VoIP Capacity Analysis in Cognitive Radio System

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Abstract—In this letter, we analyze a voice over IP (VoIP) capacity in a cognitive radio system. We formulate the system as a two-dimensional discrete time Markov chain (DTMC). The VoIP traffic and wireless channel in the cognitive radio system are described as a Markov modulated Poisson process (MMPP) model and a Markov channel model, respectively. We demonstrate various numerical and simulation results, such as packet dropping probability and VoIP capacity.

Index Terms—Cognitive radio, VoIP service, Markov modulated Poisson process, discrete-time Markov chain.

I. INTRODUCTION

COGNITIVE radio is a promising and challengeable technology for maximizing radio resource utilization in a future wireless communication system because conventional systems exploit most available frequency bands for wireless communications and these frequency bands are not always fully utilized in general [1]. In addition, VoIP traffic (VoIP) will be an essential service in the future because, through VoIP technology, wireless users can utilize voice services more cheaply. Therefore, supporting as many voice users as possible while using limited radio resources is a very important issue that could be a key to the success of the future systems [2][3].

However, they did not consider the cognitive radio system. Also, in [5], Q. Bi et al. analyzed the VoIP capacity of 1xEV-DO system. In this letter, we analyze VoIP capacity in a cognitive radio system through a queuing model based on the MMPP traffic model and a Markov channel model. Here, VoIP packets can be transmitted when the wireless channel is not utilized by a primary user. To the best of our knowledge, the VoIP capacity analysis has not been studied yet in the cognitive radio system.

The remainder of this letter is organized as follows. In Section II, we present the VoIP traffic model based on a Markov modulated Poisson process (MMPP) traffic model to analyze VoIP performance. However, they did not consider the cognitive radio system. In Section II, we present the VoIP traffic model based on a Markov modulated Poisson process (MMPP) traffic model to analyze VoIP performance. In Section III, we analyze the VoIP capacity in a cognitive radio system. In Section IV, we demonstrate numerical and simulation results and conclude this letter.

II. VOIP TRAFFIC AND CHANNEL MODELING

1) VoIP Traffic Modeling: In general, we can formulate the VoIP traffic of a single user as a simple on-off model. The probability that the status of the users is inactive (= off) in the simple on-off model can be obtained by

\[ p_{\text{off}} = \beta^{-1}/(\alpha^{-1} + \beta^{-1}) \]

and \( p_{\text{on}} = 1 - p_{\text{off}} \). Here, \( 1/\alpha \) and \( 1/\beta \) are the mean values of the on and off periods which are distributed exponentially. Furthermore, all the traffic generated by the VoIP users in the cell can be modeled as a two-state MMPP model [6]. This MMPP model is highly suitable for formulating the multi-user VoIP traffics because the MMPP captures the interframe dependency between consecutive frames. Here, the transition rate matrix \( (R) \) and the Poisson arrival rate matrix \( (A) \) of the MMPP can be expressed as follows:

\[
R = \begin{bmatrix}
r_1 & r_2 \\
r_2 & -r_1 
\end{bmatrix}, \quad A = \begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_2 
\end{bmatrix}. \tag{1}
\]

In order to utilize the MMPP model, we should match the MMPP parameters \( (r_1, r_2, \lambda_1, \lambda_2) \) in Eqn. (1) with the parameters of the simple on-off model \( (\alpha, \beta) \). We here adopt the IDC (index of dispersion for counts) matching technique because it yields adequate results for matching parameters and has appropriate computation complexity compared with other matching techniques [4]. Then, \( r_1, r_2, \lambda_1, \lambda_2 \) in Eqn. (1) can be calculated by

\[
r_1 = \frac{2(\lambda_2 - \lambda_{\text{avg}})(\lambda_{\text{avg}} - \lambda_1)^2}{(\lambda_2 - \lambda_1)^2}, \quad r_2 = \frac{2(\lambda_2 - \lambda_{\text{avg}})^2(\lambda_{\text{avg}} - \lambda_1)}{(\lambda_2 - \lambda_1)^2}, \quad \lambda_1 = A \cdot \frac{\sum_{j=0}^{N_{\text{act}}=1} \pi_j}{\sum_{j=0}^{N_{\text{act}}=2} \pi_j}, \quad \lambda_2 = A \cdot \frac{\sum_{j=0}^{N_{\text{act}}=2+1} \pi_j}{\sum_{j=0}^{N_{\text{act}}=2} \pi_j}.
\]

Here, \( N \) is the total number of VoIP users in the system, and \( A \) is the emission rate in the on-state \( (A = \frac{1}{T_{\text{basic}}}) \). \( T_{\text{basic}} \) is a frame duration of voice codec, and the average arrival rate is \( \lambda_{\text{avg}} = N \times A \times p_{\text{on}} \). Also, the average number of active users is \( N_{\text{act}}=1 = \lfloor N \times p_{\text{on}} \rfloor \), and the steady-state probability of an one-dimensional Markov chain when considering \( N \) independent simple on-off voice users can be calculated by

\[ \pi_i = \sum_{j=N \times p_{\text{on}}}^{N-1} C_i \cdot p_{\text{on}} \cdot (1 - p_{\text{on}})^{N-i}. \]

Moreover, \( IDC(\infty) \) is given in [6].

2) Channel Modeling in Cognitive Radio System: In a cognitive radio system, a wireless channel can be modeled as a two-state Markov process, as shown in Fig. 1 [7]. An occupied state means that the wireless channel is utilized by a primary user. Given that the channel status is ‘Occupied’, the cognitive user cannot use the channel. In this letter, we assume that there are ‘\( M \)’ wireless channels. Then, the transition

\[ \begin{bmatrix}
(p_{00}) & (p_{01}) \\
(p_{10}) & (p_{11}) 
\end{bmatrix} \]

\[ \begin{bmatrix}
\pi_0 & \pi_1 
\end{bmatrix} \]

is given in [6].

Fig. 1. Markov channel model for cognitive radio system.
probability \(P_{m,n}\) that there are \(m\) unoccupied channels \((x_c)\) in the current frame, and there will be \(n\) unoccupied channels in the next frame can be represented by

\[
P_{m,n} = \sum_{x'=\max(0,m-n)}^{\min(m,M-n)} \binom{m}{x'} P_{01}^{x'} P_{00}^{m-x'} (M-m) P_{10}^{y'} P_{11}^{M-m-y'}. \tag{2}
\]

Here, \(y' = n + m - x'\), where \(x'\) and \(y'\) denote the numbers of channels whose status are altered from ‘Unoccupied’ to ‘Occupied’ and from ‘Occupied’ to ‘Unoccupied’, respectively.

### III. System Modeling

1) Assumptions: Perfect information for channel occupancy of primary users is available in the base station (BS), and one VoIP packet can be transmitted through one unoccupied wireless channel. If all the channels are used by the primary users, the VoIP packets cannot be sent. Also, we assume that there are no packet retransmissions. That is, unreceived and destroyed packets are not sent again.

2) System Modeling: We can formulate a multi-user queuing model for VoIP services in a cognitive radio system as a two-dimensional DTMC. In other words, we can analyze the behavior of queuing packets in the BS with this DTMC model. Then, the transition matrix \((P)\) can be defined as

\[
P = \begin{bmatrix}
A_{0,0} & A_{0,1} & \cdots & A_{0,l_{max}} \\
A_{1,0} & A_{1,1} & \cdots & A_{1,l_{max}} \\
\vdots & \ddots & \ddots & \vdots \\
A_{l_{max},0} & A_{l_{max},1} & \cdots & A_{l_{max},l_{max}}
\end{bmatrix}
\tag{3}
\]

where \(l_{max}\) is maximum queue length and submatrix \((A_{i,j})\) is expressed as

\[
A_{i,j} = \begin{bmatrix}
B_{(i,0),(j,0)} & B_{(i,0),(j,1)} & \cdots & B_{(i,0),(j,M)} \\
B_{(i,1),(j,0)} & B_{(i,1),(j,1)} & \cdots & B_{(i,1),(j,M)} \\
\vdots & \ddots & \ddots & \vdots \\
B_{(i,M),(j,0)} & B_{(i,M),(j,1)} & \cdots & B_{(i,M),(j,M)}
\end{bmatrix}
\tag{4}
\]

In Eqn. (4), \(A_{i,j}\) represents the variation of the number of queuing packets. That is, there are \(i\) packets in the current frame, and there will be \(j\) packets in the next frame. In \(A_{i,j}\), each element \((B_{(i,m),(j,n)})\) indicates the transitions between the numbers of unoccupied channels when the number of queuing packets is changed from \(i\) to \(j\). Also, \(B_{(i,m),(j,n)}\) is a 2-by-2 matrix because our MMPP model has two phases, such as underloading and overloading. When the number of queuing packets is \(i\), given that \(k\) packets are scheduled, \((j - \max(i-k,0))\) packets should arrive so that the number of packets becomes \(j\). Hence, \(B_{(i,m),(j,n)}\) can be calculated by Eqn. (5). Here, \(P_s(k \mid x_c = m)\) is the probability that the BS serves \(k\) packets when the number of unoccupied channels is \(m\). In this letter, since we assumed that one VoIP packet can be transmitted through one unoccupied wireless channel, \(P_s(m \mid x_c = m) = 1\). In addition, in Eqn. (5), \(U\) and \(D(m)\) are the transition probability matrix and the diagonal probability matrix of the two-state MMPP model [4]. Each element of \(D(m)\) means the probability that \(m\) VoIP packets arrive at the BS during the MAC frame duration \((T_f)\) in each phase of the two-state MMPP.

Through the transition matrix \((P)\) in Eqn. (3), we can obtain the steady-state probability matrix \((\pi)\) for our two-dimensional DTMC model, which can be calculated by solving equations \(\pi \cdot P = \pi\) and \(\pi \cdot [I - \pi] = 1\). Here, \(\pi\) is ‘1’ by ‘\(2 \times (M + 1) \times (l_{max} + 1)\)’ matrix. Therefore, the probability that \(k\) VoIP packets are queued in the BS can be expressed as

\[
\pi(k) = \sum_{i=0}^{2(M+1)-1} \pi_p(2 \cdot (M + 1) \cdot k + i). \tag{6}
\]

Through Eqn. (6), the ‘1’ by ‘\((l_{max} + 1)\)’ steady-state probability matrix of the DTMC model can be obtained as follows: \(\pi = [\pi(0) \pi(1) \pi(2) \cdots \pi(l_{max})]\). Therefore, by using this steady-state probability, we can present various analytical results, such as the average number of arrived packets, the average queue-length, the average number of serviced VoIP packets, the average VoIP throughput, and the packet dropping probability. First, the average queue-length \((L_{avg})\) and the average arrival rate \((\rho)\) can be calculated by

\[
L_{avg} = \sum_{i=0}^{l_{max}} i \cdot \pi(k). \tag{7}
\]

\[
\rho = s \cdot \left( \sum_{m=0}^{N \times A_{max}} m \cdot D(m) \right) \cdot 1. \tag{8}
\]

In Eqn. (8), \(s\) is calculated by solving \(s \cdot U = s\), and 1 is a column matrix of ones. Also, \(A_{max}\) is the maximum number of packets that can arrive from each VoIP user during \(T_f\). Similar to the average arrival rate, the average number of serviced VoIP packets \((\kappa_{avg})\) can be expressed as

\[
\kappa_{avg} = \sum_{i=0}^{l_{max}} \sum_{j=0}^{M} \sum_{k=0}^{M} \min(j,k) \cdot \pi(k) \cdot P_s(j \mid x_c = i) \cdot \pi_{ch}(i). \tag{9}
\]

Here, \(\pi_{ch}(i)\) is the steady-state probability that the number of unoccupied channels are \(i\), and can be expressed as

\[
\pi_{ch}(i) = \left( \frac{M}{i} \right) \left( \frac{p_{01}}{p_{01} + p_{10}} \right)^i \left( \frac{p_{01}}{p_{01} + p_{10}} \right)^{M-i}. \tag{10}
\]

Moreover, the average throughput is represented by \(S_{avg} = \kappa_{avg} \times I_{PDU}\). Here, \(I_{PDU}\) is the size of VoIP PDU. Also, we can calculate the dropping probability of VoIP packets as follows: \(P_{drop} = 1 - \kappa_{avg}/\rho\). Hence, by using \(P_{drop}\), we can define the VoIP capacity as following

\[
C_{VoIP} = \arg \max N \in \{ N \mid 1 - \kappa_{avg}/\rho \leq P_{imit} \}. \tag{11}
\]

Here, \(P_{imit}\) is the upper threshold of the packet dropping probability for VoIP services.

### IV. Numerical and Simulation Results

We evaluated the VoIP performance of the cognitive radio system by using MATLAB. We included all the essential factors required for performance evaluation, such as round-robin...
In addition, the average throughput grows linearly according to variation of channel occupancy probability (pₒ). In this letter, we assume that Pₒlim = 0.02. Here, pₒ means the probability that the primary users utilize wireless channels, and also denotes the steady-state probability when channel status is ‘Occupied’. For a low loading condition, the MMPP based numerical results have lower packet dropping probabilities than the on-off based simulation results mainly due to the characteristic of the IDC matching technique [4].

Moreover, given that the total number of channels is 30, the overhead caused by control signaling is 30% [8], and pₒ = 0.5, we can obtain both downlink and uplink capacity through our analysis method. As shown in Table I, when DL/UL frame ratio is 1:1, we can show that the VoIP capacity is restricted by the downlink owing to the overhead caused by control signaling. However, when the DL/UL ratio is 2:1, the capacity is limited by the uplink due to the fact that the size of uplink frame is much smaller than that of downlink frame. Therefore, we can conclude that the VoIP capacity is determined by bottleneck-link, which can be different according to system parameters such as the DL/UL ratio.

Table I

<table>
<thead>
<tr>
<th>Capacity</th>
<th>DL/UL = 1:1</th>
<th>DL/UL = 2:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downlink capacity</td>
<td>13</td>
<td>24</td>
</tr>
<tr>
<td>Uplink capacity</td>
<td>33</td>
<td>21</td>
</tr>
</tbody>
</table>

The VoIP capacity can be varied according to the variation of channel-state transition probability, as shown in Fig.4. Here, the steady-state transition probabilities of the Markov channel models for all the cases are the same. However, if the pₒ1 and pₒ0 are larger, the state transitions occur more frequently. Thus, the VoIP capacity could be smaller. On the other hand, we can apply our formulation to VoIP admission control in the cognitive radio system. If the channel occupancy variations are dynamic, we can adjust the maximum supportable number of users based on the results demonstrated in this letter.

Furthermore, given that the total number of channels is 30, the overhead caused by control signaling is 30% [8], and pₒ = 0.5, we can obtain both downlink and uplink capacity through our analysis method. As shown in Table I, when DL/UL frame ratio is 1:1, we can show that the VoIP capacity is restricted by the downlink owing to the overhead caused by control signaling. However, when the DL/UL ratio is 2:1, the capacity is limited by the uplink due to the fact that the size of uplink frame is much smaller than that of downlink frame. Therefore, we can conclude that the VoIP capacity is determined by bottleneck-link, which can be different according to system parameters such as the DL/UL ratio.

References


