Moire topography by slit beam scanning

Seung-Woo Kim and Hyun-Goo Park

A new method of moiré topography is suggested in which a slit beam is used in a scanning mode to generate moiré fringes. One remarkable feature of this method is that, as opposed to existing shadow and projection types, height differences between two consecutive fringes become constant so that absolute fringe orders need not be identified. This advantage makes it possible to measure three-dimensional surface profiles in an automatic manner simply by using a computer-aided image-processing technique. In addition this method can be easily implemented in a conventional coordinate measuring machine with a minimum addition of optical hardware.

Key words: Moiré topography, slit beam scanning, moiré fringes, absolute fringe orders, three-dimensional surface measurement.

Introduction

Moiré topography is a method of measuring threedimensional surface profiles by using moiré fringes that appear when two gratings of similar frequencies are superimposed. In recent years much research has been done on its practical application, and as a result shadow-type1-3 and projection-type moiré topography4-6 are well established. With both types moiré fringes become contour lines of surfaces in specific geometrical conditions. However, the height difference between two consecutive fringes does not remain constant; thus absolute fringe orders are identified with much difficulty. Because of this problem the automatic implementation of moiré topography with the aid of the computer is still a problem, especially on engineering surfaces with sharp or discontinuous cuts7 9 . We present a new type of moiré topography where scanning a slit beam is used for illumination so that the problems associated with absolute fringe orders are eliminated completely. Although many uses for slit beam scanning are found in other studies,10-12 it has not yet been used in the area of moiré topography. For convenience the method described in this study is referred to as slit beam scanning moire topography. This method can fit well in a conventional coordinate measuring machine so that its practical applications can be realized effectively with a minimum addition of optical hardware.

Principles of Slit Beam Scanning

The optical configuration of slit beam scanning may be configured as in Fig. 1. The object to be measured is illuminated with a plane-type slit beam light source attached on a motion control system. The light
source moves in a linear direction so that the entire surface of the object is scanned by the slit beam. During scanning the CCD camera captures the patterns of the slit beams deformed on the object at every incremental pitch of light movement. All the line patterns monitored are then put together into a single image frame by electronic means. For a complete measurement two conjugate grating patterns are needed to generate moire fringes so that two separate scanning processes are performed with different projection angles.

For fringe analysis let the xyz-coordinate system be defined as specified in Fig. 2. The x axis denotes the direction of scanning, while the z axis is the principal optical axis of the camera. If the slit beam is held vertically to the x-z plane, its plane equation can be written for the first scanning as

\[ z = (x - n_1 p_1) \cot \theta_1. \]  

In the above equation \( n \) is defined as the integer line number indicating the scanning order, while \( p \) and \( \theta \) represent the scanning pitch and projection angle, respectively. In the same way the beam equation for the second scanning is also obtained with its line number \( n_2 \) and projection angle \( \theta_2 \) as

\[ z = -(x - n_2 p_2) \cot \theta_2. \]  

![Fig. 1. Optical configuration of slit beam scanning.](image)
The intersection trajectory of the two planes of Eqs. (1) and (2) forms a moiré fringe whose absolute order \( n \) is decided by

\[
    n = n_2 - n_1.
\]

(3)

Fig. 2. Coordinate system of the moiré topography of slit beam scanning.

Fig. 3. Equiorder planes and the coordinate system.
From Eqs. (1)-(3) the line numbers \( n \), and \( n_2 \) can be eliminated readily, and then the height of the \( n \)th order fringe \( z_n \) can be determined in the form of

\[
z_n = Ax + Bn, \tag{4}\]

in which \( A \) and \( B \) are the constants that are obtained, respectively, as

\[
A = \frac{p_2 - p_1}{p_2 \tan \theta_1 + p_1 \tan \theta_2}, \tag{5}\]

\[
B = \frac{(p_1 - p_2)p_1 \tan \theta_2 + p_2(p_2 \tan \theta_1 + p_1 \tan \theta_2)}{(p_2 \tan \theta_1 + p_1 \tan \theta_2)(p_2 \tan \theta_1 + p_1 \tan \theta_2)}. \tag{6}\]

As can be seen from Eq. (4), in this general case of slit beam scanning, the resulting fringes represent straight planes inclined with respect to the \( x \) axis as illustrated in Fig. 2. Fringes become contour planes as shown in Fig. 3 if an identical pitch is taken for the first and second scannings, i.e., \( p_1 = p_2 = p \). This condition makes the constant \( A \) of Eq. (5) zero, and thus the height \( z_n \), remains independent of the \( x \) coordinate. As results the height \( z_n \), becomes a linear function of the absolute order \( n \) as

\[
z_n = n \left( \frac{p}{\tan \theta_1 + \tan \theta_2} \right). \tag{7}\]

In this case the height difference \( \Delta z \) between the \((n - 1)\)th and \( n \)th fringes can then be decided by

\[
\Delta z = z_n - z_{n-1}. \tag{8}\]
This height difference of Eq. (9) is defined as the height pitch of fringes. One remarkable feature of the slit beam scanning type is found in the fact that the height pitch becomes constant regardless of fringe orders as opposed to the conventional shadow and project methods. Absolute fringe orders need not be identified, as the relative height difference between two arbitrary points can be measured simply by counting the number of fringes across the two points.

**Three-Dimensional Profile Measurement**

Once we identify the z coordinates of the object surface by using Eq. (7), its x and y coordinates can be determined in a readily manner by considering the

\[
\Delta z = \left( \frac{p}{\tan \theta_1 + \tan \theta_2} \right).
\]

(9)
optical geometry of the camera. Let the XYZ and \(XmYmZm\) coordinate systems be defined as illustrated in Fig. 3. The \(XYZ\) coordinate system is a simple translation of the \(xyz\) system, i.e.,

\[
X = x, \quad Y = y, \quad Z = z + h,
\]

where \(h\) represents the camera distance. Assuming that the camera is a pin hole type of infinite perspective, we decide the actual coordinates of a point \((Xm, ym)\) on the image plane by

\[
X = \frac{Z}{am} x_m, \quad Y = \frac{Z}{am} y_m,\]

in which \(am\) denotes the principal distance of the camera lens. Then, by substituting Eqs. (10)-(12) into Eqs. (13) and (14), the \(x\) and \(y\) coordinates are obtained as

\[
x = \frac{z + h}{am} x_m, \quad y = \frac{z + h}{am} y_m.\]

### Applications and Discussion

This slit beam scanning moiré topography can be readily implemented on a conventional coordinate measuring machine with the minimum addition of optical hardware. The experimental setup made in this study is shown in Fig. 4. A slit beam projector is simply placed in the probe position of the machine, while the object to be measured is fixed on the worktable. The projector produces a plane-type slit beam of 0.8-mm width made by a series of cylindrical lenses from a He-Ne laser source. The necessary scanning process is performed with the aid of existing machine control without additional hardware. A 512 x 512 CCD camera is used with an image digitizing board interfaced to an IBM PC 386 computer. An exemplary measurement is illustrated in Fig. 5. The object measured is a human-face-shaped clay model with many sharp segments as shown in Fig. 5(a). Two gratings of deformed lines are obtained as
in Figs. 5(b) and 5(c) by scanning the slit beam with a 1.5-mm pitch. These two gratings are transformed into binary images by the thresholding process and then overlapped by AND operation so that moiré fringes of Fig. 5(d) are observed. Finally a threedimensional profile is constructed as shown in Fig. 5(e). This example proves that the slit beam scanning moiré topography is effective even for sharp surfaces.

Another example is demonstrated in Fig. 6 in which two similar-sized cups are compared directly with each other. Since the height differences between two consecutive moiré fringes are constant, the pattern of resulting moiré fringes for a given object does not vary with its location with respect to the camera. The two moiré fringes obtained from the cups should therefore be identical with each other if they are exactly the same in size. If there is any discrepancy between the two cups, it can be identified directly simply by subtracting one fringe from the other as illustrated in Fig. 6(c).

Conclusion

A slit beam scanning method of moiré topography has been described in which the height differences between two consecutive fringes become constant. This remarkable feature gives many advantages in applying moiré topography to practical three-dimensional measurements of surface profiles. As opposed to existing shadow and projection types, absolute fringe orders need not be identified and surface profiles can be obtained in an automatic manner simply by using a computer-aided image processing technique. In addition this method can be implemented easily in a conventional coordinate measuring machine with a minimum addition of hardware for computer-aided image processing.
References


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