Period doubling and chaos in a directly modulated laser diode

Chang-Hee Lee, Tae-Hoon Yoon, and Sang-Yung Shin

Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, P.O. Box 150, Chongunngi, Seoul, Korea

(Received 16 August 1984; accepted for publication 8 October 1984)

It is shown theoretically that the directly modulated laser diode, with the modulation frequency of its injection current comparable to the relaxation oscillation frequency, exhibits period doubling route to chaos as the modulation index of current is increased. The effect of spontaneous emission factor on the chaotic behavior in the laser diode is also studied.

Chaos in optical devices and laser systems has been of major interest in recent years, since Ikeda et al. first predicted it in an optical ring cavity. Chaotic behavior in a nonautonomous laser system has been experimentally observed in a Q-switched CO$_2$ laser by Arecchi et al., and it has also been modeled theoretically. When the relaxation time of the population inversion is much larger than the memory time of the induced dipoles, the Maxwell–Bloch equations for a single mode laser may be reduced to the familiar rate equations by adiabatically eliminating the polarization. In this situation it was found that chaotic behavior arises in a single mode laser when its losses are periodically modulated with a frequency comparable to the relaxation oscillation frequency. To demonstrate the existence of chaotic behavior in a laser with a modulated pump, the semiconductor laser diode will provide a good example. There have been several reports on chaotic behavior occurring in a semiconductor laser diode that has an external feedback. Though some instabilities have been known to exist in laser diodes without any external feedback, the period-doubling route to chaos has not been reported.

Dynamics of semiconductor laser diodes can be described in terms of the following nonlinear rate equations for the electron density $N$ and the photon density $S$:

$$\frac{dN}{dt} = I - eV - \frac{N}{t_1} - A(N - N_0)S, \quad (1)$$

$$\frac{dS}{dt} = A(N - N_0)S - S/t_3 + \beta N/t_4, \quad (2)$$

where $I$ is the injection current, $e$ the electron charge, $V$ the volume of active region, $t_1$ and $t_4$ are the spontaneous electron and photon lifetime, respectively. The spontaneous emission factor $\beta$ is the fraction of the spontaneous emission that is coupled to the lasing mode. The constant $A$ is the gain coefficient and $N_0$ is the minimum electron density required to obtain a positive gain. Later, we will comment on the validity of single-mode rate equations in this work.

We consider that the laser diode is modulated by a sinusoidal current superimposed on the dc bias current, i.e.,

$$I = I_b + I_m \sin(\omega t), \quad (3)$$

where $\omega$ is the angular frequency of the modulation current.

A small-signal analysis of the rate equations gives the resonance frequency $f_0$:

$$2\pi f_0 = (U_b/I_{th} - 1)(1 + AN_0t_p/t_s)^{1/2}, \quad (4)$$

where $I_{th}$ is the threshold current. The existence of this relaxation resonance frequency means that the semiconductor lasers exhibit a resonance peak in their modulation characteristics before the final high-frequency falloff. Thus we will roughly divide the frequency region into two, i.e., one larger than $f_p$ and the other smaller than $f_0$, in studying chaotic behavior of laser diodes.

Numerical calculations are performed with typical parameters of semiconductor laser diode: $t_p = 1$ ps, $t_s = 3$ ns, $A = 10^{-6}$ cm$^3$/s, $N_0 = 10^{18}$ cm$^{-3}$, $\beta = 10^{-5}$, and $I_{th} = 1.4I_{th}$.

At first the frequency of modulation current, smaller than the resonance frequency, is chosen at $f = \omega/2\pi = 0.866f_0$. As $m$ is increased so that the minimum current approaches the threshold current level, the photon density becomes spiky [Fig. 1(a)]. This feature has been utilized in generating picosecond optical pulses at a very high repetition rate. With the further increase of $m$, the first subharmonic bifurcation (or period doubling) occurs at $m = 0.354$. Typical waveforms of period 2, period 4, and "chaos" are shown in Figs. 1(b), 1(c), and 1(d).

A bifurcation diagram of the sampled peak photon density $S_+$ versus the modulation index $m$ (Fig. 2) shows period-doubling route to chaos, inverse bifurcations (or period halvings) leading to the final period 2, and hysteresis at the beginning of the final period 2. To understand the qualitative features of chaotic behavior, we have plotted in Fig. 3 return maps of sampled peak photon density, i.e., the peak photon density $S_n + 1$ in the $n + 1$th period vs the peak photon density $S_n$ in $n$th period, at three different modulation indexes. The return map in one-band chaos ($m = 0.59$), that is indicated by (a), is similar to that of nonlinear logistic equation (i.e., it has a parabolic shape) except a folding on the right-hand side. With the further increase of $m$, it evolves as shown by (b) and (c). These two are for two-band chaos on the way towards the final period 2, and they show significant changes that could be related with subsequent chaotic behavior.

For a modulation frequency lower than about the half

![FIG. 1. Waveforms of photon density at various values of modulation index. (a) Period 1 at m = 0.30, (b) period 2 at m = 0.40, (c) period 4 at m = 0.47, and (d) chaos at m = 0.59.](image_url)
of the relaxation oscillation frequency, the photon density exhibits more than one spikes in a period, like relaxation oscillations. Even in this regime, we found that there still occur period doublings and chaotic behavior.

Now, we study dynamic behavior of laser diode in the region of $f$ greater than $f_{0}$, where the small-signal modulation response decreases steeply with frequency. Numerical calculations, that have been performed at $f = 1.5f_{0}$, with all the rest of the parameters the same as before, yield a bifurcation diagram shown in Fig. 4. As we increase the modulation index to $m = 0.406$, the first period doubling occurs. At a slightly larger $m = 0.414$, suddenly the photon density does not pulsate in every period of the injection current but once in every other period. In other words, the lower level of this period 2 is almost zero, as it remains the first subharmonic bifurcated (or period doubled) solution. Such a sudden behavioral change usually accompanies hysteresis or coexistence of independent stable attractors. Thus we searched to find that in a wide region from $m = 0.114$ to $m = 0.406$, two stable attractors, the period 1 and the period 2, coexist and in the narrow region from $m = 0.406$ to 0.414 two different period 2 attractors coexist. In a numerical calculation, Tarucha and Otsuka have demonstrated pulsations of photon density during every other period. Harth and Siemsen have shown analytically the first subharmonic generation of the modulating signal at the frequency $f$ near $2f_{0}$, when the spontaneous emission factor is zero. And later it was also observed spectrally in experiment. However, it has not been realized that such a subharmonic bifurcation is the onset of period-doubling route to chaos, and that two stable attractors coexist. Figure 4 shows that as we increase the modulation cur-

![FIG. 2. Bifurcation diagram of the sampled peak photon density $S_n$ vs the modulation index $m$ of injected current at $f = 0.866 f_0$.](image)

![FIG. 3. Return maps evolving with the increase of modulation index. (a) $m = 0.59$, (b) $m = 0.685$, and (c) $m = 0.75$.](image)

![FIG. 4. Bifurcation diagram of the sampled peak photon density $S_n$ vs the modulation index $m$ of injection current at $f = 1.5 f_0$.](image)

![FIG. 5. Bifurcation diagrams drawn to study the effect of the spontaneous emission factor. (a) $f = 0.866 f_0$, $\beta = 3 \times 10^{-5}$; (b) $f = 1.5 f_0$, $\beta = 10^{-4}$.](image)
\( \beta = 10^{-5} \), and hysteresis at the beginning of the period 2 is
narrower but significant. In both cases, the spontaneous emission
factor suppresses and/or delays the chaotic behavior in
laser diodes. We also found that, for a sufficiently large value of
\( \beta \), e.g., \( 10^{-2} \), the laser diode does not exhibit any perio-
doubling bifurcations.

To study fully the chaotic behavior of laser diodes, the
multimode rate equations have to be employed. Thus we also
performed numerical calculations by employing multimode
rate equations. It was found that the total photon density of
multimode rate equations exhibits a period-doubling route to
chaos, which is qualitatively similar to that of single-mode
rate equations. Therefore, our use of single mode rate equa-
tions may be justified in demonstrating the period-doubling
route to chaos, unless we are interested in the dynamic spec-
tral property of chaos.

Finally, the chaotic behavior in the frequency domain
will be mentioned briefly. For a large-signal modulation, the
resonance peak is known to move towards lower frequency in
the modulation response. At a deeper modulation level
that involves subharmonic bifurcations and chaotic behavior,
the modulation response may be described by the dia-
gram of sampled peak photon densities versus the modula-
tion frequency at a fixed modulation index. Plotting such a
diagram at a fixed \( m \) (e.g., 0.59) with \( \beta = 10^{-5} \) in the fre-
quency range from \( 0.5f_\text{p} \rightarrow 2f_\text{p} \), we found sequentially a period-
doubling route from the period 1 to chaos, the coexistence of
two independent periodic attractors, and another complex
chaotic structure in the frequency domain. Chaotic behavior in
the frequency domain has to be explored further.

In conclusion, we have demonstrated theoretically peri-
doubling routes to chaos in a directly modulated laser
diode with practical parameters.

49, 1217 (1982).
A 89, 229 (1982).
8. W. Harth and D. Siemens, Arch. Elektron. Übertragungstech. 28, 391
(1974).
9. H. Kressel and J. K. Butler, Semiconductor Lasers and Heterojunction
11, 206 (1975).

Photochemical generation and deposition of copper
from a gas phase precursor

C. R. Jones, F. A. Houle, C. A. Kovac, and T. H. Baum
IBM Research Laboratory, San Jose, California 95193

(Received 17 August 1984; accepted for publication 17 October 1984)

The photochemical generation and deposition of copper metal
from a volatile copper coordination complex are described. Pulsed and cw ultraviolet light sources
were used to induce deposition. The chemical compositions of the films are compared for all
methods.

Copper is conspicuous in its absence from the list of
metals that have been photochemically deposited from the
gas phase and critical to many of the potential applications
of laser deposition. In a separate letter, we have de-
scribed for the first time the laser chemical vapor deposition
(LCVD) of copper from the vapor phase. In that work, we
identified a volatile copper compound that would undergo
thermal decomposition to copper metal using focused laser
light. Although the material produced in this manner is of
high quality, it is not always desirable to induce the elevated
substrate temperatures required for film growth. Photolytic
deposition provides an alternative method which can involve
little or no heating. An approach to photochemical genera-
tion and deposition of copper from a gaseous precursor
is described in this letter.

Identification of copper complex which is both volatile
and undergoes photochemically induced reduction to metal
was not straightforward. Those metal complexes that have
been used in other work as precursors to gas phase photode-
position have usually been metal carbonyls and metal alkyls
which are either formally in their zero-valent (metallic) state
and simply undergo photochemical "stripping" of their
ligands or readily undergo photon-induced metal carbon bond
breaking. However, copper carbonyl does not exist for all
practical purposes, and copper alkyls are polymeric and
nonvolatile. The photochemical generation of gas phase
copper atoms has thus far been limited to very high-tempera-
ture reactions of halide salts. In alcohol solutions, on the
other hand, the acetylacetone complexes of copper are well
known to undergo photolysis to zero-valent metal at wave-
lengths less than 260 nm. The chemical pathway for this
process is complex and not well understood. It has been de-