Analysis of Halbach magnet array 
and its application to linear motor

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Abstract

Using Halbach magnet array, magnetic flux can be enhanced on one side (strong side) of the array while the flux cancelled on the other side (weak side). Inherently, rotor of rotary motor has infinite rotational length with respect to its rotation while mover of the linear motor has finite length with respect to mover’s translation. In this paper, we propose and validate a design method. By means of the method magnetic field can be calculated theoretically. General electrical motor without slotted yoke, therefore, can be designed analytically. Using the method, the linear motor with finite mover can be designed more accurately considering the flux leakage at both end of the mover. We introduce the method in designing the linear motor with Halbach magnet array. We also investigate the difference between conventional method that does not consider the finiteness and proposed one that considers it. Using the method, the magnetic field emanated from magnet can be obtained so accurately that actuating force characteristics of the motor can be predicted more precisely. So, the method is useful in designing the linear motor.

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1. Introduction

Halbach magnet array was proposed by Halbach in 1980 for undulator or electron wiggler [1]. If there is a magnet of which magnetization direction rotates continuously, magnetic flux is constructed via superposition on one side of the magnet while the flux is destructed on the other side. Since the continuously rotating magnetization is difficult to manufacture, the ideal array is approximated with segmented...
magnet [2]. The magnet array refers to (segmented) Halbach magnet array and is presented in Fig. 1 [3]. Unrolling the quadrapole, it will be understood principle of linear Halbach magnet array. We construct the linear motor as presented in Fig. 2.

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**Fig. 1.** Cross section of a 16-piece (4-segment) rare earth material quadrapole. Inner region is strong side and outer is weak. Flux line is also presented. Unrolling the quadrapole, it will be understood principle of linear Halbach magnet array. Source: [3].

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**Fig. 2.** Linear brushless DC motor with slotless yoke and Halbach magnet array: Two linear motors are located symmetrically and square air bearing supports and guides the moving actuator.
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Using Halbach magnet array, magnetic flux can be enhanced on one side (strong side) of the array while the flux cancelled on the other side (weak side). If Halbach magnet array is used in electric motor, magnetic flux density exposed to coil becomes denser. The more magnetic flux is exposed to the coil, the more actuating force of the motor is produced by Lorentz force. If the motors can generate more actuating force, they can move more rapidly. If the motors have no slot, cogging force ripple will not be generated. The force ripple is non-linearity characteristics of electric motor. Generally non-linearity is detrimental to position control the motor. In short, the electric motor with Halbach magnet array can satisfy needs of high precision positioning actuator that is used in high precision manufacturing of semiconductor industry. The needs are high speed and high accuracy.

Conventionally, electric rotary motor is used as actuator of high precision positioning system. To move the system linearly, rotary motion must be converted to translational one. However, this converting mechanism needs additional parts that make the system be nonlinear, complicate and heavy. From the reason, linear motor is widely used in precision positioning system. It is, therefore, important to analyze and design the linear motor adopted in the precision positioning system.

Inherently, rotor of rotary motor has infinite rotational length with respect to its rotation while mover of the linear motor has finite length with respect to mover’s translation. However, many researcher have designed the linear motor without regard to the finiteness. In other words, they have modelled the linear motor assuming it has mover of infinite length as rotary motor does. Without considering the finiteness of the linear motor’s mover, design error will be arisen from flux leakage at both end of the mover.

In this paper, we propose and validate a design method. By means of the method magnetic field can be calculated theoretically. General electrical motor without slotted yoke, therefore, can be designed analytically. Using the method, the linear motor with finite mover can be designed more accurately considering the flux leakage at both end of the mover. We introduce the method in designing the linear motor with Halbach magnet array. We will also investigate the difference between conventional method that does not consider the finiteness and proposed one that considers it.

Using the method, the magnetic field emanated from magnet can be obtained so accurately that actuating force of the motor can be predicted more precisely. So, the method is useful in designing the linear motor.

2. Magnetic field analysis

2.1. Magnetic field due to magnets

In order to design undulator, wiggler or electric motor, it is fundamental to analyze the magnetic field in magnet, air gap, and coil region. Magnetization of
magnet is nearly constant in the neighborhood of operating point of the electric motor if the magnet has demagnetization characteristics as depicted in Fig. 3. The magnet is made of rare earth material such as SmCo, NdGeB and so forth. Usually the permeability of the magnet presented in Fig. 3 is nearly same as the value of air, \( m_0 \). For the magnet, the demagnetization characteristics are according to Eq. (1) in the neighborhood of operating point. If the remanence of the magnet is \( B_r \), magnetization, \( M \) is \( B_r / \mu_0 \).

\[
B = \mu_0 (H + M)
\] (1)

For convenience of analysis, magnetic vector potential is introduced as Eq. (2). Applying Eqs. (1) and (2) to Maxwell’s equations, Laplace’s equation (3) is obtained. In derivation of the Eq. (3), the system is assumed to be operating in low frequency. In the equation \( M \neq 0 \) in magnet region, however, \( M = 0 \) in other region.

\[
B = \nabla \times A
\] (2)

\[
\nabla^2 A = -\mu_0 (J_f + \nabla \times M)
\] (3)

If there is an interface between medium, \( a \) and \( b \), boundary condition on the interface is given as Eqs. (4) and (5). Superscripts, \( a \) and \( b \) mean the quantities are in the medium, \( a \) and \( b \), respectively.

\[
n \cdot (B^a - B^b) = 0
\] (4)

\[
n \times (B^a - B^b) = K_f + n \times (\mu_0 M^a - \mu_0 M^b)
\] (5)

\( K_f \) is source surface current density in the interface and \( J_f \) is source current density in the concerned region. If there are only magnet, yoke, and air in the region, the above two current density is zero [4].

![Fig. 3. Demagnetization curve of rare earth magnetic material (Nd–Fe–B) at \( T = 20 \) °C \( H_c = 993 \) kA/m, \( Br = 1.23 \) T, \( (BH)_{\text{max}} \approx 305 \) GT A/m. Source: [9].](image-url)
To solve the field equation (5), it is assumed that there is no variation of all the quantities in the direction perpendicular to paper and permeability of stator yoke is infinite. The magnetic flux has no tangential component on the interface between a medium with infinite permeability and the other one with finite one.

Superposition principle can be applied on condition that every material’s properties are linear and the concerned region is under same boundary conditions. In order to obtain the magnetic field due to the array of segmented magnets, therefore, it is needed to calculate magnetic field due to each magnet firstly, and then to superpose the field. So, the magnetic field due to a single magnet will be given in following description.

A concerned analytical model of single magnet and stator yoke is presented in Fig. 4(a) and (b). The length of mover i.e. magnet array is finite with respect to $x$-directional motion of the linear motor in contrast with the length of stator yoke which has infinite length with respect to the motion. Fig. 5(a) and Fig. 6(a) are analytical models of single magnet with $y$- and $x$-directional magnetization. Introducing image magnet as depicted in Fig. 5(b) and Fig. 6(b), the boundary condition on the yoke surface will be satisfied. The magnetic field generated by the single magnet above the stator yoke, therefore, can be obtained by superposing the magnetic field due to real magnet and the field due to image magnet in free space [5]. To calculate the field due to the magnet in free space, Fourier transform is applied while Fourier series is usually used in conventional analysis that assumes the mover has infinite length. The field due to a $y$-directional magnetized magnet in free space will be given as following equations.
\[
B_x = -\frac{\mu_0 M}{4\pi} \left[ \log \frac{(x - W_1/2)^2 + (A - y)^2}{(x + W_1/2)^2 + (A - y)^2} - \log \frac{(x - W_1/2)^2 + y^2}{(x + W_1/2)^2 + y^2} \right]
\]
in region I, II and III

\[
B_y = -\frac{\mu_0 M}{2\pi} \left[ \tan^{-1} \frac{x + W_1/2}{A - y} - \tan^{-1} \frac{x - W_1/2}{A - y} \\
+ \tan^{-1} \frac{x + W_1/2}{y} - \tan^{-1} \frac{x - W_1/2}{y} \right]
\]
for \( M_y \)
in region I and III

\[
B_y = -\frac{\mu_0 M}{2\pi} \left[ \tan^{-1} \frac{x + W_1/2}{A - y} - \tan^{-1} \frac{x - W_1/2}{A - y} \\
+ \tan^{-1} \frac{x + W_1/2}{y} - \tan^{-1} \frac{x - W_1/2}{y} \right] + \mu_0 M_y(x) \text{ in region II}
\]

(6)

According to a similar way, the field due to a \( x \)-directional magnetized magnet in free space will be also given as the following equations for all region.

Fig. 5. Analytic model of a single magnet with \( y \)-directional magnetization. (a) Real magnet above stator yoke, (b) the magnet and its image.
On upper surface of coil, \( y = \Gamma \), the magnetic field will be as Fig. 7(a) and (b) by superposing the field due to real magnet and image magnet. This field is the solution of analytical model in Figs. 5(a) and 6(a). In Fig. 7(a) and (b), the field calculated by the finite element analysis is also presented. The two results coincide very well.

If there is a magnet with an arbitrary magnetization, the field emanated from the magnet is superposition of the field due to a magnet with \( x \)- and \( y \)-component of the magnetization as depicted in Fig. 8.

According to the analysis as mentioned above, the magnetic field generated by the 2-, 3-, and 4-segmented Halbach magnet array is obtained. The results are compared with the field solution by finite element analysis as presented in Figs. 9(a), 10(a) and 11(a). The model of finite element analysis is depicted in the figures as a region enclosed by dashed line. In the comparison, it is assumed that the array has infinite

\[
B_x = \frac{\mu_0 M}{2\pi} \left[ \tan^{-1} \frac{x + W/2}{\Delta - y} - \tan^{-1} \frac{x - W/2}{\Delta - y} + \tan^{-1} \frac{x + W/2}{y} - \tan^{-1} \frac{x - W/2}{y} \right] \quad \text{for } M_x
\]

\[
B_y = -\frac{\mu_0 M}{4\pi} \left[ \log \frac{(x - W/2)^2 + (\Delta - y)^2}{(x + W/2)^2 + (\Delta - y)^2} - \log \frac{(x - W/2)^2 + y^2}{(x + W/2)^2 + y^2} \right]
\]
Fig. 7. Validity of proposed model: magnetic flux of magnet above yoke ($M = 796$ kA/m, width $W_1$, $W_2 = 1.27$ cm) for (a) $\gamma$-directional magnetization and (b) $x$-directional magnetization.

Fig. 8. Arbitrary magnetization direction in permanent magnet: superposition of $x$- and $\gamma$-directionally magnetized magnets.
length with respect to the \( x \)-directional motion of the linear motor. In other words the comparison is performed between the field due to array with enough length by
the method and the field by finite analysis of the model in the dashed line. To account the infiniteness, appropriate boundary condition such as flux normal, flux parallel condition must be applied to dashed line in finite element analysis. If the length of magnet array of proposed model is long i.e. the array has enough number of pitches, the field in central pitch will be nearly same as finite element analysis result as depicted in Figs. 9(b), 10(b) and 11(b). Therefore, the results validate the proposed method and shows that the method can be also applied to infinite motor.

Using the proposed method, the magnetic flux leakage in the both end of the mover can be so precisely calculated that we can design the linear motor more effectively. The field due to finite magnet array and due to infinite array is compared in Fig. 12. If length of the array is one pitch, prior researchers usually use only magnetic flux on the pitch of the stator coil to calculate actuating force, however, we use the flux on the pitch and on the other region as well. The flux on the other region is concerned as to the flux leakage in the prior research. The calculation of magnetic force will be described in Chapter 3.

2.2. Magnetic field due to stator coil current

The stator coil current on stator yoke is presented in Fig. 13. In contrast with the mover with Halbach magnet array, the stator is long enough to be assumed that it is infinitely long. Using Eq. (3), the magnetic fields can be developed in similar way. Supposing current density can be represented as a Fourier series as Eq. (8), magnetic field due to stator coil current will be a Fourier series of which coefficient is as Eq. (9). In the equation, subscript $a$, $b$, and $c$ means each phase of the coil current.
As mentioned above, total magnetic field is superposition of the field due to magnets and stator coil current.

\[ J_A = \sum_{n=-\infty}^{\infty} J_{an} e^{-jk_n x}, \quad \text{where} \quad k_n = \frac{\pi}{\text{pole pitch}} \]

\[ J = \sum_{n=-\infty}^{\infty} \left( J_{an} + J_{bn} + J_{cn} \right) e^{-jk_n x} = \sum_{n=-\infty}^{\infty} J_n e^{-jk_n x} \]

(8)

As mentioned above, total magnetic field is superposition of the field due to magnets and stator coil current.

\[ A_{n,III} = \frac{\mu_0}{\gamma_n k_n} J_n e^{-j\alpha_y} \sinh k_n \Gamma \]

\[ B_{nx,III} = -\frac{\mu_0}{k_n} J_n e^{-j\alpha_y} \sinh k_n \Gamma \]

\[ B_{ny,III} = j \frac{\mu_0}{\gamma_n} J_n e^{-j\alpha_y} \sinh k_n \Gamma \]

(9)
3. Actuating force

Force acting on an object in magnetic field is integral sum of stress on surface of the object. The stress, \( \tau \) is inner product of surface normal vector, \( n \) and Maxwell stress tensor, \( T \) [6]. Maxwell stress tensor is presented in Eq. (10) where \( \delta \) is identity tensor. The stress on upper surface of coil is as Eq. (11). Assuming the stator length is infinite, actuating force that exerted on the mover is integral sum of the stress and in the opposite direction of the force.

\[
T = BH - \frac{1}{2} \mu_0 H \cdot H \delta = \frac{1}{\mu_0} \begin{bmatrix} B_x^2 - \frac{1}{2} B_y^2 & B_x B_y & B_x B_z \\ B_x B_y & B_y^2 - \frac{1}{2} B_z^2 & B_y B_z \\ B_x B_z & B_y B_z & B_z^2 - \frac{1}{2} B_x^2 \end{bmatrix} \tag{10}
\]

\[
\tau = T \cdot n = \frac{1}{\mu_0} \begin{bmatrix} B_x B_y & B_x^2 - \frac{1}{2} B_y^2 & 0 \end{bmatrix}^T \tag{11}
\]

In the proposed method, the stress must be integrated over the overall upper surface of stator coil to obtain the actuating force. The actuating force is interaction between the stator coil current and magnetic flux of the magnet array [7]. As the \( x \)-directional distance gets longer from mover to a pitch, the magnetic flux of the magnet array becomes so smaller as depicted in Fig. 12 that integration over the pitch of stator coil approaches to zero. The stress is, therefore, integrated over the upper surface of several stator coil pitches in the neighborhood of mover position in practical calculation.

The infinite model has the infinite number of the magnet array pitches in contrast with the model mentioned above. Actuating force of infinite model can be obtained by integrating the stress over the surface of one pitch and by multiplying by the number of all magnet pitches with finite length. If the length of magnet array of proposed model is long i.e. the array has enough number of pitches, the field in central pitch will be nearly same as the ideal model with the infinite number pitches.

4. Results and conclusion

In case of ideal Halbach magnet array, the magnetic field is sinusoidal in \( x \)-direction in strong side while vanished in weak side. In case of segmented Halbach magnet array, however, the magnetic field is deviated from the ideal case in a certain extend. The more segmentation number is in a pitch of the array, the smaller deviation from the ideal sinusoid is in the field in the strong side. The commutation ripple will be generated in actuating force if the stator coil energized by ideally sinusoidal phase current.

Generally, the commutation force, which is ripple generated by mismatching the current and the field due to magnet array, is constituted by \( 60^\circ/n \)-period harmonics (\( n \) is integer) [8]. The force ripple is compared in Fig. 14 when the finite model and the infinite model are respectively applied to linear motor with a mover of finite length.
In the figure, the segmentation number of magnet in one pole pitch is 4 and the length of the mover is $1\frac{1}{8}$ times the pitch of the motor. Using the proposed method, the force ripple with $60^\circ/n$-period is recognized as main component of the commutation ripple. Using the infinite method, however, the force ripple with $60^\circ/n$-period can not be described for the magnetic field at both ends of the mover is disregarded. In other words the proposed method enable us to predict the force ripple or force constant more precisely than the conventional infinite model.

An analytical method is proposed to obtain magnetic field emanated from general magnet array. The method can be applicable to every slotless electrical motor. Using the method, therefore, accurate design will be obtained and batch process can be easily applicable to the design, for example, optimal design, dynamic simulation and etc.

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