Synergy/Perforation Control for 16-QAM in Orthogonal Code Hopping Multiplexing

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Abstract—Orthogonal code hopping multiplexing (OCHM) is a statistical multiplexing scheme designed to accommodate a large number of allowable downlink channels in code division multiple access (CDMA) systems. However, OCHM may yield a worse bit error rate (BER) performance than the conventional CDMA systems due to code collisions. Therefore, a synergy/perforation operation, which is a control scheme for mapping code-colliding modulation symbols to specific constellation points on the $I-Q$ plane, is needed to compensate for BER degradation, and this synergy/perforation operation should be defined separately with modulation types. Previously, synergy/perforation control schemes for quadrature phase shift keying (QPSK) and binary phase-shift keying (BPSK) have been proposed, but there has been no study of 16-quadratic-amplitude modulation (16-QAM) in OCHM. We propose two synergy/perforation control schemes for 16-QAM in OCHM and a log-likelihood ratio conversion scheme for 16-QAM in OCHM. The proposed control schemes are evaluated by an analytical approach and simulations and are compared to the previous scheme for QPSK modulation. We show that the proposed control scheme has a performance similar to that of QPSK modulation in terms of the mean synergy ratio for code-colliding modulation symbols and additional required $E_b/N_0$.

Index Terms—Log-likelihood ratio (LLR), modulation, orthogonal code-hopping multiplexing (OCHM), perforation, synergy, 16-quadratic-amplitude modulation (QAM).

I. INTRODUCTION

In the initial call setup process of conventional code division multiple access (CDMA) systems, a base station (BS) allocates a codeword to each mobile station (MS), which attempts a downlink connection. The BS spreads downlink data with an allocated codeword and multiplexes the spread data with other data. In second-generation systems, this code allocation and multiplexing scheme is effective in a voice-dominated traffic environment. However, in third-generation (3G) and beyond-3G wireless communication systems, packet-type traffic in the downlink will grow significantly. Packet-type traffic usually exhibits high burstiness and low activity. Therefore, if the conventional code channel allocation and multiplexing mechanism is used in 3G and beyond-3G systems with many inactive periods from packet call setup to termination, a waste of orthogonal codeword (OC) resources will occur. This resource waste and an increasing demand for downlink channels in 3G and beyond-3G systems will cause a lack of available downlink channels. A multiscrambling code in wideband CDMA [1] and a quasi-orthogonal function in cdma2000 [2] are used to mitigate this shortage of codeword resources. However, these techniques cause a partial loss of orthogonality among codewords.

In place of a completely new physical layer technique for packet-type traffic, orthogonal code hopping multiplexing (OCHM) was proposed by Park and Sung [3] as a new code channel allocation and multiplexing scheme to efficiently support packet-type traffic using OCs. Statistical multiplexing from code hopping can reduce waste of orthogonal code resources and provide more available downlink channels. However, the bit error rate (BER) performance of OCHM can be worse than the conventional scheme due to code collisions. Code collisions cause synergies if all the symbols experiencing the same code collisions have the same symbol values. Otherwise, they cause perforations. Therefore, a control scheme for synergies/perforations that maps code-colliding modulation symbols to specific constellation points on the $I-Q$ plane is needed to compensate for the degradation in BER performance. This synergy/perforation control should be defined differently for different modulation types.

OCHM has been studied and improved through our several previous papers. Performance comparisons between OCHM and high-data-rate (HDR) systems have been performed in the synchronous downlink [4], and various operation modes for OCHM have been proposed [5]. Multirate transmission for OCHM has been introduced [6], and OCHM capacity has been analyzed [7]. Channel coding at the OCHM receiver is important for the recovery of symbols lost due to code hopping, and the FER performance of OCHM has been evaluated [8], [9]. However, prior work has dealt with OCHM only for BPSK and QPSK modulations. Thus far, there has been no attempt.
to analyze 16-quadratic-amplitude modulation (16-QAM) in OCHM. With 16-QAM, synergy/perforation control is expected to be difficult and complex because there are more constellation points than for BPSK and QPSK, and a symbol represents more bits than the number of constellation dimensions. Since 16-QAM is one of the key techniques as a higher order modulation for achieving high transmission rates, a synergy/perforation control scheme and proper constellation mapping for 16-QAM in OCHM are required.

We propose two synergy/perforation control schemes. We also propose a new log-likelihood ratio (LLR) conversion scheme for 16-QAM in OCHM, which is essential for channel decoding of 16-QAM symbols at the receiver. The performance of the proposed synergy/perforation control schemes and the LLR conversion scheme is evaluated by an analytical approach and simulations and is compared to previous QPSK results.

The rest of this paper is organized as follows. The previously proposed OCHM scheme is briefly introduced in Section II. A synergy/perforation control scheme in BPSK and QPSK is described, and its performance is mathematically evaluated in Section III. Two synergy/perforation control schemes and a proper LLR conversion scheme for 16-QAM in OCHM are presented in Section IV. The performance of 16-QAM in OCHM with the proposed control schemes is evaluated by an analytical approach and simulation in Section V. Finally, conclusions are presented in Section VI.

II. OCHM

Code hopping schemes based on spread spectrum systems have been studied [10]–[12]. One advantage of these code hopping schemes is code diversity [12]. In a multicell environment, cochannel interference in the downlink is averaged, and better BER performance can be expected. All conventional studies regarding code hopping are focused mainly on code diversity, with no collisions among hopping patterns (HPs). However, OCHM is distinguished from conventional code hopping methods because code hopping in OCHM allows collisions and includes downlink multiplexing to increase the utilization of code channels.

Fig. 1 shows the basic operations of conventional orthogonal code division multiplexing (OCDM), which is the conventional CDMA, and OCHM. Each box indicates a modulation symbol with a duration of $T_s$. User information bits are encoded by channel coding and then mapped to modulation symbols (Fig. 1). Before multiplexing and transmission, modulation symbols are spread by OCs.

In OCDM [Fig. 1(a)], the OC of each MS is allocated by a BS at the initial call setup time and is maintained during a call. For example, a BS spreads the modulation data streams of MSs #a, #b, $\ldots$, #g with OCs #A, #B, $\ldots$, #G, respectively. The spread data streams are multiplexed, and the multiplexed data stream is transmitted. Then, each MS receives and de-spreads the code-multiplexed data stream with its own OC. In Fig. 1(a), the dotted boxes for MSs #c and #g represent the inactive periods that cause a waste of OC resources. Less activity in the downlink channels causes more OC resources to be wasted.

In OCHM [Fig. 1(b)], a BS spreads the modulation symbols of each MS using different OCs in every symbol time. For example, MS #f uses a different OC for each modulation symbol based on a HP indexed by #f. HP is defined as a set of OCs. A variable MS-specific HP is generated based on an MS identifier such as an electronic serial number (ESN) at an initial channel allocation time. No additional signaling for HP is needed during a call in OCHM. From the hopping of OCs, a BS can use all codewords and statistically multiplex more downlink channels than the given number of OCs. A statistical multiplexing gain can thus be achieved.

OCHM increases the utilization of OCs and accommodates more downlink channels than OCDM without additional signaling overhead during a call connection or session. However, when spread modulation symbols are multiplexed, code collisions may occur during one symbol time among downlink channels due to the allocation of the same codeword in the HP. Code collisions are illustrated as X-marked boxes in Fig. 1(b). However, a BS can monitor all downlink channel information during multiplexing and manage undesirable code collisions before transmission.

When a code collision among the HPs of active downlink channels occurs at a symbol time, the comparator and controller in the OCHM transmitter detect the code collision and perform either a synergy or a perforation for only the active downlink channels. In BPSK modulation, we can define synergy and perforation as follows.

1) **Synergy**: If modulation symbols with the same code collision(s) have an identical symbol value (1 or 0 for BPSK), all the corresponding data symbols are transmitted.

2) **Perforation**: If all the modulation symbols with the same code collision(s) do not have an identical symbol value, then none of the corresponding data symbols during a symbol time of $T_s$ are transmitted.
Each MS receives the code-multiplexed data stream with or without synergies and perforations and despreads the stream based on the HP allocated at the initial call setup time. Each MS has no information about code collisions (synergy/perforation) because there is no additional signaling during a call connection.

In addition to hopping of OCs, synergy/perforation is a key concept in OCHM. This synergy/perforation control performs only at the BS transmitter regardless of modulation types. However, it is differently defined and applied according to modulation types for better system performance.

### III. Synergy/Perforation Control for BPSK and QPSK in OCHM

In order to obtain a statistical multiplexing gain, OCHM allows code collisions among downlink channels and handles the code collisions using synergy and perforation. If modulation symbols related to a code collision have different constellation points (different symbol values), perforation should neutralize the modulation symbol values of the code-colliding symbols (no transmission in BPSK) because the symbol values biased to one of the constellation points cause difficulty in recovering the modulation symbols. On the other hand, synergy is applied if the modulation symbols related to a code collision have the same constellation point. The synergy operation enables the corresponding MSs experiencing the same code collision to share all the transmit power. Thus, a synergy operation results in a transmission power gain.

Synergies and perforations must be applied differently according to modulation types. Fig. 2 shows two examples of synergy/perforation control in BPSK. We assume that MSs indexed by #f and #b experience a code collision, and have different modulation symbols at a symbol time from \((n + 2)T_s\) to \((n + 3)T_s\), as shown in Fig. 1(b). The corresponding MSs experience a perforation, and no signal is received during the symbol time, as shown in the perforation case of Fig. 2. On the other hand, if two modulation symbols experiencing a code collision have the same value of “1” at a symbol time from \((n + 7)T_s\) to \((n + 8)T_s\), as shown in Fig. 1(b), the transmitted signal during the symbol time is a cumulative value of the signals assigned for the corresponding modulation symbols. This event is here called a synergy.

With QPSK, there are four distinct constellation points. These are selected independently by 2-bit indices that are separately mapped onto the I- and Q-axes. Thus, QPSK operates as two BPSK modulations. Fig. 3 shows two examples of synergy/perforation control in QPSK. Suppose MSs #b and #f have QPSK modulation symbols of 01 and 10, respectively. Then, each bit pair of the I- and Q-axes is different, and a perforation for 2 bits is required. On the other hand, two QPSK modulation symbols of 11 and 10 with the same code collision results in a 1-bit synergy on the I-axis and a 1-bit perforation on the Q-axis, as shown in the second example of Fig. 3. Therefore, five additional QPSK constellation points exist with two points located on both the I- and Q-axes, plus one zero point.

If code collisions occur only between two downlink channels at a symbol time in BPSK, half of the code collision symbols experience synergies, and the other half experience perforations because BPSK has only two modulation symbols (1 or 0). Thus, the synergy probability for a single bit, \(P_{s,1,BPSK}\), is \(1/2\). Since only a 1-bit synergy exists in BPSK, the synergy ratio \(\lambda_{s,BPSK}\), which is defined as the ratio of the number of synergy symbols to the total number of code-colliding symbols, is the same as the synergy probability for 1 bit and is expressed as

\[
\lambda_{s,BPSK} = 1(\text{symbol}) \times P_{s,1,BPSK} = \frac{1}{2}.
\]

A synergy ratio of 1/2 indicates that half of the code-colliding symbols experience synergy.

In QPSK, 2-bit indices represent I and Q independent channels. Thus, a modulation symbol may partially experience...
both synergy and perforation. If a code collision is assumed to occur only between two downlink channels at a symbol time, two types of synergy probability exist. First, if 2 bits of each QPSK modulation symbol is related to synergy, the synergy probability for 2 bits, \( P_{s,2,\text{QPSK}} \), is 1/4. Second, if only 1 bit of a QPSK modulation symbol experiences synergy, the synergy probability for 1 bit, \( P_{s,1,\text{QPSK}} \), is 2/4. Since 2 bits indicates one modulation symbol in QPSK, the synergy ratio is expressed as

\[
\lambda_{s,\text{QPSK}} = 1\text{ (symbol)} \times P_{s,2,\text{QPSK}} + \frac{1}{2} \text{ (symbol)} \times P_{s,1,\text{QPSK}} = \frac{1}{2}.
\]  

(2)

The synergy ratio for code-colliding modulation symbols in QPSK is the same as the ratio in BPSK because of independent \( I \)- and \( Q \)-axes.

A channel decoder in the receiver can recover error symbols from the fading channel and additionally perforated symbols. Link-level simulation results show that a larger transmission power level is required in OCHM than in OCDM to compensate for the perforated symbols [3]. Thus, a decrease in the synergy ratio for code-colliding symbols indicates that the transmitter requires a larger \( E_b/N_0 \) value to satisfy a given BER requirement. In the next section, we propose two new synergy/perforation control schemes for 16-QAM in OCHM.

IV. PROPOSED SYNERGY/PERFORATION CONTROL SCHEMES FOR 16-QAM

16-QAM is an attractive modulation scheme with a high spectral efficiency. However, as an amplitude-phase-based scheme, 16-QAM is more sensitive to channel variation than phase-based modulation schemes such as BPSK and QPSK. Thus, amplitude compensation is important in 16-QAM for maintaining the required BER performance in fading environments. Conventional power control mechanisms [13], which have been used for BPSK and QPSK, have a limitation for 16-QAM due to feedback delay and errors. Estimation errors in amplitude and phase compensation in 16-QAM cause severe degradation in BER performance [14]. Thus, a pilot-symbol-assisted modulation (PSAM) scheme [15] can be an effective choice to compensate for amplitude and phase variation in 16-QAM. In PSAM, a pilot signal is continuously or periodically transmitted to all MSs in a cell and is used as a reference signal for amplitude and phase compensation. While transmitted signal constellations for MSs are different in amplitude due to power control in BPSK and QPSK, the transmitted signal constellations are identical in PSAM. Each MS demodulator scales the received signal to normalize channel gain so that its received decision region is the same as the transmitted signal constellation. We assume PSAM in 16-QAM, and thus, the transmitted signal constellations for all MSs are identical.

A. Synergy/Perforation Control Scheme I

Synergy and perforation operations in 16-QAM are more complex than in BPSK and QPSK. Since more constellation points increase the probability that code-colliding symbols have different constellation points, perforations occur more frequently in 16-QAM than in BPSK or QPSK. BS transmission power must be increased in order to compensate for performance degradation due to an increased number of perforations. In addition, various constellation mappings are possible because each 16-QAM symbol consists of 4 bits. Therefore, it is hard to find the appropriate constellation points for the synergy and perforation of modulation symbols.

The first synergy/perforation control proposal for 16-QAM in OCHM, which is called Scheme I, is a BPSK/QPSK-like synergy/perforation control scheme. For simple explanation, we only consider \( I \)-axis bits since the \( I \)- and \( Q \)-axes are independent. Fig. 4 shows two examples of synergy/perforation control on the \( I \)-axis. If two downlink channels experience a code collision at a symbol time, Scheme I determines the synergy when two modulation symbols in the two downlink channels related to the corresponding code collision have the same constellation point on the \( I \)-axis. The synergized symbol is then mapped to their own constellation points, as shown in Fig. 4(a). A perforation occurs when the code-colliding symbols have different constellation points on the \( I \)-axis. The perforated symbols are then mapped to the zero point. In Scheme I, as shown in the perforation for BPSK and QPSK, the zero point on the \( I \)-axis is only a perforation point [Fig. 4(b)].

Considering both the \( I \)- and \( Q \)-axes, the synergy for 4 bits occurs when two modulation symbols with the same codeword in the two downlink channels have the same constellation point among 16 candidates. Thus, the synergy probability for 4 bits, \( P_{s,4,\text{16-QAM}} \), is 1/16. The synergy for 2 bits occurs when two modulation symbols with the same codeword in the two downlink channels have the same constellation point at only one of the \( I \)- and \( Q \)-axes. Thus, the synergy probability for 2 bits, \( P_{s,2,\text{16-QAM}} \), is 6/16. Based on these two synergy probabilities, the synergy ratio for code-colliding modulation symbols in 16-QAM (Scheme I) is derived as

\[
\lambda_{s,\text{16-QAM}} = 1\text{ (symbol)} \times P_{s,4,\text{16-QAM}} + \frac{1}{2} \text{ (symbol)} \times P_{s,2,\text{16-QAM}} = \frac{1}{4}.
\]  

(3)
From this approach, only a quarter of code-colliding modulation symbols experience synergies. Therefore, it is expected that the BER performance degradation for 16-QAM is larger than for both BPSK and QPSK, which have synergy ratios of 1/2.

**B. Synergy/Perforation Control Scheme II**

For mitigating the BER degradation in Scheme I for 16-QAM, a new scheme that can increase the synergy ratio for code-colliding modulation symbols is proposed. Fig. 5 shows a gray-coded 16-QAM constellation for this new synergy/perforation control scheme. The first two bits indicate the \(I\)-axis position, and the last two bits indicate the \(Q\)-axis positions. The X-marked circles (\(\bigotimes\)) indicate the possible positions of 16-QAM symbols with synergy and perforation. There are synergized/perforated points on the \(I\)- and \(Q\)-axes and on the \(I-Q\) plane.

For simplification, we only consider \(I\)-axis bits. Fig. 6 shows three examples of synergy/perforation control on the \(I\)-axis. Boundaries between points \(a/b\) and \(c/d\) are determined by the first bit, which can be called the most significant bit (MSB). Points \(a\) and \(b\) or \(c\) and \(d\) are separated by the second bit, which can be called the least significant bit (LSB). Three types of synergy/perforation may occur on the \(I\)-axis if a code collision is assumed to occur only between two downlink channels at a symbol time. First, if two modulation symbols related to a code collision in the two downlink channels have the same constellation point, \(d\) among four points [Fig. 6(a)], each code-colliding symbol experiences a synergy for both MSBs and LSBs, and the synergized constellation point is \(d\). Second, if two modulation symbols related to a code collision in the two downlink channels have the same \(I\) and different \(Q\) [Fig. 6(b)], the synergy for MSBs and the perforation for LSBs occur, and the synergized/perforated constellation point is the midpoint between the two code-colliding symbols. Third, if the perforation for both MSBs and LSBs exist when MSBs are different between the two modulation symbols, the perforated constellation point has a zero value [Fig. 6(c)]. For the \(Q\)-axis, the mapping rule is the same as for the \(I\)-axis. A 2-D expansion of the mapping rules in the \(I\) and \(Q\)-axes is a complete 16-QAM constellation mapping, as shown in Fig. 5.

For example, if we assume that two downlink channels experience a code collision during a symbol time, and one of the two modulation symbols has a point indexed by \((1111)\), the synergy for 4 bits occurs when the other code-colliding symbol has the same constellation point. The synergy for 3 bits occurs when the other code-colliding symbol has a point indexed by either \((1011)\) or \((1110)\). Similarly, the synergy for 2 bits occurs when the other code-colliding symbol has a point indexed by \((0011), (0111), (1010), (1100)\), or \((1101)\). The synergy for 1 bit occurs when the other code-colliding symbol has a point indexed by \((0010), (0110), (1000), (1001)\). Thus, the synergy probabilities of the above four cases are calculated as \(P_s,4,16\)-QAM = 1/16, \(P_s,3,16\)-QAM = 2/16, \(P_s,2,16\)-QAM = 5/16, and \(P_s,1,16\)-QAM = 4/16.

Therefore, the synergy ratio for code-colliding modulation symbols in Scheme II can be expressed as

\[
\lambda_{s,16\text{-QAM}} = 1\text{symbol} \times P_{s,4,16\text{-QAM}} + \frac{3}{4}\text{symbol} \times P_{s,3,16\text{-QAM}} + \frac{2}{4}\text{symbol} \times P_{s,2,16\text{-QAM}} + \frac{1}{4}\text{symbol} \times P_{s,1,16\text{-QAM}} = \frac{3}{8}.
\] (4)
Fig. 7. Proposed LLR conversion scheme for 16-QAM. (a) LLR function of MSB, \(d_4\). (b) LLR function of LSB, \(d_3\).

Code-colliding modulation symbols have a synergy ratio of 3/8. Compared with the results in (3), the synergy ratio for code-colliding modulation symbols is improved in 16-QAM.

C. LLR Conversion Scheme for the Proposed Synergy/Perforation Control in 16-QAM

In addition to the proposed synergy/perforation control in the transmitter, an appropriate LLR conversion scheme is also needed for 16-QAM in the receiver. In a soft-input decoder, such as a turbo decoder, the channel demodulator output is generally demapped and used as an input value for the decoder. In 16-QAM, the four signal classes \((-3D/2), (-D/2), (D/2), (3D/2)\) exist on the I- and Q-axes, where \(D\) is the distance between adjacent signal classes. Each signal class is mapped to a bit index of 1 or 0. For example, for the MSB in Scheme II, the four signal classes \((-3D/2), (-D/2), (D/2), (3D/2)\) are mapped to 0, 0, 1, and 1, respectively. Therefore, the LLR values are obtained as the logarithm of the ratio of the likelihood values on the signal classes mapped to bit index 1 and the likelihood values on the signal classes mapped to bit index 0 [16], [17].

We assume to transmit \(d = (d_4, d_3, d_2, d_1)\) equiprobably and receive \(y = (y_I, y_Q)\), where \(d_4\) and \(d_3\) are the MSB and the LSB, respectively, on the I-axis. Since the four signal classes of the MSB are mapped to bit indices 0, 0, 1, and 1, the LLR function for the MSB is derived as

\[
L(d_4|y_I) = L(y_I|d_4)
= \log \frac{\frac{1}{2} P(y_I|d_4 = \frac{D}{2}) + \frac{1}{2} P(y_I|d_4 = -\frac{3D}{2})}{\frac{1}{2} P(y_I|d_4 = -\frac{D}{2}) + \frac{1}{2} P(y_I|d_4 = \frac{3D}{2})}.
\]

In 16-QAM with control scheme II, a zero point is used for a perforation value when two MSBs are different. Therefore, the zero point should have unbiased information for the code-colliding symbols. However, the zero point of control scheme II with gray-coded mapping does not have a zero LLR value for the LSB since the bit indices are 0, 1, 1, and 0. Thus, the LLR function for the LSB must be modified.

In Regions 1 and 4, we can obtain approximated LLR equations from (7). In Region 1, \(P(y_I|d_3 = (-D/2))\) and \(P(y_I|d_3 = (-3D/2))\) are used for the LLR calculation since \((-D/2)\) and \((-3D/2)\) are the closest constellation points mapped to bit indices 1 and 0, respectively. Likewise, in Region 4, the LLR is obtained
respectively, as shown in Fig. 8(b). This modified LLR function is the same as in Fig. 7(b) in Region 3. Therefore, the LLR line in Region 3 is expressed as

\[
L(d_3|y_1) = L(y_1|d_3) = \log \frac{1}{2} P (y_1|d_3 = \frac{-D}{2}) + \frac{1}{2} P (y_1|d_3 = \frac{3D}{2})
\]

\[
\approx \log \frac{1}{2} P (y_1|d_3 = \frac{-D}{2}) + \frac{1}{2} P (y_1|d_3 = \frac{3D}{2}) = \frac{D y_1}{\sigma^2} \quad \text{(Region 3)}.
\]

(V) PERFORMANCE EVALUATION OF 16-QAM IN OCHM WITH THE PROPOSED CONTROL SCHEMES

A. Effect of Code Collisions Related to Three or More Downlink Channels

For simple descriptions in the previous sections, code collisions were assumed to occur only between two downlink channels at a symbol time. However, if three or more downlink channels have the same codeword at a symbol time, a code collision can also occur. From the viewpoint of a specific downlink channel related to the code collision, this is a code collision among multiple downlink channels. When we focus on code collisions only between two downlink channels at a symbol time, the synergy ratios for code-colliding modulation symbols in QPSK and 16-QAM using the proposed control scheme II are 1/2 and 3/8, respectively, as derived in (2) and (4). However, code collisions related to three or more downlink channels decrease the synergy ratio. To evaluate the effect of code collisions related to three or more downlink channels, two factors have to be considered: degradation due to code collisions related to three or more downlink channels and the code-collision probability among three or more downlink channels in a given environment. In this section, we evaluate the above two factors independently for QPSK and 16-QAM and then combine both results.

First, the conditional synergy ratio for code-colliding modulation symbols is analyzed. Here, “conditional” means that we assume that code collisions among \( n_c \) downlink channels occur at a symbol time. In other words, a downlink channel experiences a code collision with other \((n_c - 1)\) downlink channels at a symbol time. The conditional synergy probabilities for 2 bits, \( P_{s,2,\text{QPSK}|n_c} \), and for 1 bit, \( P_{s,1,\text{QPSK}|n_c} \), and the conditional synergy ratio for code-colliding modulation symbols for a code collision among \( n_c \) downlink channels, \( \lambda_{s,\text{QPSK}|n_c} \), are, respectively, expressed as

\[
P_{s,2,\text{QPSK}|n_c} = \left( \frac{1}{4} \right)^{n_c-1}
\]

\[
P_{s,1,\text{QPSK}|n_c} = \frac{(2n_c-1)}{4^n_c-1} \times 2
\]

\[
\lambda_{s,\text{QPSK}|n_c} = 1 \times P_{s,2,\text{QPSK}|n_c} + \frac{1}{2} \times P_{s,1,\text{QPSK}|n_c} = \left( \frac{1}{2} \right)^{n_c-1}.
\]
In 16-QAM, since the calculation of the conditional synergy probabilities is more complex than in QPSK, the calculation is focused only on the $I$-axis. The $I$- and $Q$-axes in 16-QAM are independent of each other, and the result from the $I$-axis is the same as the result from both the $I$- and $Q$-axes. On the $I$-axis, 2 bits represents one symbol, and the synergy for 1 bit and 2 bits can occur. If a preselected downlink channel experiences a code collision with other ($n_c - 1$) downlink channels at a symbol time, a synergy for 2 bits occurs when the other ($n_c - 1$) downlink channels have the same constellation point as the preselected downlink channel, and a synergy for 1 bit occurs when the other ($n_c - 1$) downlink channels have one of the two constellation points indexed by the same MSB, except the case when all $n_c$ downlink channels have the same constellation points. Therefore, the conditional synergy probabilities for 2 bits on the $I$-axis, $P_{s,2,16-QAM,I|n_c}$, and for 1 bit on the $I$-axis, $P_{s,1,16-QAM,I|n_c}$, and the conditional synergy ratio for code-colliding modulation symbols in 16-QAM with the proposed control scheme II, $\lambda_{s,16-QAM|n_c}$, are, respectively, expressed as

$$
P_{s,2,16-QAM,I|n_c} = \left( \frac{1}{4} \right)^{n_c-1}
$$

$$
P_{s,1,16-QAM,I|n_c} = \frac{2^{n_c-1}}{4} 
$$

$$
\lambda_{s,16-QAM|n_c} = \lambda_{s,16-QAM,I|n_c}
$$

$$
= 1(\text{symbol}) \times P_{s,2,16-QAM,I|n_c} + \frac{1}{2}(\text{symbol}) \times P_{s,1,16-QAM,I|n_c}
$$

$$
= \frac{2^{n_c-1} + 1}{2 \times 4^{n_c-1}}
$$

(12)

where $\lambda_{s,16-QAM,I|n_c}$ is the conditional synergy ratio for code-colliding modulation symbols on the $I$-axis when a downlink channel experiences a code collision with other ($n_c - 1$) downlink channels at a symbol time.

Fig. 9 shows the conditional synergy ratios for (11) and (12) as $n_c$ increases. Even when $n_c = 3$, the conditional synergy ratio for code-colliding modulation symbols in 16-QAM with the proposed control scheme II is 0.15. Thus, code collisions for large $n_c$ values yield a larger BER performance degradation than for $n_c = 2$. However, the code collision probability among three or more downlink channels is generally extremely low, especially for a low channel activity and a large number of OCs. The code-collision probability among $n_c$ downlink channels at a symbol time occurs, we consider an $N_{OC}$-axis, $M$ allocated orthogonal downlink channels, and a mean activity value of $\bar{\nu}$ for all allocated downlink channels. The code-collision probability among $n_c$ downlink channels can be determined from the number of active downlink channels at the time and the number of available codewords that can be used by the active downlink channels. We assume that channel activity follows a Bernoulli process. With $k$ active channels, the code-collision probability between two downlink channels is the product of the probability for choosing $(k-1)$ active channels among $(M-1)$ and the probability for choosing one code-colliding channel among $(k-1)$ and is expressed as

$$
p_c(2,k) = \left( \frac{M-1}{k-1} \right) \bar{\nu}^{k-1}(1-\bar{\nu})^{M-k} \cdot \left( \frac{k-1}{1} \right) \left( \frac{1}{N_{OC}} \right)^{n_c-1} \left( 1 - \frac{1}{N_{OC}} \right)^{k-n_c}
$$

(13)

Therefore, the code-collision probability between two downlink channels is the summation of the code-collision probabilities with 2, 3, ..., $M$ active channels. This probability can be expressed as

$$
p_c(2) = \sum_{k=2}^{M} p_c(2,k).
$$

(14)

From an extension of (13) and (14), the code-collision probability among $n_c$ downlink channels is derived as

$$
p_c(n_c) = \sum_{k=n_c}^{M} \left\{ \left( \frac{M-1}{k-1} \right) \bar{\nu}^{k-1}(1-\bar{\nu})^{M-k} \cdot \left( \frac{k-1}{n_c-1} \right) \left( \frac{1}{N_{OC}} \right)^{n_c-1} \left( 1 - \frac{1}{N_{OC}} \right)^{k-n_c} \right\}
$$

$$
\simeq \left( \frac{M-1}{n_c-1} \right) \left( \frac{\bar{\nu}}{N_{OC}} \right)^{n_c-1} \left( 1 - \frac{\bar{\nu}}{N_{OC}} \right)^{M-n_c}
$$

(15)

where $p_c(n_c)$ is the probability of selecting $(n_c - 1)$ downlink channels that meet the two conditions of “active” and “having the same codeword as the preselected channel.”

Fig. 10 shows the code-collision probability. We consider two cases of channel activity and three different cases for the number of allocated orthogonal downlink channels. For a given value of $n_c$, large values of $\bar{\nu}$ and $M$ yield relatively high code-collision probabilities. Note in Fig. 10 that the code-collision
probability is greatly reduced as \( n_c \) increases from 2 to 3. For \( n_c \geq 4 \), the code-collision probabilities of all cases are lower than 0.02.

Combining (11), (12), and (15), we can derive the mean synergy ratio for code-colliding modulation symbols including the effect of code collisions among three or more downlink channels. The mean synergy ratio for the code-colliding modulation symbols \( \bar{\lambda}_{s,\text{QPSK}} \), and \( \bar{\lambda}_{s,\text{16-QAM}} \) is expressed as

\[
\bar{\lambda}_{s,\text{QPSK}} = \frac{\sum_{n_c=2}^{M} p_c(n_c) \cdot \lambda_{s,\text{QPSK}|n_c}}{p_c,\text{OC}}
\]

\[
\bar{\lambda}_{s,\text{16-QAM}} = \frac{\sum_{n_c=2}^{M} p_c(n_c) \cdot \lambda_{s,\text{16-QAM}|n_c}}{p_c,\text{OC}}
\]  \hspace{1cm} (16)

where \( p_c,\text{OC} \) is defined as the code-collision probability among multiple downlink channels and is calculated as \( \sum_{n_c=2}^{M} p_c(n_c) \).

Fig. 11 shows the mean synergy ratio versus the mean channel activity for QPSK and 16-QAM if we consider the effect of code collisions among multiple downlink channels. The two horizontal lines at 1/2 (QPSK) and 3/8 (16-QAM) correspond to the mean synergy ratio values if we only consider code collisions between two downlink channels. The actual mean synergy ratios for code-colliding modulation symbols are related to BER performance degradation since a small number of synergies yields a large number of perforations, which demands a slightly larger power for maintaining the given quality of service. If we consider a typical data channel activity of 0.1 for packet transmission, the difference in the mean synergy ratio for both cases is very small. Even for \( M = 128 \) and \( \bar{\nu} = 0.3 \), the mean synergy ratios for code-colliding modulation symbols are reduced to less than 0.1.

**B. Additional Required \( E_b/N_0 \) in 16-QAM for the Proposed Control Schemes**

OCHM requires slightly larger \( E_b/N_0 \) values than OCDM to compensate for perforated modulation symbols in order to maintain the same BER (see Section III). Since the number of perforated symbols in 16-QAM is larger than in QPSK from (2)–(4), 16-QAM may require a larger \( E_b/N_0 \) value than QPSK under the same code-collision probability. In this section, we investigate the required power under a given mean synergy ratio for code-colliding modulation symbols.

The performance of the proposed synergy/perforation control schemes for 16-QAM is evaluated through simulation. From the simulation, BER performance of the proposed schemes is estimated. At a transmitter, information bits are encoded by turbo coding and are mapped to constellation points. With the constellation mapping, a comparator determines whether each modulation symbol code-collides with other modulation symbols by comparing the HPs of multiple downlink channels. Code-colliding symbols are controlled using the rules of QPSK and 16-QAM (Schemes I and II). In 16-QAM, the proposed LLR conversion scheme is used at the receiver. The detailed simulation environment is described in Table I.

**A performance measure that we used in this simulation is the additional required \( E_b/N_0 \) for a BER value of 0.001. The reference values for the required \( E_b/N_0 \) are set for the \( E_b/N_0 \) values of QPSK and 16-QAM in an OCDM system. Thus, the additional required \( E_b/N_0 \) is needed to compensate for BER performance degradation from the viewpoint of energy, compared with the reference BER result. As the code-collision
probability increases, the number of perforated modulation symbols increases, and additional power is required to obtain a given BER value of 0.001.

Fig. 12 shows the performance of QPSK and 16-QAM for Schemes I and II. For a code-collision probability of 0.2 and a code rate of 1/3, QPSK and 16-QAM with the proposed control scheme II require an additional 0.8 and 1.3 dB of power, respectively, whereas 16-QAM with Scheme I needs an additional 3.4 dB of power. From (2) and (4), a few more perforated symbols in 16-QAM with Scheme II yield 0.5 dB of difference, compared to QPSK. For a code rate of 1/4, the difference in the additional required $E_b/N_0$ between QPSK and 16-QAM (Scheme II) is smaller than 0.2 dB in the entire range of code-collision probabilities. For 16-QAM in Scheme I, poor BER performance is shown because many code-colliding symbols result in perforations. From the result shown in Table II [3], OCHM can accommodate more than 200 downlink channels with 64 OCs, a mean channel activity of 0.1, and a code-collision probability of 0.2. In this case, if the data symbols of each downlink channel are encoded with a code rate of 1/4 and are modulated by the proposed 16-QAM, the additional required $E_b/N_0$ of OCHM is less than 0.7 dB.

### C. Required $E_b/N_0$ With a Given Input Data Traffic Load

In previous studies [3], [5], the code-collision probability was calculated under the condition of $N_{OC}$ OCs, $M$ allocated downlink channels, and a mean channel activity of $\bar{v}$. In

\begin{equation}
\gamma = \frac{M \cdot \bar{v}}{N_{OC} \cdot 1}.
\end{equation}

Fig. 13 shows the required $E_b/N_0$ value at a BER value of 0.001 for QPSK and 16-QAM (Scheme II). The required $E_b/N_0$ values of 16-QAM and QPSK increase with the normalized input load $\gamma$. With $\gamma = 0.3$, the required $E_b/N_0$ values of the proposed 16-QAM scheme do not exceed 5.1 dB with code rates of 1/3 and 1/4, and this normalized input load means that the OCHM system can accommodate 192 downlink channels of a channel activity factor of 0.1 only with 64 OCs. Compared to QPSK, the required $E_b/N_0$ value of 16-QAM is larger due to an inherited BER difference [18], and the difference is maintained even for large $\gamma$ values. However, there is a transmission rate difference between 16-QAM and QPSK. Under the assumption of the same transmission rate and the same allocated downlink channels, the mean channel activity of allocated downlink channels can be different, depending on modulation types. Since a 16-QAM symbol has four channel-encoded bits, the downlink data rate of 16-QAM is twice faster than for QPSK, and the downlink channels of 16-QAM exhibit a lower channel activity than for QPSK. This difference causes a code-collision difference between 16-QAM and QPSK, which
is expressed as \((1 - p_{c,\text{OC,QPSK}}) = (1 - p_{c,\text{OC,16-QAM}})^2\) derived in the Appendix. Therefore, the difference of the required \(E_b/N_0\) values between 16-QAM and QPSK can be reduced for the same transmission rate in OCHM.

VI. CONCLUSION

Thus far, the performance of OCHM has been evaluated based on BPSK and QPSK under the assumption that code collisions occur only between two downlink channels at a symbol time. We have proposed two new synergy/perforation control schemes for 16-QAM in OCHM and have analytically evaluated their performance by considering the effect of code collisions related to three or more downlink channels. In addition, an LLR conversion scheme has also been proposed for 16-QAM in OCHM.

For BPSK, QPSK, and 16-QAM (Schemes I and II), the expected values of the number of synergized symbols among code-colliding symbols were calculated and compared numerically. The proposed control scheme II exhibits a synergy ratio for code-colliding modulation symbols of 3/8, whereas the ratios are 1/4 and 1/2 for the proposed control scheme I and QPSK, respectively.

In order to investigate how much additional power \((E_b/N_0)\) is needed for a given expectation of the number of synergized symbols per code-colliding symbol, simulation was performed. From simulation results, BER performance was evaluated and compared for the three modulation methods, QPSK and 16-QAM with both Schemes I and II. The second synergy/perforation control scheme for 16-QAM showed a trend similar to QPSK for additional required \(E_b/N_0\) with differences smaller than 0.5 dB for a code rate of 1/4.

Finally, the required \(E_b/N_0\) values for the QPSK and 16-QAM methods were evaluated and compared for various input load. We thus determined a proper range for \(E_b/N_0\) in 16-QAM.

APPENDIX

PROOF OF THE RELATIONSHIP BETWEEN THE CODE-COLLISION PROBABILITIES OF QPSK AND 16-QAM

We prove the relation \((1 - p_{c,\text{OC,QPSK}}) = (1 - p_{c,\text{OC,16-QAM}})^2\) under the assumption that both QPSK and 16-QAM systems have an identical input data traffic load. Then, if we assume that \(\bar{\nu}\) denotes the mean channel activity value of QPSK modulation, the mean channel activity value of the 16-QAM system is \(\bar{\nu}/2\) in an average sense since bits per symbol in 16-QAM is twice larger than in QPSK. Therefore, the code-collision probabilities of QPSK and 16-QAM can be expressed as [5]

\[
p_{c,\text{OC,QPSK}} = 1 - \left(1 - \frac{\bar{\nu}}{N_{OC}}\right)^{M-1}
\]

\[
p_{c,\text{OC,16-QAM}} = 1 - \left(1 - \frac{\bar{\nu}/2}{N_{OC}}\right)^{M-1}
\]

(A.1)

where \(N_{OC}\) is the number of OCS, and \(M\) is the number of allocated orthogonal downlink channels.

Let \(y = (1 - p_{c,\text{OC,16-QAM}}), x = (1 - p_{c,\text{OC,QPSK}}), \) and \(t = (M - 1)\). Then, we can express (A.1) as

\[
y = \left(\frac{1 + x^\frac{1}{2}}{2}\right)^t.
\]

(A.2)

If we take logarithms on both sides of (A.2), \(\log y = t \cdot \log \left(\frac{1 + x^\frac{1}{2}}{2}\right)\). Finally, we can obtain the following relation by L'Hospital's rule:

\[
\lim_{t \to \infty} \log y = \lim_{t \to \infty} \frac{\log \left(\frac{1 + x^\frac{1}{2}}{2}\right)}{t}
\]

\[
= \lim_{t \to \infty} \frac{2 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \log x}{1/t}
\]

\[
= \frac{1}{2} \cdot \log x.
\]

(A.3)

Therefore, \(y = \sqrt{x}\) as \(t\) goes to infinity. This relationship is equivalent to

\[
(1 - p_{c,\text{OC,QPSK}}) = (1 - p_{c,\text{OC,16-QAM}})^2.
\]

(A.4)

REFERENCES


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