then, when the session $i$ becomes backlogged at the $m$-th time, the updated $D_i(b^m_i;\cdot)$ becomes a $K$ piecewise linear curve as follows:

$$D_i(b^m_i; t) = \min_{k=1}^K \left\{ C^m_i + \rho^k_i t \right\} \quad \text{for} \quad t \geq b^m_i + d_i,$$

where

$$C^m_i = \begin{cases} \sigma^1_i - \rho^1_i d_i - \rho^1_i b^1_i, & m = 1, \\ \min \left\{ C^{m-1}_i, \sigma^m_i - \rho^m_i d_i - \rho^m_i b^m_i + r^m_i(b^m_i) \right\}, & m \geq 2. \end{cases}$$

**Proof of Theorem 6.** If the service curve $S_i(\cdot)$ is allocated for the session $i$, the deadline curve $D_i(\cdot)$ is updated at each backlogged time point $b^m_i$ as follows:

$$D_i(b^m_i; t) = \min \left\{ D_i(b^{m-1}_i; t), r^m_i(b^m_i) + S_i(t-b^m_i) \right\} \quad \text{for} \quad t \geq b^m_i + d_i. \tag{33}$$

Then, we prove the theorem by structural induction on the number of backlogged times. As the base step, for $t \geq b^1_i + d_i$,

$$D_i(b^1_i; t) = S_i(t-b^1_i) = \min_{k=1}^K \left\{ \sigma^k_i + \rho^k_i(t-b^1_i-d_i) \right\} = \min_{k=1}^K \left\{ C^{1}_i + \rho^k_i t \right\}. \tag{34}$$

By the induction hypothesis, for $t \geq b^m_i + d_i$,

$$D_i(b^m_i; t) = \min_{k=1}^K \left\{ C^m_i + \rho^k_i t \right\}. \tag{33}$$

Then, as the induction step, for $t \geq b^{m+1}_i + d_i$,

$$D_i(b^{m+1}_i; t) = \min \left\{ D_i(b^m_i; t), S_i(t-b^{m+1}_i) + r^m_i(b^{m+1}_i) \right\} = \min \left\{ \min_{k=1}^K \left\{ C^m_i + \rho^k_i t \right\}, \min \left\{ \sigma^m_i + \rho^m_i(t-b^{m+1}_i-d_i) + r^m_i(b^{m+1}_i) \right\} \right\} = \min \left\{ \min_{k=1}^K \left\{ C^m_i + \rho^m_i(t-b^m_i-d_i) + r^m_i(b^m_i) \right\} \right\} = \min_{k=1}^K \left\{ C^m_i + \rho^k_i t \right\}. \tag{35}$$

Now let us see how to compute a deadline of a packet from each updated deadline curve. Let the last backlogged period of session $i$ be $b^m_i$. Consider the $k$-th packet $p^k_i$ from session $i$ that has arrived after the time $b^m_i$. Let us denote the deadline of the packet $p^k_i$ by $D_i^{k}$. Let $A^k_i$ be the accumulated input traffic amount from the session $i$ until the $k$-th packet.

Then, when the session $i$ becomes backlogged at the $m$-th time, the updated $D_i(b^m_i; \cdot)$ becomes a $K$ piecewise linear curve as follows:

$$D_i^{k}(b^m_i; t) = \min \left\{ D_i^{k}(b^{m-1}_i; t), r^m_i(b^m_i) + S_i(t-b^m_i) \right\} \quad \text{for} \quad t \geq b^m_i + d_i, \tag{33}$$

where

$$C^m_i = \begin{cases} \sigma^1_i - \rho^1_i d_i - \rho^1_i b^1_i, & m = 1, \\ \min \left\{ C^{m-1}_i, \sigma^m_i - \rho^m_i d_i - \rho^m_i b^m_i + r^m_i(b^m_i) \right\}, & m \geq 2. \end{cases} \tag{32}$$

Note that if we save $-C^{m-1}_i / \rho^m_i$ for $k = 1 \cdots K$ at the previous update, $\max \{ -C^{m-1}_i / \rho^m_i, -(\sigma^m_i + r^m_i(b^m_i)) / \rho^m_i, +b^m_i + d_i \}$ requires a constant time since both $b^m_i$ and $r^m_i(b^m_i)$ are fixed at the last backlogged time $b^m_i$. Thus, $-C^{m}_i / \rho^m_i$ can be calculated in $O(1)$ at each backlogged time point.

From Eqs. (32) and (39), we can derive pseudo-codes for enqueuing and dequeuing modules in Figs. 9 and 10, respectively. The notations used in the codes are summarized in Table 1. In the codes, the variable $v^k_i$ keeps $-C^{m}_i / \rho^m_i$ at each backlogged time point. $w^0_i$ is $r^m_i(b^m_i)$ if $b^m_i$ is the last backlogged time point. Note that receive_packet() can be performed in $O(K)$ complexity. Since, in most cases, $K$ is restricted to a small number, say 2 or 3, we can regard that the function is performed in a constant time. In addition, transmit_packet() can be performed in $O(\log N)$ complexity, where $N$ is the number of sessions.

6. Comparison of the SC-EDF with the multi-rate service discipline

We compare the SC service discipline with the SC-EDF’s to the multi-rate service discipline in terms of network utilization and scalability.

As mentioned in Section 1.1, the multi-rate service discipline is a GPS service discipline that guarantees a traffic specification characterized by multiple rates [17]. Each incoming packet of a session is given a time-stamp from
the traffic rate. Packets are transmitted in an increasing order of the time-stamps. Ties are broken arbitrarily. Originally, the multi-rate service discipline was proposed for ATM networks which belong to a fixed-sized packet environment. If the multi-rate service discipline is used in a variable-sized environment, transmissions can be either preemptive or non-preemptive. Only in this section, we assume that packets are transmitted preemptively to focus on the comparison.

To compare with the multi-rate service discipline, we set the SC-EDF Si(t) for session i as following:

\[ Si(t) = \begin{cases} 0, & 0 \leq t \leq d_i; \\ bi(t - d_i), & t > d_i, \end{cases} \]

where \( bi(t) \) is the traffic rate function of the session in the multi-rate service discipline and \( d_i \) is the worst-case tolerable delay. Note that we can consider \( bi(t) \) as the output function of the regulator in the underlying RC-EDF service discipline. As mentioned in Section 2.1, \( bi(t) \) is a \( K(\sigma, \rho) \) function, i.e., \( bi(t) = \min_{k=1}^{K} (\sigma_k + \rho_k t) \). We compare network utilization first. We show, in two steps, that the SC service discipline with the SC-EDF’s can achieve strictly higher network utilization than the multi-rate service discipline. We first show that if a set of sessions is admitted by the multi-rate service discipline, the set is also admitted by the SC service discipline. Then, we show that there exists a set of sessions that are admitted by the SC service discipline, but cannot be admitted by the multi-rate service discipline.

Let us see the first step in detail. In the multi-rate service discipline, the end-to-end delay bound to a session is distributed to each service discipline that the session passes through as local delay bounds. Each session uses the traffic function and the distributed local delay bound at each service discipline. Thus, the admission of each session is localized to the respective service disciplines. Note that, in the case of a single service discipline, the SC service discipline with the SC-EDF’s has the highest network utilization since the RC-EDF service discipline is optimal in this case [7].

---

### Table 1

<table>
<thead>
<tr>
<th>Notations used for pseudo-codes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variable</strong></td>
</tr>
<tr>
<td>( v_i ) ( i ) ( 1 ) ( \cdots ) ( K )</td>
</tr>
<tr>
<td>( FLAG_i )</td>
</tr>
<tr>
<td>( t )</td>
</tr>
<tr>
<td>( w_i^0 )</td>
</tr>
<tr>
<td>( w_i )</td>
</tr>
<tr>
<td>( A_i )</td>
</tr>
</tbody>
</table>

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Fig. 9. Enqueueing module for the SC service discipline: On arrival of packet \( p \) to session \( s \), receive _packet\( (s, p) \) is called.

### Function

- **earliest()**: Get a packet with the earliest deadline among all head packets.
- **size(p)**: Return the size of the packet \( p \).
- **session_id(p)**: Returns the session id of the packet \( p \).
- **dequeue(s)**: Extract the head packet from the queue of session \( s \).
- **send(p)**: Send the packet \( p \) to the link.
we conclude that if a set of sessions is admitted by the network composed of the multi-rate service disciplines, the set is also admitted by the network with the SC service disciplines.

We now show the second step with an example. Consider two sessions, session 1 and session 2, that try to enter a service discipline. Both sessions have the same traffic rate, but different worst-case tolerable delays. Specifically, let $b_i(t) = \min\{1 \text{ Mbt}, \ 30 \text{ Kb} + 400 \text{ Kbt}\}$ for $i = 1, 2$, and $d_1 = 30 \text{ ms}$, $d_2 = 120 \text{ ms}$. We assume that the link capacity is 1 Mbt. Both sessions are admitted if the service discipline is the SC service discipline with the SC-EDF’s, since $\sum_{i=1}^2 b_i(t - d_i) \leq 1$ Mbt. However, session 1 cannot be admitted by the multi-rate service discipline. The reason is as follows. Since both sessions have the same traffic rate, the actual delay bounds to both sessions are identical. In this case, the bound is 55 ms if we follow the Lemma 3.3 presented in [17]. Therefore, the delay requirement to session 1 cannot be satisfied.

Now, let us turn to the comparison of scalability. The multi-rate service discipline requires $O(K)$ complexity to compute the time-stamps of each packet. In addition, $O(\log N)$ complexity is required for maintaining a priority queue where $N$ is the number of packets in the queue. As mentioned in Section 5, the SC service discipline requires $O(K)$ complexity to compute the deadline of each packet. The SC service discipline can maintain a priority queue either for the head packets from each session or for all the packets from all sessions. We consider the case that the SC service discipline maintains one queue for all the packets to compare with the multi-rate service discipline. In this case, the SC service discipline requires $O(\log N)$ complexity for the priority queue where $N$ is the number of packets in the queue. Therefore, both have the same scalability.

7. Conclusion

In this paper, we propose the SC service discipline with SC-EDF’s for packet networks. It achieves very high network utilization. (It can achieve the same utilization as the RC-EDF service discipline.) It has $O(K)$ deadline calculation complexity if the traffic rate function of each session is a $K(t,p)$ function. Different from the RC-EDF service disciplines, it doesn’t need regulators at all. Thus, it has better scalability than the RC service disciplines and is work-conserving. Compared to the GPS service disciplines including the multi-rate service discipline, it can achieve strictly higher network utilization.

Acknowledgements

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From Eqs. (45) and (44),
\[ A_i(t) - A_i(s) \geq S_i(t - s). \]  
(46)
Since, the time \( s \) is not greater than the last backlogged time point, the packet arrival time \( a \) is greater than the time \( s \), i.e., \( a > s \). In addition, since session \( i \) has the traffic envelope function \( b_i(t) \),
\[ b_i(a - s) \geq A_i(a) - A_i(s). \]  
(47)
From Eqs. (46) and (47),
\[ b_i(a - s) \geq S_i(t - s) = S_i(a - s + d). \]  
(48)
To satisfy Eq. (48),
\[ d \leq \min \{ A : A > 0 \text{ and } b_i(a - s) \leq S_i(a - s + A) \} \leq \max \min \{ A : A > 0 \text{ and } 1b_i(k) \leq S_i(k + A) \}. \]  
(49)
From Eq. (49), for the packet \( p \), the worst-case delay becomes \( \max_{k \geq 0} \min \{ A : A > 0 \text{ and } b_i(k) \leq S_i(k + A) \} \). Note that, however, all the packets including the packet \( p \) have this delay bound since Eq. (49) is independent of the packet departure time \( t \), which is a special packet departure time for the packet \( p \). \( \square \)

**Proof of Theorem 3.** Consider a time \( t \) that a packet \( p \) from session \( i \) departs the router. Since, \( S_i(t) \) is guaranteed for session \( i \), there exists a time \( s, s < t \), which is the beginning of one of session \( i \)'s backlog periods such that
\[ W_i(t) \geq A_i(s) + S_i(t - s). \]  
(50)
Let us denote the backlog of session \( i \) at the packet departure time \( t \) by \( B_i(t) \). First of all, we derive the worst-case backlog at the time \( t \). (Later, we consider all packet departure times and also the times that are not packet departure times.) The backlog amount at the time \( t \) is the arrived amount until the time \( t \) minus the transmitted amount until the time \( t \). Thus,
\[ B_i(t) = A_i(t) - W_i(t) \leq A_i(t) - A_i(s) - S_i(t - s) \leq A_i(s, t) - S_i(t - s) \leq b_i(t - s) - S_i(t - s) \leq \max_{k \geq 0} \{ b_i(k) - S_i(k) \}. \]  
(51)
From Eq. (51), the worst-case backlog at the packet departure time \( t \) becomes \( \max_{k \geq 0} \{ b_i(k) - S_i(k) \} \). Note that, however, this backlog bound holds for all packet departure times since Eq. (51) is independent of the time \( t \), which is a special packet departure time. Now, we consider all times including those when no packets depart the router. Only one packet departs the router between packet departure times. Thus, for all times, the worst-case backlog amount cannot exceed \( \max_{k \geq 0} \{ b_i(k) - S_i(k) \} + \max \) to satisfy Eq. (51) at all packet departure times. \( \square \)

**Proof of Theorem 5.** We show that if there exists any packet that is not served until its deadline plus \( \frac{f_{\text{max}}}{r} \), Eq. (10) does not hold.

Let the packet \( m \) be the first packet that is not served until its deadline \( D^m \) plus \( \frac{f_{\text{max}}}{r} \). Let \( \tau^* \) be the latest time when the router is empty. Note that since the router is always empty at time zero, \( \tau^* \) always exists. Let the packet \( s \) be the last packet of which the deadline is greater than \( D^m \) and which is served in the time interval \( (\tau^*, D^m + \frac{f_{\text{max}}}{r}) \). Note that the packet \( s \) does not always exist. We consider the case that such a packet \( s \) exists in Part I and the other case in Part II.

**Part I:** Let’s denote the service start time and the length of the packet \( s \) by \( \tau^* \) and \( l^* \), respectively. Since, the SC service discipline transmits packets in an increasing order of the deadlines, it follows that \( \tau^* < D^m \). Since, the SC service discipline is continuously backlogged during the time interval \( (\tau^*, D^m + \frac{f_{\text{max}}}{r}) \), the total service amount during the interval becomes \( r(D^m - \tau^*) + \frac{f_{\text{max}}}{r} \).

Let \( T(\tau^*, D^m) \) be the amount of the packets which arrive during the interval \( (0, D^m + \frac{f_{\text{max}}}{r}) \), have deadlines that are not greater than the time \( D^m \), and have not been transmitted by the time \( \tau^* \). Then, \( T(\tau^*, D^m) \) plus \( l^* \) becomes the traffic amount to be transmitted during the time interval \( (\tau^*, D^m + \frac{f_{\text{max}}}{r}) \). Since the packet \( p \) was not transmitted until the time \( (D^m + \frac{f_{\text{max}}}{r}) \),
\[ T(\tau^*, D^m) + l^* > r(D^m - \tau^*) + \frac{f_{\text{max}}}{r}. \]  
(52)
To derive the maximum of \( T(\tau^*, D^m) \), two lemmas are presented below.

**Lemma 1.** Let’s partition the set of sessions to \( P_1 \) and \( P_2 \) as follows: \( P_1 \) is the set of sessions such that, in the session, (i) the backlog is empty at the moment \( \tau^* \) and, (ii) only the packets with deadlines that are not greater than \( D^m \) are transmitted by the time \( \tau^* \). All the other sessions are included in \( P_2 \). Then, only the packets from the sessions in \( P_1 \) contribute to \( T(\tau^*, D^m) \).

**Proof.** Let a session \( j \) be in the set \( P_2 \). At the time \( \tau^* \), the session \( j \)'s queue may not be empty, or at least one packet of the session with a deadline that is greater than \( D^m \) is transmitted by the time \( \tau^* \). Consider the first case. Let a packet \( p^* \) be in the session \( j \)'s queue at the time \( \tau^* \). If the deadline of the packet were not greater than \( D^m \), the packet would have been transmitted instead of the packet \( s \). Thus, the deadline of the packet \( p^* \) is greater than \( D^m \). Consider the second case. Since packets are transmitted in an increasing order of the deadlines, after the time \( \tau^* \), all packets have deadlines greater than \( D^m \). \( \square \)

**Lemma 2.** Let \( N_i(t) \) be the amount of packets from session \( i \) that have deadlines less than or equal to time \( t \). Suppose that a packet \( p_i^k \) is the last packet that have such a deadline during the \( m \)-th backlogged period of session \( i \). Then,
\[ N_i(t) \leq \min_{s \in [B_i(t)]} \{ p_i^k(s) + S_i(t - s) \}. \]  
(53)
**Proof.** From the condition of \( p_i^k \) in the lemma, \( D_i^m \leq t \) and \( N_i(t) = \sum_{j=1}^{\infty} |l_j| \). That is,
\[ N_i(t) = \sum_{j=1}^{k} I_j^i \]
\[ \leq \min_{s \in (D_{in})^i} \{ r_{in}(s) + S_i(D_{out}^i - s) \} \]  
(54)
\[ \leq \min_{s \in (D_{in})^i} \{ r_{in}(s) + S_i(t - s) \}. \]  
(55)

Eq. (54) follows from Eq. (6). Eq. (55) holds since \( S_i(\cdot) \) is a non-decreasing function. \( \square \)

Now, with Lemma 1 and 2, let’s derive the maximum of \( T(t', D^m) \).
\[ T(t', D^m) \leq \sum_{i \in P_1} [N_i(D^m) - r_{in}(t')] \]  
(56)
\[ = \sum_{i \in P_1} [N_i(D^m) - r_{in}(t')] \]  
(57)

Eq. (56) follows from the definition of \( T(t', D^m) \), Lemma 1 and 2. Eq. (57) follows since, from Lemma 1, session \( i \) is empty at the time \( t' \). Note that
\[ \sum_{i \in P_1} [N_i(D^m) - r_{in}(t')] \leq \sum_{i \in P_1} \min_{r \in (D_{in})^i} \{ r_{in}(s) + S_i(D_{out}^m - s) - r_{in}(t') \} \]  
(58)
\[ \leq \sum_{i \in P_1} r_{in}(t') + S_i(D_{out}^m - t') - r_{in}(t') \]  
(59)
\[ = \sum_{i \in P_1} S_i(D_{out}^m - t') \]  
(60)
\[ \leq \sum_{i = 1}^N S_i(D_{out}^m - t'), \]  
(61)

Eq. (58) follows from Lemma 2. Eq. (59) holds from the fact that session \( i \) is empty at time \( t' \) for any \( i \in P_1 \). (If the time \( t' \) is the beginning of one of session \( i \)'s backlogged periods, Eq. (59) holds. Otherwise, there exists either a time which is greater than the time \( t' \) and is the beginning of one of session \( i \)'s backlogged periods or no such a time after \( t_x \). In either case, Eq. (59) also holds since \( S_i(\cdot) \) is a non-decreasing function.) From Eqs. 52, 57, and 61,
\[ \sum_{i = 1}^N S_i(D_{out}^m - t') + \eta' > T(t', D^m) + \eta' > r(D_{out}^m - t') + r_{in}(t') \]
That is, \( \sum_{i = 1}^N S_i(D_{out}^m - t') > r(D_{out}^m - t') \), which contradicts Eq. (10).

**Part II:** In this case, we set \( t' = t^* \) and \( T(t', D^m) \) becomes the traffic amount to be transmitted during the time interval \( (t', D_{out}^m + r_{in}(t')) \). Then, it is easily derived that
\[ T(t', D^m) > r(D_{out}^m - t') + r_{in}(t'). \]  
(62)

**Lemma 1 and 2** can still be applied to this case. By following the same procedure as in Part I, we can get the following:
\[ T(t', D^m) \leq \sum_{i = 1}^N S_i(D_{out}^m - t'). \]  
(63)

From Eqs. (62) and (63), \( \sum_{i = 1}^N S_i(D_{out}^m - t') > r(D_{out}^m - t') + r_{in}(t') \), which contradicts Eq. (10). \( \square \)

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