Precoder and Decoder Design of Cognitive Radio Non-Regenerative Full-Duplex Relay with Zero-Forcing Method

Chang-Jae Chun, Dong-Woo Lim, Jae-Hwan Lee and Hyung-Myung Kim

Abstract—Full-duplex relay (FDR) is one of the efficient relaying methods because FDR can receive and transmit signals over a shared channel simultaneously. Cognitive radio (CR) is an emerging technology improving the spectrum utilization efficiency. In CR networks, transmit signal power should be limited under the acceptable interference power at the primary user because cognitive user uses the licensed spectrum with the primary user. In this paper, we find the optimal decoder and the precoder at the relay with sum power constraint for capacity maximization in the CR non-regenerative FDR networks. Zero-forcing method is used for removing the loop interference. To derive the closed form solution, it is assumed that the interference from the relay to the primary user is zero. The simulation results showed that the proposed method has a higher achievable rate than the optimal CR half-duplex relay (HDR).

Keywords—Cognitive radio, full-duplex relay, zero-forcing.

I. INTRODUCTION

Radio spectrum is a valuable resource for wireless communications. We need to use the spectrum efficiently because the spectrum is not infinite resources. According to federal communications commission (FCC), spectrum utilization depends on the region or time and there exist unused licensed spectrum bands [1]. Cognitive radio (CR) is one of the promising techniques for increasing the utilization of the saturated spectrum by using the licensed frequency bands. Therefore, in CR system, cognitive user control the transmit power because the interference power at the primary user should be lower than the threshold. In order to achieve the same rate with the smaller transmit power, relay can be used in CR networks. There are two kind of relay transmission methods. The one is HDR and the other is FDR. The FDR can transmit more symbols than the HDR due to instantaneous reception and transmission. However, loop interference at the relay node degrades the system capacity because the transmit power at the relay is greatly larger than the received signal power at the relay. Thus, it is not practical to control the interference power below the threshold similar to the CR system. In order to solve the critical loop interference problem, many methods are proposed to eliminate the loop interference [2]-[5]. In CR system, the methods of the interference power control for capacity maximization is proposed in [6]. Also [7] proposed the null space-based transmission scheme for zero interference at the primary user.

In this paper, we consider the CR FDR networks. Using the relay and the full-duplex transmission are suitable methods for CR. Because CR is sensitive to the transmit power level, both the relay and the full-duplex reduce the transmit power under same transmission rate. We assume that the relay node has multiple antennas which are not partitioned into the receive and the transmit antennas. It is possible that all the antennas are shared for both transmission and reception because the electromagnetic fields are independent each other [2][8]. We propose the precoder and the decoder at the relay node for capacity maximization in the CR FDR networks using zero-forcing method.

II. SYSTEM MODEL AND PROBLEM FORMULATION

This paper considers a CR non-regenerative FDR network with single primary user and a single pair of secondary source, relay and destination as shown in Fig. 1. The primary user, the source and the destination has a single antenna, whereas the relay has \( N \) antennas which are not partitioned. The source has full CSI between source and the primary user and the relay also has full CSI of loop, relay-destination and relay-primary user channels. All the terminals share the same bandwidth for transmission and no direct link exists between the secondary source and the secondary destination. We consider the sum power of the source and the relay nodes. Sum interference power at the primary user is also considered because the source and the relay transmit the signal instantaneously. in. It is assumed that the loop interference should be zero because large capacity degradation is occurred when loop interference interfere the signal receive at the relay.
A. Signal Model

The data transmission takes place over one time slot due to the full-duplex relay. If we denote the decoder and the precoder as \( \mathbf{d}_o = P \mathbf{d} \) and \( \mathbf{p}_o = P \mathbf{p} \), respectively finding the \( \mathbf{d}_o \) and \( \mathbf{p}_o \) is the same as designing \( \mathbf{d} \) and \( \mathbf{p} \) with power allocation where \( P_s \) and \( P_r \) are the source and the relay transmit power and \( \| \mathbf{p} \| = d \| \mathbf{d} \| = 1 \). The received signal at the relay, the destination and the primary user at time instant \( t \) are given by

\[
\begin{align*}
    y_r(t) &= P \mathbf{d}^H \mathbf{h}_r x_s(t) + P \mathbf{d}^H \mathbf{H}_r \mathbf{p} x_r(t) + n_r(t) \\
    y_d(t) &= P \mathbf{h}_r^H \mathbf{p} x_r(t) + n_d(t) \\
    y_{pu}(t) &= P g_s x_s(t) + P g_r \mathbf{p} x_r(t) + n_{pu}(t) \\
    x_r(t) &= f \{ y_r(t-1) \}
\end{align*}
\]

where \( y_r, y_d \) and \( y_{pu} \) are the received signal at the relay, the destination and the primary user, respectively. \( x_r \) is the transmitted signal at the source with transmit power \( P_r \) and the covariance \( E \{ x_r x_r^H \} = I \). \( x_r \) is the transmitted signal at the relay with transmit power \( P_r \) with the covariance \( E \{ x_r x_r^H \} = I \). \( f \{ \cdot \} \) is an arbitrary function that generates transmitted data at the relay. \( \mathbf{p} \in \mathbb{C}^{N \times 1} \) and \( \mathbf{d} \in \mathbb{C}^{N \times 1} \) are the relay precoder and decoder vector respectively. \( \mathbf{h}_r \in \mathbb{C}^{N \times 1} \), \( \mathbf{h}_s \in \mathbb{C}^{1 \times N} \), and \( \mathbf{H}_r \in \mathbb{C}^{N \times N} \) are the channels of the source-relay, relay-destination links and relay loop, respectively, and it is assumed that all the channel elements are distributed as \( \mathcal{CN}(0,1) \). \( g_1 \) and \( g_2 \) are the channels of the source-primary user and relay-primary user, respectively, and the elements of the channels are distributed as \( \mathcal{CN}(0,1) \). \( n_r, n_d \) and \( n_{pu} \) are the noise at the relay, the destination and the primary user, respectively, and they are distributed as \( \mathcal{CN}(0,1) \).

B. Problem Formulation

Assuming that the loop interference nullification is perfect \( \mathbf{d}^H \mathbf{H}_r \mathbf{p} = 0 \) and that \( \mathbf{p} \) and \( \mathbf{d} \) are given, the end-to-end capacity of the full-duplex non-regenerative relay network can be expressed as [9]

\[
C = \log_2 \left( 1 + \frac{P_r P_s |\mathbf{d}^H \mathbf{h}_r |^2 |\mathbf{h}_r \mathbf{p}|^2}{P_r |\mathbf{d}^H \mathbf{h}_r |^2 + P_r |\mathbf{h}_r \mathbf{p}|^2 + 1} \right)
\]

The loop interference is zero and there are primary user interference constraints in this system. The interference at the primary user consists of the signals from the source and the relay due to the FDR system. These conditions can be expressed as

\[
\mathbf{d}^H \mathbf{H}_r \mathbf{p} = 0 \quad \text{and} \quad |P g_s + g_r \mathbf{p}|^2 \leq P_{th}
\]

where \( P_{th} \) is the minimum interference power at the primary user.

We can now formulate the capacity maximization problem (P1) with zero interference constraints as

\[
\begin{align*}
\text{maximize} & \quad C = \log_2 \left( 1 + \frac{P_r P_s |\mathbf{d}^H \mathbf{h}_r |^2 |\mathbf{h}_r \mathbf{p}|^2}{P_r |\mathbf{d}^H \mathbf{h}_r |^2 + P_r |\mathbf{h}_r \mathbf{p}|^2 + 1} \right) \\
\text{subject to} & \quad P_r + P_s \leq P_t \\
& \quad \mathbf{d}^H \mathbf{H}_r \mathbf{p} = 0 \\
& \quad |P g_s + g_r \mathbf{p}|^2 \leq P_{th}
\end{align*}
\]

The problem (P1) is hard to solve directly due to the interference power constraint at the primary user. The optimal solution can be obtained by full searching iterative method. If we assume that there are many primary users near the CR user or the primary system is very sensitive to the interference, we can consider the new system that the interference power from the relay to the primary user is zero. In new system, we can find the optimal solution in closed form. The modified problem is given by (P2)

\[
\begin{align*}
\text{maximize} & \quad C = \log_2 \left( 1 + \frac{P_r P_s |\mathbf{d}^H \mathbf{h}_r |^2 |\mathbf{h}_r \mathbf{p}|^2}{P_r |\mathbf{d}^H \mathbf{h}_r |^2 + P_r |\mathbf{h}_r \mathbf{p}|^2 + 1} \right) \\
\text{subject to} & \quad P_r + P_s \leq P_t \\
& \quad \mathbf{d}^H \mathbf{H}_r \mathbf{p} = 0 \\
& \quad g_r \mathbf{p} = 0 \\
& \quad g_s |\mathbf{g}|^2 \leq P_{th}
\end{align*}
\]

III. RELAY PRECODER AND DECODER DESIGN

A. Relay Precoder Design

We can find the precoding vector \( \mathbf{p} \) using the third constraint \( g_r \mathbf{p} = 0 \) in (P2). The interference channel vector \( \mathbf{g}_2 \) can be separated as following singular value and singular vectors. This separation is same as singular value decomposition (SVD) for a matrix.

\[
\mathbf{g}_2 = u_{\mathbf{g}_2} \lambda_{\mathbf{g}_2} \mathbf{V}_2^H
\]

where \( u_{\mathbf{g}_2} \) is a scalar, \( \lambda_{\mathbf{g}_2} \in \mathbb{C}^{N \times N} \) and \( \mathbf{V}_2 \in \mathbb{C}^{N \times N} \).

Therefore, we can find that the N-1 singular vectors, \( \mathbf{v}_{\mathbf{g}_2,2}, \ldots, \mathbf{v}_{\mathbf{g}_2,N} \), are orthogonal to \( \mathbf{g}_2 \) where \( \mathbf{V}_{\mathbf{g}_2} = [\mathbf{v}_{\mathbf{g}_2,1}, \ldots, \mathbf{v}_{\mathbf{g}_2,N}] \). If we use one of the N-1 vectors which are orthogonal to \( \mathbf{g}_2, \mathbf{g}_2 \mathbf{v}_{\mathbf{g}_2,i} = 0 \) for \( i = 2, \ldots, N \). Now we choose the orthogonal vector for maximizing the capacity in N-1 orthogonal vectors. The objective function in (P2) is an increasing function of \( |\mathbf{h}_r \mathbf{p}| \). Therefore we can find the precoder maximizing \( |\mathbf{h}_r \mathbf{p}| \) with the constraints as \( g_r \mathbf{p} = 0 \).
\[
p_{\text{opt}} = \max_{\mathbf{v}_{g_{i,j}}} |h_{i,j}| \quad \text{for } i = 2, \ldots, N \quad \text{(5)}
\]

\[
s.t. \quad g_{i,j} \mathbf{v}_{g_{i,j}} = 0
\]

**B. Relay Decoder Design**

The relay decoding vector \( \mathbf{d} \) can be found by the second constraint \( \mathbf{d}^H \mathbf{H} p_{\text{opt}} = 0 \) in (P2). \( \mathbf{H} p_{\text{opt}} \) can be separated as \( \mathbf{H} p_{\text{opt}} = \mathbf{U} \lambda_{i} \mathbf{v}_{i} \) where \( \mathbf{U}_{i} \in C^{N \times N} \), \( \lambda_{i} \in C^{N \times 1} \) and \( \mathbf{u}_{i} \) is a scalar.

To eliminate the loop interference, \( \mathbf{d} \) should be one of the vectors in \( \mathbf{u}_{i,2,\ldots,\mathbf{u}_{i,N}} \) which is orthogonal to \( \mathbf{H} p_{\text{opt}} \) where \( \mathbf{U}_{i} = [\mathbf{u}_{i,1},\ldots,\mathbf{u}_{i,N}] \). Because the objective function in (P2) is also increasing function of \( |\mathbf{d}^H \mathbf{h}| \), the decoder vector for capacity maximization is

\[
\mathbf{d}_{\text{opt}} = \max_{\mathbf{u}_{i,j}} |\mathbf{u}_{i,j}^H \mathbf{h}| \quad \text{for } i = 2, \ldots, N \quad \text{(6)}
\]

\[
s.t. \quad \mathbf{u}_{i,j}^H \mathbf{H} p_{\text{opt}} = 0
\]

**C. Sum Power Allocation**

We want to find the source power \( P_{s} \) and the relay power \( P_{r} \) to maximize the capacity of the cognitive relay networks with zero-forcing decoder and precoder \( \mathbf{d}_{\text{opt}} \) and \( \mathbf{p}_{\text{opt}} \). The fourth constraint \( P_{s} | g_{1} |^2 \leq P_{th} \) in (P2) is the interference power constraint at the primary user. Therefore, we should control the transmit power \( P_{s} \) to meet the constraint with the CSI of the source-relay link. The following lemma helps us to simplify the problem (P2).

**Lemma 1**: Finding the optimal solution of the problem (P2) is the same as solving (P2) without the constraint \( P_{s} | g_{1} |^2 \leq P_{th} \) and considering it later.

**Proof**: Let the optimal relay power without the constraint \( P_{r} | g_{1} |^2 \leq P_{th} \) be \( \overline{P}_{r} \) and \( \overline{P}_{r} = P_{r} - P_{th} \). If \( P_{r} | g_{1} |^2 \leq P_{th} \), \( \overline{P}_{r} \) and \( \overline{P}_{r} = P_{r} - P_{th} \) are the optimal power. However, if \( \overline{P}_{r} | g_{1} |^2 > P_{th} \), we should find the power allocation that is different from \( \overline{P}_{r} \) and \( \overline{P}_{r} \). Two cases need to be considered. First, the source transmit with maximum power as \( P_{s} = P_{th} / |g_{1}|^2 \). In this case, the relay also uses maximum power as \( P_{r} = P_{r} - P_{th} / |g_{1}|^2 \) because objective function in (P2) is an increasing function of \( P_{r} \). Second, we can consider the case that the source transmits the signal with slightly lower power than the maximum power as \( P_{s} = P_{th} / |g_{1}|^2 - \alpha, (\alpha > 0) \) in \( 0 < P_{s} < P_{t} \). However, the capacity is maximized when \( P_{s} \) closes to \( \overline{P}_{r} \) because the second term in the log has one or two zero differentiation values between \( 0 \leq P_{s} < P_{t} \) and the value at \( P_{s} = 0 \) and \( P_{s} = P_{t} \) is zero (it will be known at the next part of this paper).

From the lemma 1, the optimal solution when \( P_{r} | g_{1} |^2 > P_{th} \) is given by

\[
P_{r} = P_{r} - \frac{P_{th}}{|g_{1}|^2} \quad \text{(7)}
\]

Finding the power for maximizing the objective function in (P2) is the same as finding the power for maximizing \( P_{s} | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |^2 / (P_{r} | \mathbf{d}^H \mathbf{h} |^2 + P_{r} | \mathbf{h}, \mathbf{p} |^2 + 1) \) because \( \log \) function is an increasing function. With the simplified objective function and the lemma 1, we can consider a simplified problem (P3):

\[
\begin{align*}
\max_{P_{r}, P_{s}} & \quad f(P_{s}, P_{r}) = \frac{P_{s} P_{r} | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |^2}{P_{r} | \mathbf{d}^H \mathbf{h} |^2 + P_{r} | \mathbf{h}, \mathbf{p} |^2 + 1} \\
\text{s.t.} & \quad P_{r} + P_{s} \leq P_{th}, \\
& \quad \mathbf{d}^H \mathbf{h}, \mathbf{p} = 0, \\
& \quad \mathbf{g}, \mathbf{p} = 0
\end{align*}
\]

(P3)

The function \( f(P_{s}, P_{r}) \) is zero at \( P_{r} = 0 \) and \( P_{s} = P_{r} \). When \( P_{r} = P_{r} / 2 \), \( f(P_{r}) \geq 0 \). Therefore, at least there is one point satisfying \( f(P_{r}) = 0 \) and \( f(P_{r}) \geq 0 \) between \( P_{r} = 0 \) and \( P_{r} = P_{r} \). Thus for finding the optimal power of (P3), the second derivative of the objective function can be obtained as

\[
\frac{d}{dP_{r}} f(P_{r}) = \left\{ P_{s} | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |^2 (| \mathbf{d}^H \mathbf{h} |^2 - | \mathbf{h}, \mathbf{p} |^2) \right\}
\]

\[
-2P_{r} | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |^2 (P_{r} | \mathbf{d}^H \mathbf{h} |^2 - 1)
\]

\[
+P_{r} | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |^2 (P_{r} | \mathbf{d}^H \mathbf{h} |^2 + 1)
\]

\[
\left( P_{r} (| \mathbf{h}, \mathbf{p} |^2 - | \mathbf{d}^H \mathbf{h} |^2) + P_{r} | \mathbf{h}, \mathbf{p} |^2 + 1 \right)^2
\]

where \( P_{s} = P_{r} - P_{r} \).

By equating (8) to zero, we obtain the following two values of \( P_{s} \), when \( | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |, \) as

\[
P_{s,2} = P_{s} | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |^2 + 1 + \sqrt{P_{s} | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |^2 + P_{s} | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |^2 + P_{r} | \mathbf{h}, \mathbf{p} |^2 + 1}
\]

\[
| \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |^2
\]

for \( \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |, \)

\[
P_{s,2} = P_{s} / 2
\]

for \( | \mathbf{d}^H \mathbf{h} / | \mathbf{h}, \mathbf{p} |, \)

\[
\text{(9)}
\]
where the total power is \( P_t = P_r + P_s \).

If \( P_r | g_i |^2 > P_s \), the relay power allocation \( P_r = P_t - P_s | g_i |^2 \) by lemma 1.

IV. SIMULATION RESULTS

The elements of all the channels and noises are generated as \( CN(0,1) \). We calculate the achievable rates and the capacity of the proposed and other schemes. Monte-Carlo simulation with 10000 randomly generated channels and noises are implemented and the average achievable rates and capacity are plotted versus the total power in SNR values. The number of relay antenna is four in all simulation results. The proposed method is compared with three schemes which are CR HDR with maximum ratio combining (MRC) decoder and zero-forcing precoder, optimal CR HDR and FDR with zero-forcing decoder and pre-MRC precoder.

Lemma 2 : It is assumed that there is a CR HDR networks which is the same as the proposed system model without the loop channel. Then, the optimal decoder and the precoder are obtained as \( d = h_i / |h_i|^2 \) and \( p \), respectively, as in [6, Theorem 2] that is the optimal beamforming vector for capacity maximization in CR MISO networks.

Proof: \( f(P_r) \) in (P3) is an increasing function of \( |d^H h_i|^2 \) or \( |h_i p|^2 \) with arbitrary power \( P_r \) and \( P_r = P_t - P_s \). Also, determination of the decoder is independent on the precoder design result. Therefore, the optimal decoder is MRC as \( d = h_i / |h_i|^2 \). In [6], the optimal beamforming vector maximizes the received SNR at the destination. In our problem, \( |h_i p|^2 \) is SNR with unit noise power at the destination. Thus, the precoder maximizing \( |h_i p|^2 \) in the proposed network is the same as the beamforming vector founded in [6].

We can find the capacity of the CR HDR networks using lemma 2 and proposed power allocation as (10).

Fig. 2 compares the achievable rates for four schemes with four relay receive and transmit antennas. As a result, the proposed scheme always has higher achievable rate than those for three CR relay schemes. Although performance gap between the optimal CR HDR scheme and the proposed scheme is small, the proposed scheme do not interfere the primary user while the primary user in CR HDR network suffers from the non-zero interference. Fig. 3 shows that the achievable rates depend on the design method for \( d \) and \( p \). We also know that if the relay has more antennas, the performance gap increases. This implies that as the number of antenna increase, the chance to select the vector close to the optimal vector increases. The optimal vectors for (5) and (6) are \( h_i / |h_i| \) and \( h_i / |h_i| \), respectively. Fig. 4 and Fig. 5 show the achievable rates with different interference power constraints. The loop interference and the interference from the relay to the primary user are zero in the proposed scheme. However there are some interference from the source to the primary user. Therefore, the achievable rate decreases with low interference power threshold. The achievable rates are crossed with high interference power threshold, \( P_s = 15dB \) in Fig. 5, because the beamforming at the transmitter and MRC at the receiver come to be more efficient than the zero-forcing methods.
V. CONCLUSION

In this paper, we consider the CR non-regenerative FDR with zero loop interference. The CR FDR system controls two sources of interference; one is loop interference and the other is the interference from the relay to the primary user. We designed the zero-forcing decoder and precoder at the relay node so that the power allocation can be optimized. From the simulation results, the proposed scheme outperforms the optimal CR HDR scheme in capacity.

REFERENCES


Fig. 5. Comparison of the achievable rates with $P_0 = P_{a,1} = P_{a,2} = 15\text{dB}$.