Similarity measure between fuzzy sets and between elements

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Abstract: Two similarity measures are proposed: one for the similarity between fuzzy sets and the other between elements in fuzzy sets. With an example, it is shown that the proposed measures can be used in the behavior analysis in an organization.

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1. Introduction

Many measures of similarity between fuzzy sets have been proposed in the literature and some measures have been used in system analysis and linguistic approximation [1, 4, 5]. Zwick has reviewed 19 measures and compared their performance in an experiment [3].

Let us consider an example of behavior analysis. There is an observation of a class. In the class, there are 4 members \((x_1, x_2, x_3, x_4)\) and 3 groups \((A, B, C)\). A member is involved in groups with the membership degrees as shown in Table 1.

On the observation, we can have two types of questions:

(Type 1) ‘At what degree can the groups \(A\) and \(B\) be cooperated?’ or ‘What is the guaranteed minimum level of cooperation between groups \(A\) and \(B\)?’

(Type 2) ‘At what degree can \(x_2\) and \(x_3\) be in the same group?’ or ‘What is the level of their friendship?’

For these questions, we can use some similarity measures between fuzzy sets and between elements such as geometric distance similarity and Hausdorff similarity [3, 6]. These similarity measures are based on the Minkowski distance and thus the similarities represent the global distances between fuzzy sets and between elements. Therefore, they are not appropriate for the above questions and some measures are needed which can represent the guaranteed minimum similarity between sets and the maximum level similarity between elements.

Therefore, in this paper, we will propose a new similarity measure between fuzzy sets and a similarity measure between elements in fuzzy sets. We can see that the proposed measures are useful to analyze the behavior of the groups.

A fuzzy set \(A\) in the universal set \(X\) is defined by the membership function \(\mu_A(x)\), for \(x \in X\) [2].

2. Similarity measure

2.1. Similarity between fuzzy sets

We define the similarity measure \(S(A, B)\) between fuzzy sets \(A\) and \(B\) as follows:

\[
S(A, B) = \max_{x \in X} \min(\mu_A(x), \mu_B(x))
\]

We can see some properties of the measure.

1. \(S(A, B)\) is the maximum membership degree in the intersection \(A \cap B\).
2. The similarity degree is bounded \(0 \leq S(A, B) \leq 1\).
3. If \(A\) and \(B\) are normalized, and \(A = B\)
   \[
   S(A, B) = 1.
   \]
   If \(A \cap B = \emptyset\)
   \[
   S(A, B) = 0.
   \]
4. The measure is commutative
   \[
   S(A, B) = S(B, A).
   \]
When the set \( A \) and \( B \) are crisp sets,
\[
S = 0 \quad \text{if} \; A \cap B = \emptyset, \quad S = 1 \quad \text{if} \; A \cap B \neq \emptyset.
\]

For example, consider two fuzzy sets \( A \) and \( B \) defined in \( X = \{x_1, x_2, x_3, x_4\} \) as shown in Table 2. In this example, we can have the similarity
\[
S(A, B) = \max \left\{\min(0.4, 0.6), \min(0.8, 0.3), \min(0.9, 0)\right\} \\
\quad = \max[0.4, 0.3, 0] \\
\quad = 0.4.
\]

### 2.2. Similarity between elements

The similarity measure between two elements \( x, y \in X \) in fuzzy sets \( A_i \in X, \; i = 1, \ldots, n \), is defined as follows:
\[
S_e(x, y) = \max \min(\mu_{A_i}(x), \mu_{A_j}(y)).
\]

This measure satisfies the following properties:
1. \( 0 \leq S_e(x, y) \leq 1 \).
2. If \( x = y \), \( S_e(x, y) = 1 \),
   if \( \exists A_i \) such that \( x \in A_i, \; y \in A_i \) then
   \( S_e(x, y) = 0 \).
3. \( S_e(x, y) = S_e(y, x) \).

For example, consider the elements \( x_1, x_2, x_3 \) and \( x_4 \) in fuzzy sets \( A_1 \) and \( A_2 \) in Table 3.

We have the similarity
\[
S_e(x_1, x_4) = \max[\min(0.2, 0.5), \min(0.5, 1)] \\
\quad = \max[0.2, 0.5] \\
\quad = 0.5, \\
S_e(x_1, x_2) = 0.5, \\
S_e(x_1, x_3) = 0.
\]

### 3. Application and conclusion

To illustrate a possible interpretation of the measures, let us reconsider the previous example of the observation of a class.

We have the similarity between \( A \) and \( B \), \( S(A, B) = 0.4 \). A possible interpretation of the similarity is that the group \( A \) and \( B \) can be cooperated by the coordination of \( x_1 \) with the degree at least 0.4. This similarity can be an answer for the type 1 question.

The similarity between elements \( x_2 \) and \( x_3 \) is \( S_e(x_2, x_3) = 0.8 \). We can think that the members \( x_2 \) and \( x_3 \) are commonly involved in the same group \( (A) \) with the degree at least 0.8. This measure can be also an answer for the type 2 question.

In conclusion, we have proposed two measures: one measures the similarity between fuzzy sets and the other between elements in fuzzy sets. The proposed measures can also work in the crisp sets. The operators max and min can be replaced by t-conorm and t-norm operators respectively. We have seen that the proposed measures can be useful in the behavior analysis in an organization.

### References


