Localisation and map building of mobile robots are key technologies in implementing autonomous navigation systems. Because of many efforts in the past decade it has proved that simultaneous localisation and map building (SLAM) is possible both theoretically and practically. Features or landmarks obtained from sensor measurements are registered and associated in SLAM if they are sustainable and correctly matched ones. To have such features it is essential to use a sensor system that provides precise range measurements. Typical sensors for SLAM includes the laser range finder (LIDAR), ultra sonic and vision sensor. LIDARs provide very accurate and long range measurements whereas measurements from ultrasonic sensors are shorter and coarse due to cone shaped beam patterns. In this article, a structured light system, which combines laser and vision is thus considered for use with structural displacement measurement. This article proposes a multiple dual laser-vision system to be used as a structural displacement measurement system. The proposed dual laser-vision system is quite similar to the structured light system but it solves accurate relative pose instead of range map of the environment. Some simulations were done to determine the minimal configuration and to show the effectiveness of the system for the long span structural displacement monitoring.

**Keywords:** structural health monitoring; displacement; structured light; robot; laser-vision

1. Introduction

Civil infrastructures are continuously exposed to various dynamic external loads such as traffic, earthquakes, gusts, waves, and so on. Thus the structural health monitoring has become an important research topic for continuous assessment and evaluation of structural safety and integrity. Conventional approaches use accelerometer, strain gauge, PZT sensors, GPS, and so on. Even though it is one of the most important descriptors of structural behaviour under all the potential disturbances either from natural or man-made origin (Lee et al. 2007), displacement has not been used popularly because of the current lack of sensors that are reliable, accurate, easy to use and have real-time capability for monitoring of large structures because of the inaccessibility and the huge size of the civil infrastructures.

One of the most frequently employed approaches uses linear variable differential transformer (LVDT), a contact-type sensor for displacement measurement that needs stable reference points underneath, which is not so practical to be used. Thus the development of structural health monitoring system that directly measures the displacement of the structure using vision or laser-based mobile robot system can be very helpful. In solving this, we can adopt the structured light (SL) type system which is widely used in robot navigation.

Recently, the problem of simultaneous localisation and map-building, known as SLAM, has gained great attention in the mobile robotics research area. SLAM focuses on the problem of self-localisation while building a map of an unknown environment from a sequence of noisy sensor measurements is obtained from a moving robot. SLAM is considered to be a core capability for autonomous mobile robots operating in unknown environments (Smith et al. 1990).

2D laser range finders, sonar sensors and cameras are the most popular sensors in indoor SLAM implementations. They can be used separately (Gutmann and Schlegel 1996) or in combination (Castellanos and Tardos 1999). Time of flight (TOF) laser sensors such as SICK laser provide very long and accurate range measurements. Although some error models of TOF type laser sensors were proposed the range measurements are accurate enough for segmentation without preprocessing. On the contrary sonar sensors have wide beam patterns hence inducing large uncertainty. In the SLAM systems developed earlier by us, an SL range sensor system is introduced that consists of a B/W camera and a line laser projector (Myung et al. 2004).
To use the structured light like system in a long distant observation, it is recommended to put the camera in the observation area instead of putting in the projecting area. By formulating this kind of configuration, a dual laser-vision system can be designed for structural displacement measurement.

In Section 2 the SL range sensor is introduced for SLAM purpose. Section 3 describes the structural health monitoring system that estimates the 6-DOF displacement using laser and vision sensors. Some preliminary simulation results will be shown to demonstrate the applicability of the proposed system.

2. Structured light system for obtaining 2D range

In this section, the principle of the SL sensor is discussed implemented for a SLAM system. As shown in Figure 1, several steps are involved in obtaining a 2D range map. The process includes:

- **Correction of lens distortion**
- **Image processing (detection of laser region)**
- **Triangulation for 2D mapping**

Figure 1. Process to obtain 2D range profiles.

Figure 2. Configuration of camera and laser projector.

Figure 3. Displacement monitoring using multiple dual laser-vision system.
2D range profile from the laser pattern captured by camera. The first step is to correct the camera image that has been distorted by the lens. The OpenCV library of Zhang’s method (Zhang 2000) provides camera’s intrinsic and extrinsic parameters as well as lens distortion model coefficients. Scale factors of the intrinsic parameters and all of the extrinsic parameters are used later in mapping the laser regions in the image to range profiles in 2D plane. The following equation is the relationship between real and distorted image coordinates in the model:

\[
\begin{align*}
\hat{x} &= x + x[k_1r^2 + k_2r^4] + [2p_1xy + p_2(x^2 + 2y^2)], \\
\hat{y} &= y + y[k_1r^2 + k_2r^4] + [2p_1xy + p_2(x^2 + 2y^2)],
\end{align*}
\]

where \(x\) and \(y\) are ideal, and \(\hat{x}\) and \(\hat{y}\) are real distorted image physical coordinates. The radial distance \(r\) is defined as \(r^2 = x^2 + y^2\). Hence the distortion model is
characterised by the four coefficients $k_1, k_2, p_1$ and $p_2$. The corrected image is then convolved with a kernel to determine the laser region. The kernel is designed in consideration of the thickness of the laser pattern in the image. After the convolution only those pixels whose convolution values are above a threshold remain as the candidate laser regions. Then the image is looked into column by column so that only one laser region is selected in each column. If there is more than one region in a column, the largest region is selected. Finally, in each column, the region is averaged in row values to be mapped in 2D plane.

Figure 2 shows the relationship between the camera frame $\Sigma_{X,Y,Z}$ and that of the laser projector $\Sigma_{X',Y',Z'}$. A camera’s longitudinal displacement along $Y_p$ is annotated with $P_Y$, and angular displacement along $X_p$ with $a$. The following equations are used to map the laser pattern in the image to the 2D plane.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos a & -\sin a \\ 0 & \sin a & \cos a \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} + P_Y$$

$$\begin{bmatrix} x_c' \\ y_c' \end{bmatrix} = \begin{bmatrix} x_s \\ y_s \end{bmatrix} T_{x_c'y_c'z_c'}$$

$$y' = 0.$$

In the above equation $z'$ is the longitudinal distance to the laser pattern while $x'$ is the lateral distance with respect to the origin of $\Sigma_{X,Y,Z}$. Laser pattern coordinates $[x_s y_s]^T$ are the coordinates in the image.

Figure 6. Simulation result of $(3 \times 3)$ configuration. True posture is $(x, y, z, \theta, \phi, \psi) = (0.01, 1.02, 100, 0.001, -0.002, 0.003).$
frame (in pixels) subtracted by the image centre. Intrinsic parameters such as image centre coordinates and scale factors \((s_x, s_y)\) along image’s \(x\) and \(y\) axes are obtained from the camera calibration stage.

3. Structural health monitoring system

3.1. Dual laser-vision system

The previous methods using vision sensor for displacement used specially manufactured optical device to concurrently measure reference and target points using one image capturing device (Olaszek 1999) or high resolution camera system (Wahbeh et al. 2003). And the most recent method used fixed observation point and landmarks that should be scaled precisely (Lee et al. 2007). To avoid those problems and to provide low-price solution, a structured light type displacement measurement system is proposed. A possible candidate for the displacement measurement of the long structure is illustrated in Figure 3. In Figure 3, the multiple laser-vision modules is proposed such that one module can act as a reference for the neighbouring modules.

Figure 4 shows the configuration of the proposed dual laser-vision system which corresponds to one module in the overall system described in Figure 3. Laser sensor \(A\), composed of at least two point lasers, projects its parallel beams to the distant screen \(B\). The distance can be quite large, i.e., 100 m or so. In screen \(B\), the images of two points are captured by camera \(B\), and the two image points can be obtained. The laser

![Figure 7](image_url)

Figure 7. Simulation result of \((2 \times 2)\) configuration. True posture is \((x, y, z, \theta, \phi, \psi) = (0.01, 1.02, 100, 0.001, -0.002, 0.003)\).
sensor B also projects its two beams to the distant screen A. Also two distinct image points are observed in screen A. It is also possible to place the camera at the front side of the screen instead of the backside to observe the projected beams.

Overall four different image points are observed in 2D screens and 8 scalar values can be obtained. Using these observed values, the relative displacement, i.e., the translations \( x, y, z \), and the rotation angles \( \theta, \phi, \psi \) around \( x \)-axis, \( y \)-axis and \( z \)-axis are to be estimated.

### 3.2. Kinematics and estimation algorithm

The kinematics defines the geometric relationship between the observed data \( m = \left[ \begin{array}{c} O \ Y^A, B \ O, B \ Y^A \end{array} \right]^T \) and estimated posture \( p = [x, y, z, \theta, \phi, \psi]^T \). Here \( O \) and \( Y \) are the projected points in screen A, and \( B \) and \( Y \) are the projected points in screen B. To derive the kinematics, the transformation matrices \( ^A T_B \) and \( ^B T_A \) can be used. As can be seen in Equation (1), the transformation matrix consists of rotations about \( x \)-axis, \( y \)-axis and \( z \)-axis and translations in \( x \), \( y \), \( z \)-axis. Refer to Figure 5 for some notations.

\[
^A T_B = T(x, y, z) R(\theta) R(\phi) R(\psi)
\]

\[
= \begin{bmatrix}
1 & 0 & 0 & x \\
0 & 1 & 0 & y \\
0 & 0 & 1 & z
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta & -\sin \theta & 0 \\
0 & \sin \theta & \cos \theta & 0
\end{bmatrix}
\begin{bmatrix}
\cos \phi & 0 & \sin \phi & 0 \\
0 & 1 & 0 & 0 \\
-\sin \phi & 0 & \cos \phi & 0
\end{bmatrix}
\begin{bmatrix}
\cos \psi & -\sin \psi & 0 & 0 \\
\sin \psi & \cos \psi & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\cos \phi & 0 & \sin \phi & 0 \\
0 & 1 & 0 & 0 \\
-\sin \phi & 0 & \cos \phi & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\cos \psi & -\sin \psi & 0 & 0 \\
\sin \psi & \cos \psi & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
(1)
\]

It is easy to obtain \( ^B T_A \) if the following relationship is used between \( ^B T_A \) and \( ^A T_B \) (Craig 2005).

\[
^B T_A = \begin{bmatrix}
^A R_{B}^T & ^A P_B \\
0 & 1
\end{bmatrix}
\]

(2)

where \( ^A R_B \) is the rotation matrix of \( ^A T_B \) which is the first \( 3 \times 3 \) components of \( ^A T_B \) and \( ^A P_B \) is the translation matrix of \( ^A T_B \) which corresponds to the last column vector of \( ^A T_B \). Using the transformation matrices \( ^A T_B \) and \( ^B T_A \), a final kinematic equation can be obtained after some mathematical manipulations.

Figure 8. Simulation result of \((4 \times 0)\) configuration. True posture is \((x, y, z, \theta, \phi, \psi) = (0.01, 1.02, 100, 0.001, -0.002, 0.003)\).
In each screen, it is assumed that the first laser projector, which projects to \(^A O\) or \(^B O\), is installed at a distance of \(Y_S\) away from the origin in \(y\)-axis direction, and the second laser projector, which projects to \(^A Y\) or \(^B Y\), is installed at a distance of \(Y_L\) away from the first laser projector as can be seen in Figure 5. If we consider the first laser projected from \(B\) to screen \(A\), the screen coordinate \(^A O\) can be obtained from the following equation:

\[
{^A O} = {^A T_B}\begin{bmatrix} 0 & Y_S \\ Z_{BA1} & 1 \end{bmatrix}, \tag{3}
\]

where \(Z_{BA1}\) is the distance from laser \(B\) to screen \(A\). As \(z = 0\) on the screen \(A\) in coordinate frame \(A\), \(z\) component of \(^A O\) vector can be set to zero to obtain \(Z_{BA1}\). By substituting the obtained \(Z_{BA1}\) to Equation (3), the actual screen coordinate \(^A O\) of the projected laser beam can be obtained. Since the second laser which projects to \(^A Y\) point is installed at a distance of \((Y_S + Y_L)\) away in \(y\)-axis direction, the screen coordinate \(^A Y\) can be obtained from the following equation:

\[
{^A Y} = {^A T_B}\begin{bmatrix} 0 & Y_S + Y_L \\ Z_{BA2} & 1 \end{bmatrix}. \tag{4}
\]

Also by setting \(z\) component of \(^A Y\) point to be zero, \(Z_{BA2}\) can be obtained. By substituting the obtained \(Z_{BA2}\) to
Equation (4), the actual screen coordinate $A^Y$ of the projected laser beam can be obtained. In a similar way, the additional constraints can be easily obtained in screen $B$ for $B^O$ and $B^Y$ as follows:

$$B^O = B^T A^O \begin{bmatrix} 0 \\ Y_s \\ Z_{AB1} \\ 1 \end{bmatrix}.$$  \hspace{1cm} (5)

By putting the four constraints altogether, the kinematics $M$ can be obtained as in Equation (6).

$$M = \begin{bmatrix}
(-\cos \theta \sin \psi - \sin \theta \sin \phi \cos \psi - z \sin \phi + x \cos \theta \cos \phi)/(\cos \theta \cos \phi) \\
(\cos \psi + z \sin \theta + y \cos \theta)/\cos \theta \\
1/10(-9 \cos \theta \sin \psi - 9 \sin \theta \sin \phi \cos \psi - 10z \sin \phi + 10x \cos \theta \cos \phi)/(\cos \theta \cos \phi) \\
1/10(9 \cos \psi + 10z \sin \theta + 10y \cos \theta)/\cos \theta \\
-(x \cos \theta \cos \psi + \cos \phi \sin \psi - x \sin \theta \sin \phi \cos \psi + y \sin \psi \cos \phi)/(\cos \theta \cos \phi) \\
-(x \cos \theta \sin \psi - \cos \phi \sin \psi \cos \phi - x \sin \theta \cos \psi \sin \phi - y \cos \psi \cos \phi)/(\cos \theta \cos \phi) \\
-1/10(10x \cos \theta \cos \psi + 9 \cos \phi \sin \psi - 10x \sin \theta \sin \psi \sin \phi + 10y \sin \psi \cos \phi)/(\cos \theta \cos \phi) \\
-1/10(10x \cos \theta \sin \psi - 9 \cos \psi \cos \phi + 10x \sin \theta \cos \psi \sin \phi - 10y \cos \psi \cos \phi)/(\cos \theta \cos \phi)
\end{bmatrix}.$$  \hspace{1cm} (6)
By using the steepest descent method, the estimation of posture $p$ can be obtained as follows:

$$\hat{p}(k + 1) = \hat{p}(k) + J_p^+ (m(k) - \hat{m}(k)),$$

(7)

where $J_p = \frac{\partial M}{\partial p}$ is the Jacobian of the kinematic equation and $J_p^+$ is the pseudo-inverse of the Jacobian.

In case there exist uncertainties in the measurement, the extended Kalman filtering scheme (Welch and Bishop 2006) can be applied to estimate the posture $p$. Since the state $p$ is assumed to be stationary in a short time interval, the system function is set to an identity matrix. In the prediction step of Kalman filter, prediction of the state ($\hat{p}$) and measurement ($\hat{m}$) can be obtained as follows:

$$\hat{p}(k + 1|k) = \hat{p}(k|k)$$

(8)

$$\hat{m}(k + 1|k) = M \cdot \hat{p}(k + 1|k)$$

$$P(k + 1|k) = P(k|k) + Q,$$

where $P$ is the error covariance matrix of the state, $Q$ is the covariance matrix of the system noise, and $M$ is the kinematic equation obtained from Equation (6). In the previous equation, $(k + 1|k)$ means a priori estimate and $(k + 1|k + 1)$ means a posteriori estimate.

In the observation step of Kalman filter, the following equation is applied:

$$v(k + 1) = m(k + 1) - \hat{m}(k + 1|k)$$

(9)

$$S(k + 1) = J_p P(k + 1|k) J_p^T + R,$$

where $R$ is the covariance matrix of the measurement noise.
Finally, the Kalman filter update step can be applied to obtain a posteriori result as follows:

\[ K(k+1) = P(k+1|k)J_p^T S^{-1}(k+1) \]  

\[ \hat{p}(k+1|k+1) = \hat{p}(k+1|k) + K(k+1)v(k+1) \]  

\[ P(k+1|k+1) = P(k+1|k) - K(k+1)S(k+1)K^T(k+1). \]

By doing so, the state error covariance \( P \) can be estimated as well as the state vector \( p \).

3.3. Simulation results

Before going further, various possible configurations had to be tested to ensure that we have minimal configuration that can solve 6-DOF posture estimation problem. At first, the total of six-lasers configuration is tested; three lasers on one side and the other three lasers on another side. Let us denote this as \((3 \times 3)\) configuration. At various settings we could conclude that this \((3 \times 3)\) configuration can estimate the 6-DOF posture perfectly. Figure 6 shows one example when the true values are set as \((x, y, z, \theta, \phi, \psi) = (0.01, 1.02, 100, 0.001, -0.002, 0.003)\). To see if \((2 \times 2)\) configuration also works, the total of four-lasers configuration is tested; two lasers on one side and the other two lasers on another side. At various settings, it is shown that this \((2 \times 2)\) configuration can also estimate the 6-DOF posture. Figure 7 shows one example when the true values are set as \((x, y, z, \theta, \phi, \psi) = (0.01, 1.02, 100, 0.001, -0.002, 0.003)\). To see if \((4 \times 0)\) configuration also works, the total of
The four-lasers configuration is tested; four lasers on one side and no lasers on another side. At various settings, it is shown that this $(4 \times 0)$ configuration could not estimate the 6-DOF posture. Figure 8 shows one example when the true values are set as $(x, y, z, \phi, \chi, \psi) = (0.01, 1.02, 100, 0.001, -0.002, 0.003)$. As Figure 8 shows, the estimation converges to other local minima, which can be regarded as a singularity problem. With various configurations, it is clear that $(2 \times 2)$ configuration is the minimal configuration that can estimate the 6-DOF posture. Thus all the following simulations were done in $(2 \times 2)$ configuration.

In the following simulations, the true values are set as $(x, y, z, \phi, \psi) = (0.1, -1.02, 100, 0.001, -0.002, 0.003)$. Using the distance of $z = 100$ m, the estimated value converged to true values very rapidly. As it can be seen in Figure 9, it converged only after 10 iterations.

When there exist random uniform measurement noises, the estimation using steepest descent method also quickly converged to the true value as can be seen in Figure 10. The Kalman filtering was also applied to the noisy measurement using $Q = 1.0e^{-6} I$ and $R = 1.0e^{-6} I$. As can be seen in Figure 11, the convergence speed to the true value was faster than the steepest descent method.

Next, the effect of dynamic disturbances is tested in $x$-axis direction by applying $x = 0.01 \sin(0.1t)$ with the same amount of random uniform measurement noise as in the previous simulation. As can be seen in Figure 12, $x$ direction estimation quickly tracks the dynamic displacement.
The Kalman filtering was also applied to the dynamic case with noisy measurement using $Q = 1.0e^{-6}I$ and $R = 1.0e^{-6}I$. As can be seen in Figure 13, the convergence speed to the true value was also faster than the steepest descent method.

4. Conclusions
In this article, a multiple laser-vision system is proposed that can measure the long span structural displacement. Using a minimal configuration of $(2 \times 2)$, it is shown that the system can solve the 6-DOF estimation problem quite efficiently using a steepest descent algorithm. When there exist measurement noises, the displacement estimation problem could be solved using extended Kalman filter (EKF). In this case, as the estimated instantaneous translation and rotation are assumed to be stationary, the system function is set to an identity matrix. And the observation matrix is the same as the kinematic equation derived earlier. It is not needed to solve the inverse of Jacobian in this EKF scheme. Instead, Kalman gain plays the role of inverse of Jacobian. The advantage of EKF is that the error covariance is also predicted. Thus the propagation of the overall error could also be estimated. The proposed system is now being implemented to be applied in real-time in real structures.

We believe that this type of robot navigation technology can have wide range of application in civil engineering as well as in home environment. For example, these robot navigation techniques can be used for the monitoring of civil structures such as concrete crack width measurement using structured light range sensor as well as the displace measurement using laser and vision sensors.

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References