Kinematic Design of a Six Degree-of-Freedom In-Parallel Manipulator for Probing Task

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Abstract

This paper presents a kinematic analysis of a parallel manipulator we have been developing for the probing task application that requires high bandwidth, active compliance, and high precision. Based on the concept of macro/micro manipulators, the parallel manipulator serves as a wrist of a macro manipulator. The developed manipulator is a class of in-parallel platforms with 3 PRPS(prismatic-revolute-prismatic-spherical joints) chain geometry. The main advantages of this manipulator, compared with the typical Stewart platform type, are the capability of pure rotation generation and the easy prediction of the moving platform motion. The purpose of this paper is to develop an efficient kinematic model which can be used for real-time control and to propose systematic methods to design the manipulator considering workspace, manipulability, resistivity, and the existence conditions of the forward kinematic solution. A series of simulation are carried out to show the kinematic characteristics and performance of the mechanism. A prototype manipulator has been built based on the kinematic design analysis.

1. Introduction

In the past two decades, there have been considerable developments in the area of parallel manipulators. Obviously, parallel manipulators offer high structural stiffness and precise positioning accuracy over serial ones.

Parallel manipulators have been studied by many researchers, usually in the configuration known as a Stewart platform[1-5]. The Stewart platform was originally designed as an aircraft simulator[1]. Since then, various applications of the Stewart platform have been investigated such as a compliance device for assembly[2], a force/torque sensor[3], an active vibration isolator[4], and a master hand mechanism for

teleoperation[5]. But in general, the Stewart platform has some disadvantages that forward kinematics is too complex to solve in real-time and the prediction of the motion of each joint is not intuitive.

Other configurations for parallel manipulators with six degree-of-freedom have also been proposed. Behi[6] developed a configuration with three legs where each leg consists of a PRPS chain. Hudgens and Tesar[7] investigated a device with six inextensible legs where each leg is driven by a four-bar mechanism mounted on the base platform. Alizade et al.[8] proposed a parallel manipulator which has three legs, mounted on moving sliders passing through a circular trajectory. Previous works in these modified Stewart platform manipulators have shown few industrial applications compared with the Stewart platform.

Most tasks carried out by robotic manipulators are involved with the interaction with the environment such as precision assembly, machining, and probing operation. As the contact occurs very fast, limited control bandwidth of conventional robot controllers restricts their active force control. Concept of the macro/micro manipulator has been proposed as one of the methods to solve the contact problem[9]. A micro manipulator is attached to a macro conventional robot serially. The micro manipulator, generally a specialized device in the form of fingers or a wrist, is well adapted to active force control due to its low inertia. Some of the micro manipulator has parallel structures[10]. Most of their structures are based on the Stewart platform. The actuators of the manipulators have been electrical rams and pneumatic actuators.

This paper presents a kinematic analysis of a parallel manipulator we have been developing for industrial applications that require high control bandwidth, active compliance, and high precision as shown in Fig. 1. The developed manipulator is a class of in-parallel platforms with 3 PRPS chains geometry whose design is based on the Behi mechanism[8]. The main advantages of this

manipulator, compared with the Stewart platform type, are the capability to produce pure rotation and to predict the motion of the moving platform intuitively. Also, this manipulator has simple kinematic characteristics compared with the Stewart platform. Therefore, controlling in real-time is possible due to less computational burden. Although the basic structure of the developed mechanism is similar to Behi platform, this paper emphasizes the following contents that have not been addressed by others: a simple kinematic model of the 3 PRPS type parallel manipulator which can be used for real-time control and systematic methods to design the manipulator considering workspace, manipulability, resistivity, and the existence range of the forward kinematic solution.

Fig. 1. A photograph of the proposed parallel manipulator

2. Inverse and Forward Kinematics

The developed mechanism consists of a base plate, a top plate, three horizontal actuating links, and three vertical actuating links. The three horizontal links provide three degree-of-freedom, that are one degree of orientational freedom and two degrees of translational freedom. The three vertical links provide three degree-of-freedom, that are two degrees of orientational freedom and one degree of translational freedom.

The notation used to describe the kinematics of the proposed mechanism is shown in Fig. 2. The fixed global coordinates called the base frame (X, Y, Z) is located at the center of the mass of the base plate. Another reference coordinate, called the top frame (x, y, z), is located at the center of the mass of the top plate. The coordinate (x, y, z) with respect to the base frame (X, Y, Z) can be described by the vectors of the homogeneous transformation matrix T :

$${}^o T = (\bar{n}, \bar{o}, \bar{a}, \bar{p}) = \begin{bmatrix} n_1 & o_1 & a_1 & X_c \\ n_2 & o_2 & a_2 & Y_c \\ n_3 & o_3 & a_3 & Z_c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $(\bar{n}, \bar{o}, \bar{a})$ and (X_c, Y_c, Z_c) are the orientation vector and the position of the top platform center with respect to the base frame (X, Y, Z), respectively. The top plate is connected to the vertical links with ball joints B_i which are equally spaced at 120 degrees and at a radius r from the center of the top platform. The other ends of the vertical links are connected to the horizontal links through equally spaced pin joints P_i at a radius R from the center of the base platform.

Fig. 2. Kinematic structure with coordinate assignment

The inverse kinematics problem can be briefly stated as: for a given position and orientation of the top platform, compute the actuating length of each link. From the relationship that the links P_1B_1 , P_2B_2 , and P_3B_3 are constrained by the pin joints, we can obtain the actuating lengths of the horizontal links S_i as follows:

$$\begin{aligned} S_1 &= n_2 r + Y_c \\ S_2 &= \frac{\sqrt{3}}{4} n_1 r + \frac{1}{4} n_2 r - \frac{3}{4} o_1 r - \frac{\sqrt{3}}{4} o_2 r - \frac{\sqrt{3}}{2} X_c - \frac{1}{2} Y_c \\ S_3 &= -\frac{\sqrt{3}}{4} n_1 r + \frac{1}{4} n_2 r - \frac{3}{4} o_1 r + \frac{\sqrt{3}}{4} o_2 r + \frac{\sqrt{3}}{2} X_c - \frac{1}{2} Y_c \end{aligned} \quad (1)$$

Then, the lengths of the vertical links L_i can be obtained from the the following vector analysis:

$$\overline{OO'} + \overline{O'B_i} = \overline{OP_i} + \overline{P_iB_i} \quad \text{for } i = 1, 2, 3. \quad (2)$$

$$\begin{aligned}
L_1^2 &= X_C^2 + Y_C^2 + Z_C^2 + r^2 + R^2 + S_1^2 - 2RX_C - 2SY_C \\
&\quad + 2m_1(X_C - R) + 2m_2(Y_C - S_1) + 2m_3Z_C \\
L_2^2 &= X_C^2 + Y_C^2 + Z_C^2 + r^2 + R^2 + S_2^2 + RX_C - \sqrt{3}RY_C \\
&\quad + \sqrt{3}S_2X_C + S_2Y_C + (X_C + \frac{1}{2}R + \frac{\sqrt{3}}{2}S_2)(\sqrt{3}o_1 - n_1)r \\
&\quad + (Y_C - \frac{\sqrt{3}}{2}R + \frac{1}{2}S_2)(\sqrt{3}o_2 - n_2)r + (\sqrt{3}o_3 - n_3)rZ_C \\
L_3^2 &= X_C^2 + Y_C^2 + Z_C^2 + r^2 + R^2 + S_3^2 + RX_C + \sqrt{3}RY_C - \sqrt{3}S_3X_C \\
&\quad + S_3Y_C - (X_C + \frac{1}{2}R - \frac{\sqrt{3}}{2}S_3)(\sqrt{3}o_1 + n_1)r \\
&\quad - (Y_C + \frac{\sqrt{3}}{2}R + \frac{1}{2}S_3)(\sqrt{3}o_2 + n_2)r - (\sqrt{3}o_3 + n_3)rZ_C
\end{aligned} \quad (3)$$

If the position of the top plate is given, the solutions of the inverse kinematics are uniquely determined as shown in (1) and (3).

The forward kinematics problem can be stated as: for given actuating lengths of the extensible links, compute the position (X_c, Y_c, Z_c) and orientation vector $(\vec{n}, \vec{o}, \vec{a})$ of the top platform. The angles θ_i are defined to be the angles between the base platform and the vertical links P_iB_i (for $i = 1, 2, 3$). Since the distance between any two adjacent ball joints is $\sqrt{3}r$, the vector relations can be expressed as:

$$\vec{OP}_i + \vec{P}_iB_i + \vec{B}_iB_j = \vec{OP}_j + \vec{P}_jB_j \quad (\text{for } i = 1, 2, 3 \quad j = 2, 3, 1) \quad (4)$$

$$\|\vec{B}_iB_j\|^2 = (\sqrt{3}r)^2 = \|\vec{OP}_j + \vec{P}_jB_j - \vec{OP}_i - \vec{P}_iB_i\|^2 \quad (5)$$

From the above (5), the implicit relationships between θ_i and six lengths of the extensible links L_i, S_i are described as follows:

$$\begin{aligned}
f_1(\theta_1, \theta_2) &= L_1^2 + L_2^2 + L_1L_2 \cos \theta_1 \cos \theta_2 - 2L_1L_2 \sin \theta_1 \sin \theta_2 \\
&\quad - L_1 \cos \theta_1 (3R + \sqrt{3}S_2) + L_2 \cos \theta_2 (-3R + \sqrt{3}S_1) + 3R^2 - 3r^2 \\
&\quad + S_1^2 + S_2^2 + S_1S_2 + R(-\sqrt{3}S_1 + \sqrt{3}S_2) = 0 \\
f_2(\theta_2, \theta_3) &= L_2^2 + L_3^2 + L_2L_3 \cos \theta_2 \cos \theta_3 - 2L_2L_3 \sin \theta_2 \sin \theta_3 \\
&\quad - L_2 \cos \theta_2 (3R + \sqrt{3}S_3) + L_3 \cos \theta_3 (-3R + \sqrt{3}S_3) + 3R^2 \\
&\quad - 3r^2 + S_2^2 + S_3^2 + S_2S_3 + R(-\sqrt{3}S_2 + \sqrt{3}S_3) = 0 \\
f_3(\theta_1, \theta_3) &= L_1^2 + L_3^2 + L_1L_3 \cos \theta_1 \cos \theta_3 - 2L_1L_3 \sin \theta_1 \sin \theta_3 \\
&\quad - L_1 \cos \theta_1 (3R - \sqrt{3}S_3) - L_3 \cos \theta_3 (3R + \sqrt{3}S_1) + 3R^2 \\
&\quad - 3r^2 + S_1^2 + S_3^2 + S_1S_3 + R(\sqrt{3}S_1 - \sqrt{3}S_3) = 0
\end{aligned} \quad (6)$$

We thus have three nonlinear equations for three unknowns θ_i . Their solutions can be obtained numerically. To control the manipulator in real-time without additional sensor for sensing the position and orientation of the end effector installed at the top plate,

it is very important to compute θ_i fast by using L_i and S_i sensed from the position sensors of the links. In this paper, the Newton-Raphson method is used to solve the nonlinear equation. Since the initial value of θ_i is critical for the calculation time of the Newton-Raphson method, the initial value has to be guaranteed to meet the required computing time constraints within the workspace. Then, the position of the center (X_c, Y_c, Z_c) and the orientation vector $(\vec{n}, \vec{o}, \vec{a})$ of the top plate are obtained from the calculated θ_i as follows:

$$\begin{aligned}
X_C &= \frac{1}{3}(\frac{\sqrt{3}}{2}S_3 - \frac{\sqrt{3}}{2}S_2 - L_1 \cos \theta_1 + \frac{1}{2}L_2 \cos \theta_2 + \frac{1}{2}L_3 \cos \theta_3) \\
Y_C &= \frac{1}{3}(S_1 - \frac{1}{2}S_2 - \frac{1}{2}S_3 + \frac{\sqrt{3}}{2}L_2 \cos \theta_2 + \frac{\sqrt{3}}{2}L_3 \cos \theta_3) \\
Z_C &= \frac{1}{3}(L_1 \sin \theta_1 + L_2 \sin \theta_2 + L_3 \sin \theta_3)
\end{aligned} \quad (7)$$

$$[\vec{n}, \vec{o}, \vec{a}] = \begin{bmatrix} n_1 & o_1 & a_1 \\ n_2 & o_2 & a_2 \\ n_3 & o_3 & a_3 \end{bmatrix} = \begin{bmatrix} \frac{R - L_1 \cos \theta_1 - X_C}{r} & \frac{-\sqrt{3}S_2 + L_2 \cos \theta_2 - L_1 \sin \theta_2 - 3X_C}{\sqrt{3}r} & \frac{n_2 o_3 - n_3 o_2}{\sqrt{3}r} \\ \frac{S_1 - Y_C}{r} & \frac{\sqrt{3}R + S_1 - S_2 - \sqrt{3}L_2 \cos \theta_2 - 3Y_C}{\sqrt{3}r} & \frac{n_3 o_1 - n_1 o_3}{\sqrt{3}r} \\ \frac{L_1 \sin \theta_1 - Z_C}{r} & \frac{L_1 \sin \theta_1 + 2L_2 \sin \theta_2 - 3Z_C}{\sqrt{3}r} & \frac{n_1 o_2 - n_2 o_1}{\sqrt{3}r} \end{bmatrix} \quad (8)$$

where $\vec{a} = \vec{n} \times \vec{o}$.

From (6), we know that multiple solutions of the angles θ_i exist for a given set of link lengths. In other words, there are multiple possible configurations of the manipulator for a specific set of link displacements.

3. Existence of the Forward Kinematic Solution

Since a parallel manipulator consists of closed chains, there are kinematic constraints that restrict the motion of the links of the manipulator. Some solutions of the direct kinematics are not realizable according to combinations of the link displacements. In this section, we introduce conditions for checking the existence of the solution of the direct kinematics for the proposed in-parallel manipulator.

The kinematic constraints that restrict the motion of the links are described by (6). From the constraints, we determine the critical points that has an unique kinematic solution in the whole workspace. A set of equations for obtaining the points at which the rates of variation of f_1, f_2 , and f_3 are zero can be derived as follows:

$$\begin{aligned}\frac{\partial f_1}{\partial \theta_1} + \frac{\partial f_1}{\partial \theta_2} &= 0 \\ \frac{\partial f_2}{\partial \theta_2} + \frac{\partial f_2}{\partial \theta_3} &= 0 \\ \frac{\partial f_3}{\partial \theta_1} + \frac{\partial f_3}{\partial \theta_3} &= 0.\end{aligned}\quad (9)$$

Let us denote

$$\rho = \frac{r}{R}. \quad (10)$$

In considering the configuration of the proposed mechanism, we know that the solutions simultaneously satisfying (9) are only the following two cases:

$$\begin{aligned}\theta_1 = \theta_2 = \theta_3 = 0 & \quad \text{if } \rho = \frac{r}{R} \leq 1, \\ \theta_1 = \theta_2 = \theta_3 = \pi & \quad \text{if } \rho = \frac{r}{R} > 1.\end{aligned}\quad (11)$$

The above two cases represents the configurations of the mechanism whose forward kinematic solution exists uniquely. Therefore, the existence condition of the forward kinematic solution including the critical conditions of (11) can be described as follows:

$$\begin{aligned}f_i(0,0) &\leq 0 \quad \text{if } \rho = \frac{r}{R} \leq 1, \\ f_i(\pi,\pi) &\leq 0 \quad \text{if } \rho = \frac{r}{R} > 1, \quad \text{for } i = 1, 2, 3.\end{aligned}\quad (12)$$

Let's consider a special case in which the lengths of three vertical links and three horizontal links are same respectively: $L_1 = L_2 = L_3 = L$, $S_1 = S_2 = S_3 = S$. In this case, the solutions satisfying the inequality of (12) are given in forms as:

$$\begin{aligned}L &\geq R - \sqrt{r^2 - S^2} \quad \text{if } \rho = \frac{r}{R} \leq 1 \quad (-r \leq S \leq r), \\ L &\geq \left| -R + \sqrt{r^2 - S^2} \right| \quad \text{if } \rho = \frac{r}{R} > 1 \quad (-r \leq S \leq r).\end{aligned}\quad (13)$$

Fig. 3 shows the existence range of the forward kinematic solution satisfying the inequality of (13). For each ρ , (13) means that the kinematic solution exists at the upper zones of the obtained curves.

Generally, the forward kinematics of parallel manipulators doesn't have a unique solution, but multiple solutions. The simulations for the forward kinematics of the proposed manipulator result in eight distinct solutions according to the initially guessed θ_i . Fig. 4 shows the results of (4) at the conditions which $R = r = 50[\text{mm}]$, $L_1 = L_2 = L_3 = 100[\text{mm}]$, $S_1 = S_2 = S_3 = 0$,

and $-180^\circ \leq \theta_i \leq 180^\circ$ for $i = 1, 2, 3$. We know that sets of (θ_1, θ_2) , (θ_2, θ_3) , and (θ_3, θ_1) satisfying form a certain shape of band respectively. These shapes mean that there exist multiple solutions for the forward kinematics of the proposed mechanism.

Fig. 3. Existence range of the forward kinematic solution

(a) $f_1(\theta_1, \theta_2) = 0$

(b) $f_2(\theta_2, \theta_3) = 0$

(c) $f_3(\theta_3, \theta_1) = 0$

Fig. 4. Graphs of multiple existence of the forward kinematic solution

A series of numerical analysis represent the existence of the eight distinct forward kinematic solutions as shown in Fig. 5.

As noted earlier, fast computation of kinematics is very important to control a parallel manipulator in real

time. In this paper, we developed the kinematic model on a personal computer with a 80486-DX2 66MHz CPU. The inverse and forward kinematics take approximately 2 miliseconds. These results are fast enough to allow us to implement a real-time control of the proposed manipulator with a good frequency response.

Fig. 5. Configurations of eight distinct forward kinematic solutions

4. Work Space Analysis

Generally, the work space of the Stewart platform type manipulator forms a certain type of an umbrellar whose sectional area is changing according to the height[14]. This characteristics may restrict the possible applications of the platform for various tasks. The proposed manipulator has an uniform workspace without

variation of the sectional area, although its height varies. The workspace always forms a shape of a hexagonal pole irrespective of the variation of the ratio ρ . Fig. 6 shows the comparison of the workspaces of the two manipulators with $R = 50$ [mm], $\rho = 1$, $0 \leq L_i \leq 1.2R$, and $-0.1R \leq S_i \leq 0.1R$ for $i = 1, 2, 3$.

Fig. 6. Comparison of the workspaces for the Stewart platform and the proposed manipulator in case of $\rho = 1$

Fig. 7. Variation of workspace volume by varying the actuating lengths of the links d and the ρ

The relationship between ρ and the volume of the workspace by varying the actuating lengths of the links is shown in Fig. 7. Clearly, as ρ decreases, larger workspace is obtained.

5. Manipulability and Resistivity

Manipulability measure w was proposed to measure quantitatively the ability to change the position and orientation of the end-effector from the view point of the kinematics[13]. Since Jacobian matrix J of parallel manipulators is derived from the inverse kinematics, it is the inverse one of the serial ones as follows:

$$F = \begin{bmatrix} f_c \\ \tau_c \end{bmatrix} = J^T \tau \quad (14)$$

where f_c, τ_c are the force and moment of the end-effector, respectively.

As the proposed manipulator has no redundant degree-of-freedom, w is represented as follows:

$$w = \left| \det(J^{-1}) \right|. \quad (15)$$

The concept of resistivity measure, w_R , of robot manipulators was suggested as a quantitative measure of their ability in resisting the externally applied forces[13]. The resistivity measure for the proposed manipulator can be written as follows:

$$w_R = \frac{1}{w} = \frac{1}{\left| \det(J^{-1}) \right|}. \quad (16)$$

Fig. 8. Manipulability and resistivity with respect to ρ

Since the manipulability and resistivity are functions of the Jacobian, their values are changed according to configuration of the manipulator. For a configuration with $X_c = 0$, $Y_c = 0$, $Z_c = 60$ [mm] and the orientation angles of the top platform $\alpha = 0^\circ$, $\beta = 5^\circ$, $\gamma = 0^\circ$, the simulation results by varying ρ are shown in Fig. 8.

It is found from the results of Fig. 8 that ρ less than 1 results in rapid increase of the manipulability, while ρ greater than 1 results in monotonously decreasing magnitude of the manipulability. But, the resistivity shows the reverse trends compared with the case of the manipulability. Therefore, the manipulator has to be

designed as $\rho < 1$ for good manipulation. But for large stiffness, it must be designed as $\rho > 1$.

6. Design of the Proposed Mechanism

We developed a prototype of the proposed mechanism based on the design method described above. The mechanism will be used as a wrist of a robot for the probing task. The probing task requires large workspace, free manipulation, and high capability of resisting applied forces. In considering design tradeoffs among abovementioned workspace, manipulability, and resistivity, it is decided that ρ would be unity to compromise such considerations. The specification of the designed mechanism are $R = r = 15$ [mm], $S_i = 5$ [mm] and $L_i = 90 \pm 2.5$ [mm] for $i = 1, 2, 3$. In addition, these specification satisfy the condition of the existence of the forward kinematic solution.

Each joint actuator for the proposed mechanism utilizes the Lorentz force: the force generated by a current-carrying conductor in a static magnetic field as shown in Fig. 9. The designed actuator has various features as a fast response, a minimal mass, and a linearizable force generation[12]. The measured force in motion direction of the actuator 0.8 N/A. The position of the actuator is measured by an optical sensor. This sensor is composed of a diode laser, two mirrors and a PSD (position sensing device). The position sensing resolution of this device is approximately $\pm 5 \mu\text{m}$. To reflect accurately the applied forces on the links of the mechanism, an one dimensional force sensor is installed at each actuating link. The force sensor provided reading upto 10 N with a resolution of 2.22×10^{-4} N.

Fig. 9. The joint actuator with an optical sensing mechanism

7. Conclusions

We have presented a kinematic analysis of the developing 3 PRPS type parallel manipulator for the probing task application. The conditions for existence of the forward kinematic solutions are defined in terms of link displacements. The quantity ρ can be used as a design parameter for determining the link lengths satisfying the conditions. The forward kinematic

solution is not uniquely determined and eight distinct solutions exist according to the initially guessed quantities θ_i . The developed manipulator has the workspace which forms the shape of the hexagonal pole without change of the sectional area. ρ (the ratio of the top plate and the base plate size) is also an important design parameter for determining the workspace volume. We designed a parallel wrist that has a desired workspace, manipulability, and resistivity for the probing task, and built it as a prototype. Each joint actuator for the proposed mechanism is an electromagnetic linear actuator that has a fast response and a linearized force generation suitable for the probing task. Future research will be directed towards the experimental investigation for active contact control.

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