Optimal Reaction Wheel Steering Law for Saturation Avoidance

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Abstract: In general, a reaction wheel steering law with more than three wheels for three axis stabilization can be derived by using a least-squares method, especially pseudo-inverse. Since the least-squares method minimizes norm of the wheel torque vector, one of the allocated wheel angular momentum can be much larger than that of other wheels. In other words, the control torque or angular momentum can be easily saturated. The basic idea proposed in this paper is to reduce the saturation possibility. We have examined and formulated two different cost functions to resolve the saturation problems of the wheel angular rate. The appropriate cost function is selected so that the maximum angular rate of the previous step could be reduced. That is, an extra term is augmented to the conventional error cost to take into account the wheel speed distribution. The new approach is applied to attitude transfer of spacecraft using quaternion feedback control. Simulation results based upon three-axis rigid body rotational motion illustrate that the suggested methodology has a wider allocation range than that resulted from simple least-squares algorithm. In conclusion, the proposed approach can provide an attractive way to prevent satellite wheels from being saturated.

Keywords: Control allocation, Reaction wheel, Wheel saturation, Precision attitude control.

1. INTRODUCTION

Highly maneuverable capability is necessarily required for the next generation spacecraft to successfully achieve various missions. To enhance agility of spacecraft, Control Moment Gyros (CMGs) have been widely studied for the past two decades. This device can change its internal angular momentum vector and produce more effective torque on the spacecraft than the conventional reaction wheels (RWs). Even though this goodness of CMGs can resolve the current issue for agile spacecraft, CMGs have been suffering from the singularity problem.

For precision spacecraft attitude control, the Reaction Wheels (RWs) have been conventionally used for many years. Reaction wheels have a wheel spinning about a body-fixed axis whose spin speed is variable. Torque is produced on the spacecraft by accelerating or decelerating the reaction wheels. RWs do not have singular configurations and typically have simpler control laws than CMGs. Because reaction wheels are operated with nominally zero momentum, they are used primarily for absorbing cyclic torques and temporarily string momentum from the body during slew, or reorientation maneuvers.

A secular disturbance torque about the same magnitude as the cyclic term could eventually saturate the momentum storage capacity of RWs. Before a RW is saturated, the angular momentum can be adjusted by momentum dumping using gas jets or magnetic coils to release the momentum below acceptable levels. The storage capability of RWs can directly affect the agility of the spacecraft. To simultaneously satisfy the precision attitude control and agility of the spacecraft using RWs, the storage capability should be dramatically increased. If the range satisfying the saturation constraints can be expanded in the same system by changing control allocation method, the purpose as well as the fuel consumption can be satisfied and saved. [8]

In this paper, several control allocation methods are proposed to release the wheel speed during mission operations. By defining an appropriate cost function, the maximum angular rate of the previous step could be reduced. That is, an extra term is augmented to the conventional error cost for the pseudo-inverse to take into account the wheel speed distribution. The proposed allocation methods are applied to attitude transfer of spacecraft using nonlinear quaternion feedback control law in this paper. Finally, numerical studies are performed to illustrate that the suggested methodology can be an enabling allocation technology for wheel speed regulation.

2. DYNAMICS AND ATTITUDE KINEMATICS OF

Consider a typical pyramid mounting arrangement of four RWs as shown in Fig 1. The RWs rotation axes are orthogonal to the pyramid faces. Each face is inclined with a skew angle \( \beta \) from the horizontal. The total angular momentum vector \( \mathbf{h} \) of a rigid spacecraft equipped with several RWs is given by

\[
\mathbf{h} = \mathbf{h}_s + \mathbf{h}_{RW}
\]

(1)

where \( \mathbf{h}_s \) is the angular momentum vector of spacecraft in the spacecraft body-fixed axes and \( \mathbf{h}_{RW} \) is the total angular momentum due to the RW.

The momentum vector of spacecraft, \( \mathbf{h}_s \), is defined by

\[
\mathbf{h}_s = J_s \mathbf{\omega}_s
\]

(2)

where \( J_s \) is the inertia matrix of total spacecraft system, \( \mathbf{\omega}_s \) is the angular rate of spacecraft in the body-fixed axes.

Analogously, the reaction wheel momentum, \( \mathbf{h}_{RW} \), is given by

\[
\mathbf{h}_{RW} = C(\beta) J_{RW} \mathbf{\omega}_{RW}
\]

(3)
where \( C(\beta) \) is the rotational matrix defined as
\[
C(\beta) = \begin{bmatrix}
\cos \beta_1 & -\cos \beta_2 & \cos \beta_3 & -\cos \beta_4 \\
n \sin \beta_1 & 0 & -\sin \beta_2 & 0 \\
0 & -\sin \beta_1 & 0 & \sin \beta_2 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
(5)
The rotational matrix maps the angular momentum of RWs described in the RW rotation axes to angular momentum vector in the body-fixed axes. \( I_{gw} \) is the inertia matrix of RWs defined as
\[
I_{gw} = \begin{bmatrix}
I_{gw1} & 0 & 0 & 0 \\
0 & I_{gw2} & 0 & 0 \\
0 & 0 & I_{gw3} & 0 \\
0 & 0 & 0 & I_{gw4}
\end{bmatrix}
\]
(5)
and \( \omega_{gw} \) is the angular rate vector given by
\[
\omega_{gw} = \begin{bmatrix}
\omega_{gw1} \\
\omega_{gw2} \\
\omega_{gw3} \\
\omega_{gw4}
\end{bmatrix}
\]
(6)
Finally, applying the Euler formula into Eq.(1) to obtain governing equations of motion of a spacecraft installed with several RWs gives
\[
J \omega + [\omega \times J] \omega + [\omega \times C(\beta)] h_{gw} (\omega_{gw}) = -C(\beta) I_{gw} \omega_{gw}
\]
(7)
where a skew-symmetric matrix, for simplification, defined as
\[
[\omega \times] = \begin{bmatrix}
0 & -\omega_3 & \omega_2 & -\omega_1 \\
\omega_3 & 0 & -\omega_1 & \omega_2 \\
-\omega_2 & \omega_1 & 0 & -\omega_3 \\
\omega_1 & -\omega_2 & \omega_3 & 0
\end{bmatrix}
\]
(8)
The angular momentum vector of the reaction wheels is defined as
\[
h_{gw} (\omega_{gw}) = \begin{bmatrix}
I_{gw1} \omega_{gw1} \\
I_{gw2} \omega_{gw2} \\
I_{gw3} \omega_{gw3} \\
I_{gw4} \omega_{gw4}
\end{bmatrix}
\]
(9)
To formulate a nonlinear quaternion feedback law, the attitude kinematics is required. That is, the quaternion is given by
\[
q = \begin{bmatrix} q_1 \n q_2 \n q_3 \n q_4 \end{bmatrix}
\]
(10)
where \( q \) called the quaternion vector is defined as
\[
q_i = \begin{bmatrix} q_i \n d_2 \n q_i \n d_4 \end{bmatrix} = n \sin \left( \frac{\theta}{2} \right)
\]
(11)
\[
q_i = \cos \left( \frac{\theta}{2} \right)
\]
(12)
A vector \( n \) is the unit principle axis vector and \( \theta \) is the angle of rotation. There is an evident following constraint
\[
q_i^2 + q_j^2 + q_k^2 + q_i^2 = 1
\]
(13)
The kinematic equations of motion for the quaternion using the spacecraft’s angular velocity \( \omega \) are given by
\[
\dot{q} = \frac{1}{2} \Omega(\omega) q
\]
(14)
where
\[
\Omega(\omega) = \begin{bmatrix}
-\omega \times & 0 & 0 \\
0 & -\omega \times & 0 \\
0 & 0 & -\omega \times
\end{bmatrix}
\]
(15)

3. CONTROL ALLOCATION

Control allocation deals with the problem of distributing a given control demand among available sets of actuators. Figure 2 shows the diagram of the satellite attitude control allocation. The relation between desired or commanded control input and actuator’s control input is given by
\[
v = Au
\]
(16)
where \( v \) is the desired input for spacecraft attitude control, \( u \) is the actuator control input. The purpose of control allocation is to find a feasible solution \( u \). The generally used actuator control input is given by
\[
u = A^\dagger v = A^\dagger (AA^\dagger)^{-1} v
\]
(17)
Since the pseudo-inverse minimizes norm of the wheel torque vector, one of the allocated wheel torque can be much larger than that of other wheels. In other words, the control torque or wheel speed can be easily saturated. Accordingly, it is necessary to consider another method to release such a saturation problem.

3.1 Maximum norm approach

The basic idea proposed in this paper corresponds to a cost function to reduce the maximum wheel speed. The cost function can be selected for this goal so that the maximum angular speed of the previous step could be reduced. The proposed control allocation algorithm can be formulated as a quadratic programming problem. Namely,
\[
\min J = \frac{1}{2} u^\dagger W(u + A^\dagger (AA^\dagger)^{-1} v) W(u + A^\dagger (AA^\dagger)^{-1} v)
\]
(18)
such that \( v = Au \)
(19)
where
\[
u_j = -\omega_{gw_i}(t - \Delta t) / \Delta t
\]
(20)
\[
W_i = \begin{bmatrix} \max(\omega_{gw_i}) & 0 & 0 & 0 \\
0 & \max(\omega_{gw_i}) & 0 & 0 \\
0 & 0 & \max(\omega_{gw_i}) & 0 \\
0 & 0 & 0 & \max(\omega_{gw_i})
\end{bmatrix}
\]
(22)
and
\[
\max(\omega) = \begin{cases}
1 & \text{if } |\omega| > |\omega_i| \\
0 & \text{else}
\end{cases}
\]
\( i \neq j, i = 1,2,3,4, j = 1,2,3,4 \)
A design parameter \( q \) is the scaling constant. Eq.(21) means that only maximum speed wheels in previous step has weighting value. Eq.(18) is defined to minimize a norm of \( u \) vector and regulate the wheel speed. The optimal solution to the control allocation problem (18) is found to be
\[
u = (I_{gw} - W A H A W W W (\omega_{gw}) W_{1} + W A H W_{1})^{-1} v
\]
(23)
\[
H = AW A^\dagger
\]
(24)
\[
W = W_{1} W_{2} (\omega_{gw})
\]
(25)
Unfortunately, this approach could suffer from a chattering problem because the proposed cost function consists of maximum norms of the wheel speeds and constant value \( \alpha \). To reduce the chattering, a dynamic control allocation algorithm could be added to the cost function in Eq.(18). [2]

3.2 Dynamic control allocation method

Dynamic control allocation suggested by Ola Härkegård is a solution of a constrained optimization problem given by
\[
\min J = u^\dagger W(u + A^\dagger (AA^\dagger)^{-1} v) W(u + A^\dagger (AA^\dagger)^{-1} v)
\]
(26)
such to Eq.(19).

The second term of the cost function is applied to minimize the chattering problem. It is for minimizing the difference
between current input vector and previous input vector, namely, rate of inputs.[2]. By adding the cost function to the proposed cost function in Eq.(18), a new optimization problem can be formulated such that

\[
\min J = \frac{1}{2} u^T u + \frac{1}{2} (u - u_c)^T W_c (u - u_c) \tag{27}
\]

subject to Eq.(19) where \( W_i, W_s, W_c, W_e \) are the weighting matrices. The solution of (27) can be readily found to be

\[
u = F(W_i(\omega_{eq}), u_c) + W_s(t - \Delta t) + W_c(t - 2\Delta t) + W_e A H^T v \tag{28}
\]

\[F = (I_{4x4} - W^{-1}A^T H^{-1}A)W^{-1} \tag{29}
\]

\[H = AW^{-1}A^T \tag{30}
\]

\[W = W_i + W_s(\omega_{eq}) + W_c + W_e \tag{31}
\]

subject to Eq.(19) where \( W_i, W_s, W_c, W_e \) are the weighting matrices. The solution of (27) can be readily found to be

where, for simplification, new notations are defined as.

\[
A = -C(\beta) \\
v = -C(\beta)I_{4x4} \omega_{\text{eq}} = Au \tag{32}
\]

The overall block diagram for the control allocation method applied to the spacecraft attitude control problem is illustrated in Fig.2. To guarantee the performance of the proposed algorithm, various simulations are performed. The simulation parameters are as follows: The skew angle of \( \beta \) from the horizontal is assumed to be 45°. The initial quaternion elements are initially selected as \( q_0 = [0.5324 \ -0.2362 \ 0.4073 \ 0.7035]^T \). The target quaternions are set to \( q_c = [0 \ 0 \ 0 \ 1]^T \).

Fig.3 shows the desired control torque input obtained by quaternion feedback control to reorient the spacecraft from the initial attitude to the target attitude. At first, the simulation shown in Fig. 4 is performed by using pseudo-inverse allocation. It is easily seen that the maximum wheel speed of the 3rd RW, in particular, is larger than any other wheels. If the desired control torque input is larger than this simulation case, the wheel speed will be clearly saturated. Then, the attitude control performance of the spacecraft will be degraded due to the control torque degradation.

The next two simulations illustrated in Figs.(4) and (5) are for the proposed two methods. Fig.4 shows the result of the control allocation using the maximum norm approach. All of the wheels are under the permissible speed levels such that the acceptable range of required control torque could be extended. However, a chattering problem is occurred to the acceleration response of the wheels. To release the chattering problem, dynamic control allocation method proposed in this paper is applied to the spacecraft attitude control. The result is illustrated in Fig.5. One can see that the chattering problem is reduced dramatically with satisfying the wheel speed reduction.

4. SIMULATION

The proposed approaches are applied to spacecraft attitude control using quaternion feedback control. The desired control input is obtained by quaternion feedback. The proposed methods allocate control input to the four wheels. The RWs' torque control input of spacecraft installed with four RWs given by

\[
v = -C(\beta)I_{4x4} \omega_{\text{eq}} = Au \tag{32}
\]

where, for simplification, new notations are defined as.

\[
A = -C(\beta) \\
v = -C(\beta)I_{4x4} \omega_{\text{eq}} = Au \tag{32}
\]

5. CONCLUSION

In this paper, several allocation methods were proposed to release speeds of reaction wheels for precise attitude control and agility for spacecraft. The methods are based on constrained optimization theory. Various cost functions to minimize the wheel speed were proposed. Chattering problem induced from the proposed approach was also resolved by dynamic control allocation method. Consequently, the proposed algorithm in this paper could be an enabling technology to release the wheel saturation problem.

REFERENCES

Fig. 3 Desired control torque input.

Fig. 5 Response using maximum norm approach.

Fig. 4 Response using Pseudo-inverse

Fig. 6 Response using the proposed dynamic control allocation method.