Performance Analysis of Pulse Designed for the IEEE 802.15.4a standard

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Abstract—The novel pulses, which not only meet the power spectral mask of the Federal Communications Commission (FCC) and IEEE 802.15.4a standard constraint but also preserve the orthogonality at the receiver, have been proposed by Kim et al. for practical usage of Ultra-wideband (UWB) system. This paper analyzes the performance of proposed pulses with regards to important conditions in UWB system, such as the standard constraints, spectral utilization efficiency, capacity and effect on multi-path fading with different existing pulses. Simulation results demonstrate the performance analysis of the proposed pulses.

Index Terms—Ultra-wideband (UWB) system, pulse shape, IEEE 802.15.4a.

I. INTRODUCTION

Impulse Radio Ultra-wideband (IR-UWB) technology has been considered as a promising candidate for short-range indoor wireless communications due to its attractive features, such as low duty cycle, immunity against multipath fading and precise ranging/positioning capability [1]. However, conveying information over ultra-short pulses results in an extreme broad bandwidth, which inevitably overlays the spectrum occupied by many existing narrowband devices such as global positioning system (GPS) and wireless local area network (WLAN) terminals. To regulate the co-existence, Federal Communications Commission (FCC) has released the spectral mask containing power emission limits for all kinds of IR-UWB devices. In this regard, the generation of pulse shapes complying with the mandated spectral mask has become one of the key issues in IR-UWB research [2].

In addition, the ranging capability of IR-UWB has been particularly emphasized for location-aware applications in ad-hoc and sensor networks, which led to the introduction of the IEEE 802.15.4a standard. As a specification of the baseband transmitter impulse response in the standard, the transmitted pulse shape is constrained by the magnitude of its cross-correlation function with a reference pulse, the root-raised-cosine (RRC) pulse. Thus, such constraints need to be satisfied when the pulse shape for IR-UWB is designed for applying the indoor geolocation system.

However, most of the research on pulse shape has focused on meeting the FCC spectral mask, while few studies have been conducted to satisfy the IEEE 802.15.4a standard. Therefore, as a preliminary work in this regard, the novel pulse design strategy which generate multiple orthogonal pulses satisfying the IEEE 802.15.4a standard and FCC spectral mask is proposed by Kim et al.. In summary, the modified root-raised cosine (MRRC) pulse was developed as a single pulse satisfying both constraints. By modulating the MRRC pulse with orthogonal bases generated by the Gram-Schmidt orthonormal process, the multiple pulses were established. In order to maintain the orthogonality of proposed multiple pulses at the receiver, an orthogonal design method using the QR decomposition method was proposed.

As a middle step of the location performance analysis according to pulse shapes, in this paper, we analyze the performance of proposed pulses with exiting UWB pulses in various environment. The effectiveness of proposed pulses are verified in terms of standard constraint, spectral efficiency and multi-path effect.

The rest of the paper is organized as follows. Section II summaries the proposed pulse shape design algorithm that generates multiple orthogonal pulses satisfying both the IEEE 802.15.4a constraint and the FCC spectral mask. In Section III, the comparison of performance with established pulse and existing UWB pulses are performed in various conditions. Finally, Section IV concludes the paper.

II. THE PROPOSED PULSE SHAPE DESIGN

In this paper, an algorithm is proposed to generate multiple orthogonal pulses which conform not only to the IEEE 802.15.4a standard but also to the FCC spectral mask. The design of the orthogonal pulses is accomplished by the establishment of the initial pulse satisfying both constraints and then by multiple orthogonal pulse generation via Gram-Schmidt polynomial and QR decomposition. The detail process for pulse design is illustrated as follows:

• Step 1: Establishment of the basic pulse

According to the IEEE 802.15.4a transmitter specification, the transmitted pulse should have a magnitude of cross-correlation with the reference pulse greater than or equal to 0.8 for the main lobe and less than 0.3 for any side lobes. Without loss of generality, the RRC function is employed as a basic pulse shaper for the IEEE 802.15.4a standard; it is straightforward that RRC pulse has maximum correlation value with own pulse in the standard and is relatively easy to implement in the practical system. By considering the carrier frequency for FCC spectral mask and pulse duration, \( T_p \), for meeting the Nyquist

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basis function representation as follows:

\[ p_M(t) = \cos(2\pi f_c t) \frac{4\beta}{\pi \sqrt{T_p}} \cos\left((1 + \beta)\pi t/T_p + \frac{\sin((1-\beta)\pi t/T_p)}{4\pi t/T_p}\right) \]

where \( \beta \) is the roll-off factor of \( p_M(t) \).

- Step 2: Generation of multiple pulses

The multiple pulses can be generated by modulating \( p_M(t) \) with multiple basis functions derived from the Gram-Schmidt polynomials. A tractable formulation of the recursive polynomial is derived as follows:

\[ \psi_{n+1}(t) = t\psi_n(t) - \left( \frac{n^2}{(2n)^2 - 1} \right) \psi_{n-1}(t)T_p^2. \]

With the given basis function \( \{\psi_0(t), \cdots, \psi_{N-1}(t)\} \), the multiple transmitted pulses \( p_n(t) \) are generated in terms of \( n \)-th basis function representation as follows:

\[ p_n(t) = \mu_n \cdot p_M(t) \cdot \psi_n(t), \]

where \( \mu_n \) is the normalization coefficient of \( p_n(t) \).

- Step 3: Orthogonalization

In order to maintain the orthogonality at the receiver, an orthogonalization approach based on QR factorization is employed. Suppose that the receiver generates the same template pulses as \( N \) transmitted pulses, the output of the correlator bank can be represented by a \( N \)-by-\( N \) correlation matrix \( C \). Using the QR decomposition i.e., \( C = QR \), the corresponding output vector of correlator becomes the \( n \)-th row of \( Q \) represented as

\[ Q = CR^{-1} = [q_0 \ q_1 \ \cdots \ q_n \ \cdots \ q_{N-1}]^T, \]

where

\[ q_n = \begin{bmatrix} q_{n,0} & q_{n,1} & \cdots & q_{n,m} & \cdots & q_{n,N-1} \end{bmatrix}, \]

\[ q_{n,m} = \int_{T_p} p_n(t)w_m(t)dt, \]

\[ w_m(t) = \sum_{n=0}^{N-1} r_{n,m}p_n(t), \]

where \( w_m(t) = \sum_{n=0}^{N-1} r_{n,m}p_n(t) \) is the weighted mask pulse and \( r_{n,m} \) is the \( (n,m) \)-th element of \( R^{-1} \) and the superscript \( T \) denotes the matrix transpose operation. This procedure makes the receiver maintain the orthogonality without loss of effort to satisfy the restrictions.

### III. Numerical Performance Analysis and Discussion

In this section, the performance analysis and discussion to verify the effectiveness of proposed pulses are represented with existing UWB pulses. As general UWB pulses, the 5-th Gaussian pulse which is one of designed pulses to maximize the power efficiency [4] and multiple orthogonal pulses such as modified Hermite polynomial (MHP) pulse [5] and prolate spheroidal (PS) pulse [6], [7], are considered. From now on \( \phi_0(t), \phi_1(t) \) and \( \phi_2(t) \) are redefined as the proposed pulses with respective polynomial order \( n = 0, 2, 4 \) in (3).

#### A. Validation for standard constraint

The proposed pulses focus on the IEEE 802.15.4a standard to use the UWB signal in practical indoor navigation. In the standard, the transmitted pulse shape is constrained by the magnitude of its cross-correlation function with a reference pulse, the RRC pulse. Fig. 1 shows that the cross-correlation values of the proposed pulses with the RRC pulse are greater than or equal to 0.8 within the main lobe while being smaller than 0.3 in their side lobes. In contrast, MHP and PS orthogonal pulses as well as the 5-th derivative Gaussian pulse, do not satisfy the standard constraint.

#### B. Spectral efficiency

The efficient spectral utilization is one of the key objectives of pulse design and it was researched in many literatures [2], [8], [9]. Although Giannakis et al. proposed exploiting the digital FIR filter design solution and Zhang et al. proposed fine-tuning higher-order derivatives of the Gaussian pulse for the high spectral efficiency, both references did not consider the constraint released by IEEE 802.15.4a standard. On the other hand, we wish to focus on generating multiple orthogonal pulses satisfying both standard and FCC constraints, no getting only high spectral efficiency. Therefore, our proposed pulses might be inferior to [2], [8], [9] in terms of spectral utilization due to the pulse duration/bandwidth restriction for satisfying the standard constraint, but only the our proposed pulses comply with the standard constraint. We verify the corresponding result by using the NESP (Normalized Effective Signal Power) metric, which is the ratio of
the power transmitted in the designated passband $F_p$ of the spectral mask over the total power that is permissible under given mask. The NSEP is defined as follows [2]:

$$\psi = \frac{\int_{F_p} P_p(f) df}{\int_{F_p} P(f) df},$$

(6)

where, $P_p(f)$ is a spectrum of pulse shape $p(t)$ within designated passband $F_p$ and $P(f)$ is the spectrum defined in FCC.

From the Table I, we can show that only our proposed pulse complies with the standard constraint even though it has performance loss than that of conventional orthogonal pulse, MHP and PS pulse. Interestingly, if we do not consider standard constraint, our proposed pulse has much higher spectral efficiency than that of general orthogonal pulses, and it has small performance gap with 5-th derivative Gaussian pulse as a case of $(\phi_0$ and $\phi_1$) as well as a case of $(\phi_0$ and $\phi_2)$ can enhance data rate and we believe that our proposed pulses can be used for required higher data rate system in real UWB system.

**D. Multi-path path fading effect**

In this section, we present the performance analysis of proposed single in multiplath fading environment. For the simplicity, single pulse case is considered in two-path model. Let’s assume that the overlapped multipath components within one pulse duration are not resolvable. The impact of the overlapped multipath components on the employed correlation receiver is investigated with a simplified two-path model, which can be expressed as follows:

$$r(t) = \sqrt{\frac{E_b}{N_f}} \sum_{n_f=0}^{N_f-1} [s\phi_0(t-n_fT_f)] + n(t)$$

(8)

where $\mu_2 \leq T_p$ is assumed to be a random variable uniformly distributed on $[0, t_m]$, and $t_m$ represents the maximum delay. When the correlation mask is generated as $m(t) = \alpha_0\phi_0(t-\tau_0) + \alpha_1\phi_0(t-\tau_0 - \mu_2)$, the corresponding output of correlator with each frame duration, $x(t)$, is represented as follows:

$$x(n_f) = \int_{(n_f-1)T_f}^{n_fT_f} r(t)m(t)dt$$

$$= \sqrt{\frac{E_b}{N_f}} \sum_{n_f=0}^{N_f-1} s[\alpha_0^2 R(0) + \alpha_1^2 R(0) + 2\alpha_0\alpha_1 R(\mu_2)] + \eta(n_f),$$

(9)

where $R(\cdot)$ is the autocorrelation function of $\phi_0(t)$ and the filtered noise $\eta(n_f)$ is

$$\eta(n_f) = \int_{(n_f-1)T_f}^{n_fT_f} n(t)m(t)dt$$

(10)

$$= \int_{(n_f-1)T_f}^{n_fT_f} n(t)\alpha_0\phi_0(t-n_fT_f-\tau_0)$$

$$+ \alpha_1\phi_0(t-n_fT_f-\tau_0 - \mu_2)dt.$$
TABLE I
NORMALIZED EFFECTIVE SIGNAL POWER OF VARIOUS UWB PULSES

<table>
<thead>
<tr>
<th>Transmitted pulse</th>
<th>Normalized Effective Signal Power</th>
<th>Standard constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed pulse (\phi_m(t))</td>
<td>21.62 %</td>
<td>Satisfy</td>
</tr>
<tr>
<td>PS pulse</td>
<td>25.62 %</td>
<td>Do not satisfy</td>
</tr>
<tr>
<td>MHP pulse</td>
<td>26.39 %</td>
<td>Do not satisfy</td>
</tr>
<tr>
<td>Proposed pulse (Do not consider standard)</td>
<td>43.19 %</td>
<td>Do not satisfy</td>
</tr>
<tr>
<td>5-th derivative Gaussian pulse</td>
<td>51.94 %</td>
<td>Do not satisfy</td>
</tr>
</tbody>
</table>

TABLE II
ANALYSIS OF CAPACITY WHEN \(E_b/N_0 = 10dB\)

<table>
<thead>
<tr>
<th>Transmitted pulse</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single pulse ((N = 1, \epsilon_s = 1))</td>
<td>3.4594</td>
</tr>
<tr>
<td>B-PSM with (\phi_0) and (\phi_1) ((N = 2, \epsilon_s = 1.4471))</td>
<td>6.0511</td>
</tr>
<tr>
<td>B-PSM with (\phi_0) and (\phi_2) ((N = 2, \epsilon_s = 1.0370))</td>
<td>6.8237</td>
</tr>
<tr>
<td>B-PSM with MHP (n = 1) and (n = 2) ((N = 2, \epsilon_s = 1.00))</td>
<td>6.9189</td>
</tr>
</tbody>
</table>

With the simple notation \(t - n_f T_f - \tau_0 = \tilde{t}_0, t - n_f T_f - \tau_0 - \mu_2 = \tilde{t}_1\), the variance of \(\eta(n_f)\) is given by:

\[
\text{var} [\eta(n_f)] = \text{var} [n(t)\alpha_0\varphi_0(\tilde{t}_0)] + \text{var} [n(t)\alpha_1\varphi_0(\tilde{t}_1)] + 2\text{cov} (n(t)\alpha_0\varphi_0(\tilde{t}_0), n(t)\alpha_1\varphi_0(\tilde{t}_1))
\]

(11)

\[
= \alpha_0^2 N_0^2 + \alpha_1^2 N_0^2 + 2\alpha_0 \alpha_1 N_0^2 \text{cov} (\varphi_0(\tilde{t}_0), \varphi_0(\tilde{t}_1))
\]

where the covariance function of two noise variables

\[
\text{cov} (n(t)\alpha_0\varphi_0(\tilde{t}_0), n(t)\alpha_1\varphi_0(\tilde{t}_1)) = \alpha_0 \alpha_1 \int_{T_f} \int_{T_f} n(x) n(y) \varphi_0(\tilde{t}_0) \varphi_0(\tilde{t}_1) dx dy
\]

Therefore, the variance of filtered noise is still a white Gaussian noise and it has zero mean and the following variance:

\[
\text{var} [\eta(n_f)] = \frac{N_0}{2} \left( \alpha_0^2 + \alpha_1^2 + 2\alpha_0 \alpha_1 \text{R}(\mu_2) \right).
\]

(13)

Since the \(t_m\) is a random variable, the expected desired signal term from (9) and noise variance can be derived as shown in

\[
E [s] = \sqrt{\frac{E_b}{N_f}} R
\]

(14)

\[
\text{var} [\eta(n_f)] = \frac{N_0}{2} \epsilon_R
\]

(15)

where the non resolvable energy is \(\epsilon_R = \alpha_0^2 + \alpha_1^2 + 2\alpha_0 \alpha_1 \text{R}(\mu_2)\). Therefore, the average BER is given by

\[
P_e(x < 0|s = 1, \{\alpha\}) = Q \left( \sqrt{\frac{N_f E_b \epsilon_R^2}{N_f N_0/2 \epsilon_R}} \right)
\]

(16)

Therefore, BER performance depends on the pulses and their autocorrelation value of \(R(\mu_2)\), when an equal mean power is assumed for all different waveforms. Fig. 3 shows the autocorrelations, which are numerically generated and normalized for 5-th Gaussian derivative pulse, MHP pulse with \(n=1\) and the two proposed pulses \(\phi_0\) and \(\phi_1\). From that figure, it is observed that the \(R(\cdot)\) of proposed pulse \(\phi_0\) decays slower than that of the other pulses. It is known that pulses having fast-decaying autocorrelation functions are desirable to mitigate the interference, even though those pulses are more sensitive to timing jitter. The BER performance over multipath fading is presented in Fig. 4. Fig. 4 shows that the performance of \(\phi_0\) has some performance loss comparing with other pulses due to the larger effect of interference.

IV. CONCLUSIONS

In this paper, we analyze the performance of the pulses designed for IEEE 802.15.4a standard in various conditions, such as the standard constraints, spectral utilization efficiency, capacity and effect on multipath fading with different existing pulses. According to the performance comparison, it is shown that only the proposed pulses satisfy the IEEE 802.15.4a, and the performance of proposed pulses are varied by pulse order and pulse duration for standard restriction.
Fig. 3. Normalized autocorrelation of considered pulses

Fig. 4. BER performances over multipath fading

REFERENCES


