Practical design of tapered composite structures using the manufacturing cost concept

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Abstract

The tapered composite laminates are optimized by a patchwise layup design method. In this approach, the weight of tapered composite laminates under the strength constraints was minimized. For this purpose, stacking sequences and the number of plies were optimized. The design variables were the discrete ply angles such as $0^\circ$, $45^\circ$, and $90^\circ$ and the number of plies in each patch. The design results are compared with the uniform thickness laminates. It showed that the optimized layup could considerably reduce total weight of the composite laminates. Also, a new design concept – manufacturing cost – for the expenditure of manufacturing such as time and labor is proposed. The composite laminates designed by the consideration of manufacturing cost were more practical than those designed by only strength constraints.

Keywords: Expert system; Genetic algorithm; Patchwise layup design; Tapered composite laminates; Manufacturing cost

1. Introduction

In the design of aerospace structural components, a tapered composite section is frequently employed. Tapering laminate thickness brings about termination of plies at different locations. This is known as ply drop. Fig. 1 shows the geometry of a simple ply drop. The laminate tapers down from a thick section to a thin section as a result of the ply drops. The application of the tapered composite section is common in wing and fin skin structures, helicopter rotor blades, etc. In all these applications, the use of laminate tailoring results in significant saving in material and is therefore, cost effective. However, the ply drop is a discontinuity within the laminate and therefore it introduces structural difficulties like stress concentration at the drop location. Hence, the potential benefits in laminate tailoring should be compromised with a substantial reduction in the strength of the laminate.

A literature survey reveals that there have been a number of studies associated with the tapered composite laminates. Most of the studies [1–6] were conducted to investigate the stress distributions in the vicinity of the ply drop. Varughese et al. [7,8] proposed a ply drop-off element for the analysis of composite laminates with ply drop. Manne et al. [9–11] performed design optimization of the tapered composites in the light of strength, stiffness and manufacturing. Kim et al. [12–14] developed a new optimization method patchwise layup design method (PWLDM) for the weight optimization of composite laminates with ply drop. However, Kim et al. [12–14] did not consider the manufacturability of the designed structures.

The objective of this study is to design the tapered composite laminates with the low manufacturing cost using the PWLDM. For this purpose, manufacturing cost that is able to quantify the expenditure required for the manufacturing was introduced. The results of this study were compared with those of flat laminates. Additionally, for the practical application of the PWLDM, the optimized result was compared with thickness distribution in the skin of a horizontal stabilizer of General Dynamics F-16A/B.

2. Patchwise layup design method

The PWLDM has two components, an expert system shell and algorithmic parts. The algorithmic parts are
composed of a genetic algorithm and a finite element analysis program based on the first-order shear deformation theory [14]. All components are integrated within the expert system shell—C Language Integrated Production System (CLIPS) and the expert system shell controls execution of the algorithmic parts. The characteristics of the PWLDM are described in the following sections.

2.1. Advantages of PWLDM

- **Reduction of computation cost:** For the optimization of the tapered composite laminates, if the number of plies and the stacking sequences in each patch is optimized by the genetic algorithm, enormous computation cost is required. To overcome this problem, the stacking sequences in each patch are optimized by the genetic algorithm and the number of plies in each patch is optimized by the integration of the expert system shell and finite element method. Namely, the number of plies of each patch is adjusted according as the Tsai–Hill failure criterion in each patch is satisfied or not [13,14]. Hence, the computation cost can be reduced.

- **Guarantee of the continuity of fibers:** Because the stacking sequence is optimized for a patch that has a maximum number of plies, the continuity of fibers in all patches is guaranteed [13,14].

- **Various stacking methods:** In this approach, because it is possible to stack a sublaminate as well as a ply, it can be easily applied to the design of thick composites.

- **Various design rules:** In this approach, some various design rules can be applied during the design of tapered composite laminates. The design rules are related with the reduction of the manufacturing cost as well as the improvement of mechanical characteristics.

2.2. Limits of PWLDM

As mentioned above, the ply drop is a discontinuity within the laminate and therefore it introduces structural difficulties like stress concentration at the drop location. This leads to the failure of components through delamination and/or the failure of resin. In practice, the dropped plies are generally covered by one or several continuous plies, which suppress the delamination at the drop location. For the consideration of this problem, if a three-dimensional analysis considering the real smoothly staggered ply drop is adopted, enormous computation time and cost are unavoidable. In this study, therefore, the real smoothly staggered ply drop shown in Fig. 2(a) is idealized as the staircase model shown in Fig. 2(b). By combining the inplane stress components and the design rules for the reduction of interlaminar stress components, the calculation time is curtailed. However, this approach cannot consider the degradation of mechanical properties caused by the interlaminar stress components directly. Also, only symmetric laminates are considered, in this study.

2.3. Objective function

To apply the patchwise layup design method to composite design, a composite laminate should be divided into several patches. In the present study, the patches represent the elements generated by the finite element analysis. To determine the minimum weight for first ply failure strength, the number of plies and stacking sequences in each patch are optimized. In this design method, the number of plies of each patch is adjusted according as the Tsai–Hill failure criterion in each patch is satisfied or not. Therefore, the optimized laminates may nowhere fail in strength.

In this study, the objective function is the volume of tapered composite laminate and can be expressed as in Eq. (1).
Minimize 
\[ f = \sum_{np=1}^{n} (A_{np} t_{pby} N_{np}) \]
subject to
\[ \frac{\sigma_1^2}{X^2} - \frac{\sigma_1 \sigma_2}{Y^2} + \frac{\sigma_2^2}{Z^2} + \frac{\tau_{12}^2}{S^2} \leq 1 \]  
(1)

where \( A_{np}, np, N_{np}, t_{pby}, \theta, X, Y, \) and \( S \) mean the area of each patch, the number of patch, the number of plies in each patch, ply thickness, ply angle, strength in fiber direction, strength in transverse direction, and shear strength, respectively.

For the stacking sequence optimization using the genetic algorithm, the ply orientation angle on each layer of laminated composite is coded by a two-bit binary string. The objective of design rules are not only to preclude or reduce the undesirable stress coupling effects but also to depress the interlaminar stress components at the drop location. The design rules for stacking sequence are listed below:

2.4. Design rules

In this study, for the design of the stacking sequence of tapered composite laminates, some design rules were applied to various purposes. Design rules were imposed as constraints in the optimization process using a genetic algorithm. The objectives of design rules are not only to preclude or reduce the undesirable stress coupling effects but also to depress the interlaminar stress components at the drop location.

The design rules for stacking sequence are listed below:

The design rules to impede or diminish undesirable stress coupling are as follows:

- A laminate stacking sequence should be symmetric about the mid-plane to avoid extension–bending coupling [15–17].
- A laminate stacking sequence should be balanced to elude shear–extension coupling [15–17].
- \( \pm \theta^\circ \) plies should be grouped to reduce bending–twisting coupling [15–17].

The design rule to depress the interlaminar stress components at the drop locations is as follows:

- The difference of the number of plies between adjacent patches must not exceed two plies [4].

Generally, the larger the difference of the number of plies between adjacent patches becomes, the more the concentration of stress at the drop locations is increased. Thus, to reduce the concentration of stress at the drop locations, it is profitable to decrease the difference in the number of plies between adjacent patches. In this study, the difference in the number of plies between adjacent patches is limited within two plies. This design rule can be expressed as Eq. (2).

(1) If \(|N_{i+1} - N_i| \leq 2\)
Then \( \begin{cases} N_{i+1} = N_{i+1} \\ N_i = N_{i} \end{cases} \)

(2) If \(|N_{i+1} - N_i| = 3\)
and if \(N_{i+1} > N_i\) then \( \begin{cases} N_{i+1} = N_{i+1} \\ N_i = N_{i} + 1 \end{cases} \)
and if \(N_{i+1} < N_i\) then \( \begin{cases} N_{i+1} = N_{i+1} + 1 \\ N_i = N_{i} \end{cases} \)

(3) If \(|N_{i+1} - N_i| > 3\)
and if \(N_{i+1} > N_i\) then \( \begin{cases} N_{i+1} = N_{i+1} - 1 \\ N_i = N_{i} + 1 \end{cases} \)
and if \(N_{i+1} < N_i\) then \( \begin{cases} N_{i+1} = N_{i+1} + 1 \\ N_i = N_{i} - 1 \end{cases} \)

where \( N \) and \( i \) mean the number of plies and patch number in each patch.

In case of a model with a patch along the column as shown in Fig. 3(a), the Eq. (2) is applied to only the patches along the row. However, in the case of the models with several patches along the row and column as shown in Fig. 3(b), the Eq. (2) should be applied to the patches along the row and the column direction alternately. In the Fig. 3, \( N_c(i, j) \) and \( N_r(i, j) \) are the patch numbers along the column and row, respectively. For a plate consisted of \( m \) by \( n \) patch, the design procedure using Eq. (2) is as follows:

(i) Define the patch number of each patch as shown in Fig. 3.

In this case, each patch holds two identification numbers in common; one is the number along the rows and the other is the number along the columns.

(ii) Apply to the Eq. (2) for the patches generated along the row.

If the Eq. (2) is applied to patches along the row, the number of plies of each patch along the row is changed and, simultaneously, the patches along the column is changed.

(iii) Apply to the Eq. (2) for the patches along the column.

(iv) Repeat from (ii) to (iii) until the difference in the number of plies between adjacent patches is equal to or less than two plies.

3. Optimum design of composite plate

To validate the PWLDM, the thickness distribution of a composite beam made of T300/N5208 graphite/
epoxy is optimized. The beam is subjected to a uniformly distributed pressure of $P = 0.1$ kPa. The results are compared with an exact solution and the previous result [6]. The boundary conditions and the geometry of the beam are shown in Fig. 4. In this study, the optimizations of the beam are performed for 10 and 20 patches along the longitudinal direction. Fig. 5 shows that the present results can approximate to the exact solution very well. Moreover, the present result with 20 patches can obtain the weight reduction of 2% as compared with the previous result [6].

3.1. Problem statement

A square plate of $1000 \times 1000$ mm$^2$ subjected to a uniformly distributed load of $P = 1.5$ kPa on the top surface in the $z$ direction is optimized. The boundary conditions are simply supported at all sides. The patch number and the geometry of the plate are shown in Fig. 6. In this study, the optimization of the plate is accomplished for 10 by 10 patches. Also, the plate is optimized for the symmetric layup. The composite plate considered here is made of HFG CU-125NS graphite/epoxy and the material properties are shown in Table 1.

3.1.1. Optimization results – one ply layup

In the PWLDM, the genetic algorithm optimizes the stacking sequences of each patch. To carry out the genetic algorithm, the design variables should be coded as binary strings. Accordingly, the ply orientation angle on each layer of laminated composite is replaced as two-bit binary strings. The binary strings for ply orientation angles are defined as follows:

0° = 00, 45° = 01, −45° = 10, 90° = 11.

The various genetic parameters including population size ($\text{popsize}$), crossover probability ($P_c$), and mutation probability ($P_m$) are tuned by the trial calculations. The ranges of the genetic parameters used in this case are as follows:

Crossover probabilities: 0.7–0.8.
Mutation probabilities: 0.05–0.06.

If patches satisfying the failure criterion exist, only a ply is removed from the top of the patches and otherwise a ply is added as shown in Fig. 7. Fig. 8 shows the optimized thickness distribution and contour of the composite plate. As previously explained, because the optimization is fulfilled for the symmetric layup, Fig. 8 shows only the half of the optimal plate thickness configuration. The layup of a patch that has a maxi-
The maximum number of plies is $[-45/45/-45/45/-45/45]$. The volume and the maximum Tsai–Hill failure index are $0.97 \times 10^6 \text{ mm}^3$ and 0.95. In this case, the required time is about 48 h. As shown in Fig. 8, the maximum number of plies is 16 plies at the two opposite corner (the 10th and 91st). That can also be checked in Fig. 9 which shows the Tsai–Hill failure index contour of a uniform thickness plate that has the same layup as the patch with the maximum number of plies of the optimized plate. As shown in Fig. 9, the 10th and the 91st patches have the maximum Tsai–Hill failure index.

Table 1

<table>
<thead>
<tr>
<th>Material properties of HFG CU-125NS graphite/epoxy</th>
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</thead>
<tbody>
<tr>
<td>Properties</td>
</tr>
<tr>
<td>Elastic modulus in fiber direction, $E_1$</td>
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<tr>
<td>Elastic modulus in transverse direction, $E_2$</td>
</tr>
<tr>
<td>Shear modulus, $G_{12}$</td>
</tr>
<tr>
<td>Shear modulus, $G_{13}$</td>
</tr>
<tr>
<td>Shear modulus, $G_{23}$</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_{12}$</td>
</tr>
<tr>
<td>Ply thickness</td>
</tr>
<tr>
<td>Tensile strength in fiber direction, $X_T$</td>
</tr>
<tr>
<td>Compressive strength in fiber direction, $X_C$</td>
</tr>
<tr>
<td>Tensile strength in transverse direction, $Y_T$</td>
</tr>
<tr>
<td>Compressive strength in transverse direction, $Y_C$</td>
</tr>
<tr>
<td>Shear strength, $S$</td>
</tr>
</tbody>
</table>
3.1.2. Optimization results – sublaminate layup

In the previous section, only a ply is removed from or added to the top of the patches. Hereafter, a sublaminate is removed from or added to. The geometry and loading condition of the composite plate is symmetric about the symmetry lines as shown in Fig. 6. Therefore, to obtain the more simplified optimum solution than the solution obtained by the previous section, a constraint for the identical thickness distribution in all quadrants of the composite plate is imposed. The sublaminate is composed of two plies and the binary strings for ply orientation angles are defined as follows:

\[
\begin{align*}
0^\circ/0^\circ &= 00, \\
45^\circ/45^\circ &= 01, \\
-45^\circ/-45^\circ &= 10, \\
90^\circ/90^\circ &= 11.
\end{align*}
\]

Fig. 10 shows the optimized thickness distribution and the contour of the composite plate. From Fig. 10, we can see that the thickness is symmetrically distributed in all quadrants. The layup of a patch that has a maximum number of plies is \([-45/45/\pm45/\pm45\]). As shown in Fig. 10, the maximum number of plies is 16 at the corner patches (1st, 10th, 91st and 100th). Fig. 11 shows the Tsai–Hill failure index contour and the thickness distribution. The Tsai–Hill failure index contour plotted in the Fig. 11 is that of the uniform thickness plate which has the same layup as the patch with maximum number of plies of the optimized plate. As shown in Fig. 11, the 10th patch has the maximum Tsai–Hill failure index. Therefore, the patches located in the same position in the other quadrants have the same thickness distribution. Also, in the vicinity of the plate center, the Tsai–Hill failure index has maximum value and the center patches have 12 plies. The volume and the maximum Tsai–Hill failure index are \(1.02 \times 10^6 \text{ mm}^3\) and 0.66. In this case, the computation time is about 80 min. The weight and the failure index of this case result in the increase of 5% and 31% as compared with the result of the previous section. However, the computation time is reduced to one-thirty sixth and the thickness distribution of the optimized result is more simplified. From this outcome, in case of using the sublaminate layup and the geometrically symmetric condition, even if the weight and the failure index are increased, the computation
time and the manufacturing complexity can be considerably reduced. Hereafter, the design adopts the sublaminate layup and the geometrical symmetry condition.

3.2. Problem statement

In this study, a square plate with a rectangular cutout is optimized under uniformly distributed loading of $P = 1.5 \text{ kPa}$ on the top surface in the $z$ direction. The dimensions of the square plate and rectangular cutout are $1000 \times 1000 \text{ mm}^2$ and $200 \times 200 \text{ mm}^2$, respectively. The outer boundaries are simply supported. The patch number and the geometry of the plate are shown in Fig. 12. The design conditions of the plate are the same as the previous section.

3.2.1. Optimization results

Fig. 13 shows the optimized thickness distribution and contour of the composite plate. From Fig. 13, we can see that the thickness is symmetrically distributed

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Fig. 11. Tsai–Hill failure index contour and thickness distribution of composite plate.

Fig. 12. Boundary condition and patch numbering of composite plate with a rectangular cutout.

Fig. 13. Optimized thickness distribution and contour of the composite plate.
with respect to centerlines. The layup of a patch that has the maximum number of plies is $\left\{45^\circ/\pm 45^\circ/\pm 45^\circ\right\}$. The volume and the maximum Tsai–Hill failure index are $1.0 \times 10^6$ mm$^3$ and 0.90, respectively. The maximum thickness patches exist around the cutout due to the high stress concentration.

4. Design considering the manufacturing cost

In the previous sections, some various design rules are applied during the design of composite laminates. Most of the design rules are confined to the improvement of mechanical characteristics. However, besides the design rules of the mechanical viewpoint, the design rules of the manufacturing viewpoint should be applied to the design process for low manufacturing cost. When the design results that meet only the requirements in the mechanical viewpoint are very complex or sometimes impossible to fabricate, the results are useless for practical application. Therefore, a new design concept for the expenditure of manufacturing such as time and labor is proposed. The concept is named as the manufacturing cost.

4.1. Manufacturing cost

The construction of composite laminates made of unidirectional fiber reinforced plies requires two major steps: design and manufacture. During design, the layup angles of each layer are adjusted such that the laminate has the desired mechanical properties and is of lightweight. During manufacture, the laminate is fabricated by cutting and stacking prepregs, and then subjecting the stacked prepregs to elevated temperature and pressure. Generally, the more the stacked prepregs are required, the longer the cutting length is, and the larger the stacking area is, the more the manufacturing is complicated and the higher cost is required. Therefore, to calculate an index which is representative of the relative manufacturing expenditure of the layups, the manufacturing cost proposed in this study considers the number of prepregs, cutting length, and stacking area. The cutting length as shown in Fig. 14(a) is the summation of length that should be cut to meet the prepreg size required for the whole laminate. The stacking area as shown in Fig. 14(b) is the summation of area that should be covered by the prepregs to meet the thickness required for the whole laminate. For calculation of the manufacturing cost, the two examples of Fig. 15(a) and (b) are used. The two examples have the same weight. Intuitively, we can know that the case (a) is fabricated easier than the case (b). By the manufacturing cost, an index which is representative of the relative manufac-

Fig. 13. Tsai–Hill failure index contour and thickness distribution of composite plate with a rectangular cutout.

Fig. 14. Cutting length and stacking area.
turing expenditure of the layups is calculated and abstract concepts such as simple and very complex layups are quantified numerically.

To construct the laminate of Fig. 15(a), first, two prepregs of $300 \times 300$ mm$^2$ should be stacked as the bottom layers. Second, two prepregs of $300 \times 200$ mm$^2$ should be stacked, and then one prepreg of $300 \times 100$ mm$^2$ should be stacked finally as shown in Fig. 16(a). To construct the laminate of Fig. 15(b), first, one prepreg of $300 \times 300$ mm$^2$ should be stacked as the bottom layer. Second, two prepregs of $200 \times 200$ mm$^2$ and $100 \times 100$ mm$^2$ should be stacked next. Third, five prepregs of $100 \times 100$ mm$^2$ should be stacked thereon and then three prepregs of $100 \times 100$ mm$^2$ should be stacked finally as shown in Fig. 16(b).

Table 2 shows the prepreg size and number required for the fabrication of the two laminates. From Table 2, the total number of prepregs required for the fabrication in the case (a) and (b) is 5 and 15, respectively. Table 3 shows the total cutting length and the total stacking area of the above two cases. In Table 3, the total cutting length and the total stacking area have different unit system. Therefore, the two components are non-dimensionalized as shown in Fig. 17. Because all patches are equal in size, the prepreg shown in Fig. 17(a) is non-dimensionalized as shown in Fig. 17(b). The non-dimensionalized cutting length and stacking area are abbreviated to NCL and NSA. The manufacturing cost considered in this study is the summation of the number of prepregs, non-dimensionalized cutting length, and...
non-dimensionalized stacking area. Therefore, the manufacturing cost can be expressed as Eq. (3).

\[
\text{Manufacturing cost} = \text{NCL} + \text{NSA} + \text{number of prepregs required.}
\]  

(3)

The manufacturing cost for the two cases of Fig. 16 calculated by Eq. (3) is as follows:

- Case (a):
  - Number of prepregs required = 5.
  - Non-dimensionalized cutting length = 52.
  - Non-dimensionalized stacking area = 33.
  - Manufacturing cost = 5 + 52 + 33 = 90.

- Case (b):
  - Number of prepregs required = 15.
  - Non-dimensionalized cutting length = 80.
  - Non-dimensionalized stacking area = 33.
  - Manufacturing cost = 15 + 80 + 33 = 128.

From the results, the case (b) requires 30% more in manufacturing cost than the case (a). In case of the tapered composite laminate, even if the weight is equal to each other, we can know that the manufacturing cost significantly affects the overall cost.
is different along the thickness distribution in the laminae. Therefore, whenever the tapered composite laminate is designed, the design method must consider the manufacturing cost. Also it should be noted that the cost concept can be dependent on the particular cutting method (water jet cutting, laser cutting, etc.) and whether automated layup methods are employed. The present procedure gives a general indication of costs.

4.2. Optimum design of composite plate considering the manufacturing cost

For the composite design considering the manufacturing cost, the plate of the Section 3.1 is used. Fig. 18 shows the optimized thickness distribution and contour of the composite plate. The layup of a patch that has a maximum number of plies is \([45/-45/-45]_s\). The volume and the maximum Tsai–Hill failure index are \(1.12 \times 10^6 \text{ mm}^3\) and 0.63.
The design results obtained by the PWLDM are compared with those of uniform thickness as shown in Table 4. The ratio of volume and ratio of failure index are normalized by the values in case 1 and MC stands for the manufacturing cost. From Table 4, while the variable thickness laminates designed by the PWLDM can reduce the weight of the composite laminates, the manufacturing cost increases as compared with the uniform thickness laminates. Even if the case 4 has the minimum weight, it also has the maximum failure index, computation time, and manufacturing cost. Therefore, the case 4 is impractical. The case 6 designed by the manufacturing cost concept shows the weight reduction of 10% and the manufacturing cost increment of 8% only, as compared with the uniform thickness laminates. Also, the computation time of the case 6 is only one-thirty sixth as compared with the case 4. Therefore, the case 6 can be a practical design.

4.3. Optimum design of composite plate with a rectangular cutout considering the manufacturing cost

For the design of composite plate with a rectangular cutout considering the manufacturing cost, the plate in Section 3.2 is used. Fig. 19 shows the optimized thickness distribution and contour of the composite plate. The layup of a patch that has the maximum number of plies is \([\pm 45]_2/\pm 45/\pm 45\). The volume and the maximum Tsai–Hill failure index are \(1.14 \times 10^6 \text{ mm}^3\) and 0.86.

The design results obtained by the PWLDM are compared with those of uniform thickness as shown in

<table>
<thead>
<tr>
<th>Laminate types</th>
<th>Design rules</th>
<th>Stacking sequence</th>
<th>Volume ratio</th>
<th>Failure index ratio</th>
<th>Manufacturing cost (MC)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uniform thickness</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>1. Symmetry</td>
<td>[45/-45],</td>
<td>1</td>
<td>1</td>
<td>919</td>
</tr>
<tr>
<td>Case 2</td>
<td>1. Symmetry and balance</td>
<td>2. Minimize grouping of the same orientation plies</td>
<td>3. Separate (\pm \theta) plies</td>
<td>4. Avoid grouping of 90° plies</td>
<td>5. The difference of ply angle between adjacent plies must not exceed 45°</td>
</tr>
<tr>
<td>Case 3</td>
<td>1. Symmetry and balance</td>
<td>2. Minimize grouping of the same orientation plies</td>
<td>3. Grouping (\pm \theta) plies</td>
<td>4. Avoid grouping of 90° plies</td>
<td>5. Avoid locating transverse plies to reduce interlaminar stress around a hole</td>
</tr>
<tr>
<td><strong>Variable thickness</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td>1. Symmetry and balance</td>
<td>2. Grouping of (\pm \theta) plies</td>
<td>3. The difference of number of ply between adjacent patches must not exceed two plies</td>
<td>4. Geometric symmetry</td>
<td></td>
</tr>
<tr>
<td>Case 5</td>
<td>1. Symmetry</td>
<td>2. Balanced</td>
<td>3. Grouping of (\pm \theta) plies</td>
<td>4. The difference of number of ply between adjacent patches must not exceed two plies</td>
<td>5. Geometric symmetry</td>
</tr>
</tbody>
</table>
Table 5. While the variable thickness laminates designed by the PWLDM are lighter in weight, the manufacturing cost increases as compared with the uniform thickness laminates as the previous results. The cases 4 and 5 designed by the PWLDM show the weight reduction of 20% and 3%, respectively.

5. Design of a composite horizontal stabilizer under aerodynamic force

For the practical application of the PWLDM, the optimum design of a composite horizontal stabilizer is performed under the cruising condition of a typical aircraft. The shape of the composite horizontal stabilizer is similar to that of General Dynamics F-16 as shown in Fig. 20(a). Generally, the horizontal stabilizer of the fighter is composed of various structural components, such as skin, composite sine wave corrugated substructure, metallic leading edge and trailing edge, and metallic tip rib and root rib. However, in the case of the optimum design considering the whole components, enormous computation time and cost are unavoidable. In this study, the composite horizontal stabilizer is simply modeled as a cantilever composite plate. The aerodynamic force imposed on the composite horizontal stabilizer is calculated by the vortex lattice method (VLM). The cruising condition for the calculation of the aerodynamic force is as follows:

\[
\text{Mach number } = 0.7, \\
\text{Angle of attack } = 0.5^\circ.
\]

Fig. 21 shows the distribution of the aerodynamic force.

5.1. Problem statement

The weight minimization of a composite horizontal stabilizer is fulfilled under the combined strength and stiffness constraints. The patch numbers are shown in Fig. 20(b). The optimization of the plate is performed for 10 by 10 patches with symmetric layup. The composite plate considered here is made of HFG CU-125NS graphite/epoxy. The binary strings for ply orientation angles are defined as follows:

\[
00 = 0^\circ/45^\circ, \quad 01 = 90^\circ/-45^\circ, \quad 10 = 0^\circ/-45^\circ, \quad 11 = 90^\circ/45^\circ.
\]

For the stiffness constraint, the maximum deflection of the composite plate should be less than 0.15 m. The deflection of the composite plate is defined as Eq. (4).
\[ d = \sqrt{u^2 + v^2 + w^2}, \]

where \( u, v \) and \( w \) are the displacement components in the \( x, y, \) and \( z \) directions, respectively.

### 5.2. Design result

The optimized thickness distribution is compared with that of F-16 in Fig. 22. We can confirm that the optimized result is similar in the thickness distribution to the real structure as shown in Fig. 22. The layup of a patch that has the maximum number of plies is \((0/\pm 45)_{10}/0/45/0/\pm 45/0/45\). The patches around the root section have the maximum number of ply due to maximum stress distribution. On the other hand, the patches near the tip have the minimum number of plies. Also, the composite horizontal stabilizer has taper from root to tip and from edges to center section.

### 6. Conclusions

In this study tapered composite laminates are optimized using a patchwise layup design method under various design rules. The tapered composite laminates designed using the present approach can achieve a more weight effective design than uniform thickness layups. Also, a new design concept – manufacturing cost – manufacturing expenditure such as time and labor is proposed. The composite laminates designed with consideration of manufacturing costs had practical advantages over those designed using only strength constraints. Therefore, as tapered composites are designed, along with the mechanical design rules, design rules from a manufacturing viewpoint should also be applied to minimize component costs. From the practical application of PWLDM, the optimum design of a composite horizontal stabilizer was carried out under the cruising conditions. The optimized result was similar in the thickness distribution to the real structure. In present study, even though the manufacturing cost is applied to particular manufacturing method such as hand layup, the concept can be easily extended to other methods e.g. filament winding and robotic layup.

### References


[16] Composite Design Manual, Bell Helicopter TEXTRON.