Quantity Discount Model to maximize Channel Profit

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Studies on quantity discount have been in two directions since Buchanen published that suppliers provide quantity discount for price discrimination and for channel efficiency. The model on quantity discount for price discrimination has been researched in economics. On the contrary, the model on quantity discount for channel efficiency has been studied in production management with assumptions and focuses different from economics. In this paper, the differences in two areas are analyzed and a two-stage quantity discount model is suggested. This model integrates quantity discount models researched in each area. In addition, the discount rate required in each stage for implementation of this model is suggested in both supplier side and buyer side under some conditions.

Keywords: Quantity Discount, Channel efficiency, Price discrimination, Channel coordination

1. Introduction

Achieving effective coordination between supplier and buyer is a significant research area and also a concern of the current management issues. (Weng 1995) Generally, vertical integration is the easiest way to achieve the channel coordination. However, vertical integration brings out much start-up cost and is confronted with severe regulations. Therefore, vertical integration is not always a useful means for channel coordination. Because of difficulties and problems of vertical integration, many management strategies are suggested to achieve channel coordination. (Gerstner, E. and Hess, J.D. 1995) Among these strategies are franchising, competition between retailers, profit sharing, exchange of personnel, arbitration, implicit understandings from repeated interaction. (Shugan 1985, McGuire & Staelin 1983, Stern and EL-Ansary 1992) Also, the quantity discount model has been considered to be critical to achieve channel coordination.

Buchanen(1953) suggested two principal motivations for quantity discount. The one is price discrimination motivation and the other is channel efficiency motivation. Since Buchanen’s research, later studies on the quantity discount model have been divided into two directions. The former motivation on quantity discount is focused in economics(Moorthy 1987, Oi 1971, Murphy 1977) and the latter motivation is performed in

In economics, the quantity discount model has been researched as a method that supplier can maximize his/her profit making consumer surplus zero. (Buchanan 1953) It is assumed in economics that buyer’s willingness to pay for a product decreases as buyer purchases more. Nason and Della Vitta’s survey shows that consumers expect quantity discount in the form of lower unit price for larger package sizes of a given brand. (Dolan 1987)

In production management, the quantity discount model has been studied to be a means to achieve channel efficiency. If a supplier takes a larger order size, the supplier can reduce his/her operating cost, but the buyer’s cost increases. Therefore, in order to entice a larger order size, supplier should suggest quantity discount for buyer’s larger order size than EOQ. Supplier is able to offer quantity discount for channel efficiency because supplier’s profit due to buyer’s larger order size than EOQ is enough to compensate buyer’s increasing inventory holding cost. There is a general assumption that supplier’s setup cost is larger than buyer’s.

These two areas are based on different basic ideas. In economics, buyer’s demand is a decreasing function of price and buyer’s operation cost is fixed regardless of an order size. On the contrary, in production management, buyer’s demand is constant and given. (1995 Weng) The minimization problem of operating cost according to an order size is considered. As we mentioned in the above, two areas have taken different approaches on supplier’s quantity discount motivation and have studied independently.

However, relations between two models have never been studied so far. In addition, there has been little study on how can each model be integrated in practice. The purpose of this study is to form the model that integrates two quantity discount models. The points we are interested in the quantity discount model are the next three parts.

(1) What relations do these two kinds of quantity discount models have?
(2) When does supplier offer these kinds of quantity discounts?
(3) What discount rate does supplier have to offer in each quantity discount?

This paper is organized as follows. First, we introduce the basic concept on the quantity discount model in Chapter 2. Second, In Chapter 3, we analyze the differences between two areas and suggest a two-stage quantity discount model, which integrates two motivations on quantity discount. Third, we suggest the discount rate to implement two-stage quantity discount model. This discount rate is analyzed in both the supplier side and the buyer side. Finally, conclusions and future study are offered.

2. Basic Concept on Quantity Discount Model

Let us begin with a brief review on the quantity discount models. First, we will focus our attention on the model in economics. Next, we will discuss the quantity discount model of production management.

2.1 Quantity discount for price discrimination

When supplier provides quantity discount for price discrimination, how can he/she maximize his/her profit? It is possible when supplier sells the quantity at which buyer’s marginal value on the product is equal to supplier’s marginal cost and charges the price that is equal to the buyer’s total value on the quantity. In this case, consumer surplus becomes zero and supplier’s profit is maximized. This condition is presented in (Fig. 1).

![Fig. 1. Quantity discount for price discrimination](image-url)
Given that buyer’s marginal cost is equivalent to supplier’s supply price and end user’s demand curve is a linear type, buyer’s MR curve becomes supplier’s demand curve. In this case, supplier’s MR curve has the same vertical intercept as buyer’s demand curve, but its slope is doubled. It is presented in (Fig. 2).

If supplier doesn’t provide quantity discount, both the supplier’s profit and the buyer’s profit can be presented as (Fig. 2). Area 1 describes the buyer’s profit and area 2 presents the supplier’s profit.

![Figure 2. Supplier’s profit under no quantity discount.](image)

If supplier offers quantity discount, buyer purchases the quantity at which marginal revenue of channel is equal to marginal cost and the profit of channel is maximized (Fig. 3). It describes the channel profit under quantity discount. As a result, the channel’s profit under quantity discount is larger than that under no quantity discount.

Mooorthy showed that the minimal requirements for maximizing profit of channel are(1) that buyer’s marginal cost is equal to buyer’s marginal revenue at D(x*), and(2) that buyer’s marginal cost be strictly below his marginal revenue at quantity less than D(x*). (Mooothy 1987)

![Figure 3. Supplier’s profit under quantity discount.](image)

2.2 Quantity discount for channel efficiency

Buyer may take various purchasing strategies for an order size. However, in most of literatures, it is assumed that buyer determines an order size with economic order quantity (later, written as EOQ). Even though EOQ is so simple, it represents the core of the company inventory policy and is so much used when firms determine their inventory levels. (Lal & Staelin 1984)

Typically, since supplier’s setup cost is larger than buyer’s, supplier can reduce his/her operating cost if he/she takes a larger order size from buyer. However, in this case, buyer’s operating cost increases. Thus, to make buyer order with a larger size, supplier has to compensate buyer’s incremental operating cost with quantity discount, which enables the buyer to purchase lower unit price.

Crowther (1964) introduced Joint Economic Lot Sizing (JELS) problem that determines an order size minimizing operating cost of channel. Monahan (1984) derived an order size that minimizes operating cost of channel and formulated Crowther’s study. Monahan in his model derived the minimum discount with which supplier has to provide buyer if buyer orders with the size K times larger than EOQ as below.

\[ D \cdot \text{EOQ} \times \frac{(K - 1)^2}{2K^2} \]
Here, \( p \) is a purchasing unit price and \( h \) is an annual inventory holding cost as percentage of a purchasing unit price. \( S \) and \( S_a \) mean the buyer's setup cost and supplier's, respectively. \( D \) is buyer's annual demand.

Monahan also derived the optimal \( K \) where operating cost of channel is minimized as below.

\[
K^* = \sqrt{1 + \frac{S_a}{S_b}}
\]

Monahan's research obviously opens a significant research direction on the lot size problem with quantity discount. However, Monahan's following assumptions are criticized (Benton & Park 1995).

(i) The supplier's production frequency is the same as the buyer's ordering frequency (Lot-for-lot policy)
(ii) Changes in buyer's order quantities (or order frequencies) do not affect the supplier's inventory holding cost.

Since Monahan's research, his model has been developed as above assumptions are relaxed. (Rosenblatt and Lee 1985, Banerjee 1986, Joglekar 1988)

3. Two-Stage Quantity Discount Model

3.1 Basic assumptions

In this paper, three assumptions are given. First, the channel presented in this paper is the vertical channel that consists of a supplier, a buyer, and end users. Literatures can be classified as two categories whether the buyer in the channel is in a monopolistic position in market or not (Dada & Srikanth 1987). In the first category, supplier's quantity discount lowers buyer's input cost, which affects a buyer's selling price and quantity. However, in the second category, even though supplier's quantity discount reduces buyer's input cost, it doesn't affect a buyer's selling price and quantity. In this paper, like the former, buyer is assumed to be a monopolistic position in market.

Here, \( P \) is a supply price and \( X \) is a selling price.

Second, end users' demand is affected by many factors such as price, quality and delivery time. In this paper, we assume that demand is only determined by price under assumption that other factors are constant and the buyer's order size is determined by EOQ.

Finally, we assume that buyer takes one of two purchasing policies, which are purchase on an annual commitment basis, and purchase on an as-ordered basis (Sadrian & Yoon 1992). (For further details of these policies, see Sadrian & Yoon)

3.2 Model development

- **Purchasing scenario**

In this paper, a buyer's purchasing scenario is as follows.

1. Forecast annual demand according to a selling price.
2. Ask supplier to quote for a product
3. Determine an annual purchasing quantity maximizing profit based on a quoted price
4. Determine selling price to sell all quantity purchased from supplier.

In the previous purchasing scenario, differences between the annual commitment policy and the as-ordered policy are as follows. In the annual commitment policy, a buyer makes a contract with a supplier to purchase annual demand during a year and orders EOQ with proper time interval. On the contrary, in the as-ordered policy, the quantity determined in the stage 3 isn't contracted with supplier. The buyer just uses this value to decide a selling unit price and an order size.

- **Analysis of difference between two approaches**

Quantity discount is based on the quantity per order in channel efficiency motivation. In contrast to channel efficiency motivation, quantity discount for price discrimination probably requires aggregation over some time period. Most of practice seems to be an annual aggregation period (Dolan 1987). We also assume an annual aggregation period.

Thus, the contract on an annual commitment basis is a precondition of providing quantity discount for price discrimination. In other words, quantity discount for price discrimination
assumes buyer’s purchasing on an annual commitment basis. We would like to emphasize this point. The purchasing stage at which supplier offers buyer quantity discount for price discrimination is before the contract of an annual demand.

At this purchasing stage, if quantity discount is not given, supplier determines the selling quantity as the quantity at which his marginal cost is equal to marginal revenue and buyer also determines the purchasing quantity in the same way. Such an independent decision on the quantity transacted between channel members doesn’t maximize profit of channel.

This independent decision problem can be overcome by using quantity discount. Through quantity discount, supplier make buyer’s marginal cost reduce as buyer increases his/her purchasing quantity more. In other words, the quantity discount plays a role that makes the optimal selling quantity of channel equal to the buyer’s optimal purchasing quantity. Through the quantity discount, channel coordination can be achieved integrating each channel member’s decision. At this purchasing stage, supplier’s operating cost is not significant.

In the quantity discount model for channel efficiency, it is assumed that end users’ annual demand is given. According to buyer’s purchasing policy, the given annual demand may be the forecasting value or the contracted quantity on the annual demand. It means that quantity discount for channel efficiency doesn’t depend on contract based on the annual commitment.

Given the annual commitment contract, quantity discount for channel efficiency is considered after offering of quantity discount for price discrimination. Of course, quantity discount for channel efficiency can be provided without the contract based on an annual commitment.

From the supplier’s point of view, there are some differences from timing, object and purposes of quantity discounts between two approaches. Those are summarized in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Motivation for price discrimination</th>
<th>Motivation for channel efficiency</th>
</tr>
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<tbody>
<tr>
<td>Time</td>
<td>Before annual demand contract</td>
<td>After annual demand contract</td>
</tr>
<tr>
<td>Object</td>
<td>Buyer’s annual demand</td>
<td>Buyer’s order size</td>
</tr>
<tr>
<td>Purpose</td>
<td>Maximizing channel profit</td>
<td>Minimizing operating cost</td>
</tr>
</tbody>
</table>

- Two-stage quantity discount model

From what has been analyzed above, we suggest a two-stage quantity discount model that when supplier makes a contract with buyer for an annual demand, supplier provides quantity discount for price discrimination to maximize channel’s profit and after the contract of an annual demand, supplier provides quantity discount for channel efficiency to minimize channel’s operating cost.

Then, is it always possible that supplier offers two-stage quantity discount? In next section, we analyze some conditions under which a two-stage quantity discount model can be applied. These conditions are given in the form of discount rate required for the implementation of a two-stage quantity discount model.

4. Model Analysis

We suggested a two-stage quantity discount model. Then, at each stage of two-stage quantity discount, how much discount rate does supplier have to offer? Also, at each stage, how much discount rate does buyer have to get? Now, we will take a close look at these aspects.

4.1 Discount rate in quantity discount for price discrimination

In the previous section, we described that by using quantity discount, supplier is able to achieve price discrimination and maximize the profit of channel. In addition, studies on the quantity at which maximize the profit of channel were done. In this paper, we take focus on the discount rate at which maximize of the profit of channel. First, we define the cost structure of supplier’s and buyer’s. With this definition, we
can clarify supplier’s MR curve and MC curve.

- Cost structure of supplier and buyer. Notations mean the followings:
  F: supplier’s fixed cost
  f : buyer’s fixed cost
  C: supplier’s variable cost
  c : buyer’s variable cost

  We assume that supplier’s MC and buyer’s MC are constant. This assumption is general in the quantity discount model. Therefore, total cost structure of supplier is defined as F+C D. Variable cost is assumed to be constant regardless of the quantity of production. Thus, supplier’s marginal cost becomes variable cost.

  And, total cost structure of buyer’s is defined in the same way as F+c D. Here, buyer’s variable cost is assumed to be a supply price for convenience of analysis. Finally, total cost of channel can be expressed as F+f+C D. Then, marginal cost of channel becomes the same as supplier’s marginal cost.

  Under the cost structure defined in the above, demand curve, MR curve, MC curve are presented as the following figure.

  Now, we examine the discount rate with which supplier has to provide buyer to sell the quantity that maximizes profit of channel. An end user’s demand curve is assumed to be a linear function(D=a-bX).

  In case of a linear function, the profit of channel under the quantity discount for price discrimination(1+2) is expressed as

  \[ \frac{1}{4D}(a-b \cdot MC)^2 \]

  where MC is the marginal cost of channel.

  Supplier’s selling quantity is the quantity at which MR of channel is equal to MC of channel. If supplier doesn’t offer quantity discount, the selling quantity, the selling price and channel’s profit are as follows.

  buyer’s profit = \[ \frac{1}{4D}(a-b \cdot MC)^2 \]
  supplier’s profit = \[ \frac{1}{8D}(a-b \cdot MC)^2 \]
  channel’s profit = \[ \frac{3}{16D}(a-b \cdot MC)^2 \]

  It is possible for all channel members to increase their profit if supplier provides the quantity discount schedule that total charges on the quantity are less than the total value. Here, total value means profit of channel and total charges are supplier’s profit.

  Total Value–Total Charge buyer’s ≥ profit under no Quantity Discount

  If supplier offers the quantity discount satisfying the previous equation, the minimum discount rate is derived as follows.

  \[ (x-p')(\frac{a}{2} - \frac{b \cdot MC}{2}) \geq \frac{a}{16D}(a-b \cdot MC)^2 \]
  \[ p' \leq \frac{3a}{8b} + \frac{5MC}{8} \]

  \[ Discount\ Rate = \frac{p-p'}{p} - \frac{a-b \cdot MC}{4a+4b \cdot MC} \] (1)

  where p’ is the average discounted price.

  Also, the possible maximum discount rate can be calculated as the following procedures.
\[ (p' - MCO - \frac{a - b \cdot MC}{2}) \geq \frac{1}{8b} (a - b \cdot MC) \]
\[ p' \geq \frac{a}{4b} + \frac{3}{4} \cdot MC \]

\[ Discount \ Rate = \frac{p - p'}{p} = \frac{a - b \cdot MC}{2a + 2b \cdot MC} \] (2)

From equation (1) and (2), we can identify that both the minimum discount rate and maximum discount rate are the function of \( a/(b \cdot MC) \). \( a/b \) is the vertical intercept of buyer’s demand curve. This value means the price where the initial demand occurs. In other words, that is the highest price that supplier can estimate on the product.

Therefore, \( a/(b \cdot MC) \) describes how larger the possible maximum price is than marginal cost of channel. A large \( a/(b \cdot MC) \) means that a product is a highly value added thing. In case of large \( a/(b \cdot MC) \), supplier may provide the quantity discount with a high discount rate. If \( a/(b \cdot MC) \) changes from 0 to 6, equation (10) and (13) can be drawn as (Fig. 7).

Given a discount rate above the minimum, buyer takes quantity discount for price discrimination and makes a contract with supplier on the annual commitment base and also changes the purchasing quantity to the quantity maximizing the channel profit.

### 4.2 Discount rate in quantity discount for channel efficiency

Monahan derived the minimum discount rate that supplier has to provide to compensate buyer’s incremental cost resulted from a larger order size than EOQ. Furthermore, he showed the buyer’s order size that minimizes of operating cost of channel. As mentioned before, even though his model has some limitations, it opened a significant direction on quantity discount for channel efficiency. Thus, we will analyze our model based on Monahan’s model. Notations are the same as those used in the previous section.

If buyer orders with EOQ(\( Q^* \)), supplier’s setup cost is as follows.

\[ S_i \cdot \frac{D}{Q^*} \] (3)

Monahan derived the minimum discount for compensating the buyer’s loss resulted from the incremental inventory holding cost when buyer takes the order size of \( K \) EOQ rather than EOQ as following equation.

\[ P \cdot 2DS_{\text{op}} \cdot \frac{(K - 1)^2}{2K} \] (4)

In this case, supplier’s operating cost is as follows.

\[ S_i \cdot \frac{K^*}{KQ^*} + P \cdot 2DS_{\text{op}} \cdot \frac{(K - 1)^2}{2K} \] (5)

From the equation (3) and (5), supplier’s unit profit through the reduction of setup cost can be expressed as

\[ \frac{S_i}{Q^*} \cdot (1 - \frac{1}{K^*}) = \frac{2DS_{\text{op}}}{2S_iD} \cdot (1 - \frac{1}{K^*}) \]

\[ \text{Unit profit} = \frac{S_i}{S_i} \cdot \frac{S_i}{pD} \cdot \frac{K^* - 1}{P \cdot 2K^*} \]

\[ = \frac{8S_i}{8S_i} \cdot \frac{S_i}{pD} \cdot \frac{K^* - 1}{P \cdot 2K^*} \] (6)

\[ Unit \ profit = \frac{S_i}{pD} \cdot \frac{K^* - 1}{P \cdot 2K^*} \] (7)
Equation (6) means the profit per unit that supplier can gain by achieving channel efficiency. Equation (7) presents the ratio of the profit per unit to the unit price, which is the maximum discount rate that supplier can offer for channel efficiency.

From equation (5), a minimum unit discount can be calculated as

\[
\text{Minimum discount} = -\frac{D}{2Sph} \times \frac{(K^*-1)^i}{2K^*} \tag{8}
\]

The minimum discount rate is derived as follows.

\[
\frac{\text{Minimum unit discount}}{D} = -\frac{2S_p h}{pD} \times \frac{(K^*-1)^i}{2K^*} - c \frac{S_p}{pD} \tag{9}
\]

'\( h \)' is given as a constant in equation (7) and (7). And if \( K \) is given as a constant, the discount rate for channel efficiency can be represented as the function of \( \alpha \). That is, the proportion of supplier's setup cost to the volume of business between supplier and buyer determines both minimum discount rate and maximum discount rate that supplier can offer for channel efficiency.

Given \( K^*=2 \) and \( h=0.25 \), when supplier's setup cost occupies from 1% to 20% of the business volume, the minimum discount rate and maximum discount rate for channel efficiency are presented as (Fig. 8).

From the above figure, when supplier's setup cost occupies from 1% to 20% of the turnover, an available maximum discount rate is 3% to 14%. Also, the minimum discount rate required for channel efficiency is 1.8% to 8%.

4.3 Discount rate in two-stage quantity discount

In the previous section, the discount rate required in each stage of two-stage quantity discount was suggested. According to suggested rates, if \( \alpha/b \cdot MC \) is equal to 5, supplier is able to offer quantity discount with rates of 16.67% to 33.33%. With these discounts rates, supplier can induce buyer to purchase the quantity at which maximizes the profit of channel.

In addition, if supplier's setup cost occupies 10% of business volumes between supplier and buyer, supplier is able to minimize operating cost with discount rates of 5% to 10%.

Furthermore, quantity discount for channel efficiency is suggested after quantity discount for price discrimination. In

| Table 2. Possible Minimum Discount rates in two stage Quantity Discount(unit : %) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \alpha/b \cdot MC \) | 0.01 | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 |
| 1 | 1.77 | 5.95 | 5.30 | 6.37 | 7.29 | 8.10 |
| 1.8 | 7.78 | 8.78 | 10.81 | 12.06 | 13.06 | 13.93 | 14.66 |
| 2.6 | 12.68 | 14.62 | 15.82 | 16.77 | 17.59 | 18.31 |
| 3.4 | 15.17 | 17.05 | 18.22 | 19.14 | 19.94 | 20.61 |
| 4.2 | 16.88 | 18.72 | 19.86 | 20.77 | 21.55 | 22.23 |
| 5 | 18.14 | 19.96 | 21.09 | 21.98 | 22.74 | 23.42 |
| 5.8 | 19.11 | 20.90 | 22.01 | 22.90 | 23.65 | 24.52 |

| Table 2. Possible Maximum Discount rates in two stage Quantity Discount(unit : %) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \alpha/b \cdot MC \) | 0.01 | 0.05 | 0.09 | 0.13 | 0.17 | 0.21 |
| 1 | 3.06 | 6.85 | 9.19 | 11.04 | 12.62 | 14.03 |
| 1.8 | 6.91 | 20.16 | 22.17 | 23.75 | 25.11 | 26.32 |
| 2.6 | 24.60 | 27.55 | 29.37 | 30.81 | 32.04 | 33.13 |
| 3.4 | 29.50 | 32.25 | 33.95 | 35.50 | 36.45 | 37.47 |
| 4.2 | 32.89 | 35.51 | 37.13 | 38.41 | 39.51 | 40.48 |
| 5 | 35.37 | 37.90 | 39.46 | 40.69 | 41.74 | 42.68 |
| 5.8 | 37.37 | 39.72 | 41.24 | 42.43 | 43.46 | 44.37 |
other words. Both price and quantity given in quantity discount for channel efficiency reflect quantity discount for price discrimination. With this consideration, we suggest both maximum discount rate and minimum discount rate in the following tables. If \( a/(b \cdot MC) \) is equal to 5 and \( b/(p \cdot MC) \) is equal to 0.05, supplier is able to offer quantity discount with rates of 19.96% to 37.9%.

5. Conclusion

5.1 Summary of conclusions

In this paper, we analyzed the differences between research on the quantity discount model in economics and research on the quantity discount model in production management. Through these analyses, we found that some assumptions given in each area are different, which includes the time that each quantity discount is offered, qualifying units, objective. In these differences, what has to be noticed is that the quantity discount for price discrimination is offered before the contract on an annual demand, but the quantity discount for channel efficiency is offered after the contract on an annual demand or without the contract on an annual demand.

Based on these analyses, a two-stage quantity discount model is suggested in which supplier provides the quantity discount for price discrimination to maximize the profit of channel if supplier makes a contract with buyer for an annual demand and after an annual demand contract, supplier provides the quantity discount for channel efficiency to minimize the operating cost of channel. Each model on quantity discount was proved to be useful to improve the profit of channel in the previous studies. Therefore, we suggest that a two-stage quantity discount model integrates the performance of each model and maximizes the profit of channel.

And some conditions for implementation of two-stage quantity discount model are suggested in the form of discount rate required in each stage. Under some assumptions, buyer has to be guaranteed with discount rate of 19.96% at least and supplier may offer discount rate to 37.9% in the two-stage quantity discount model.

5.2 Contributions and further studies

In this paper, we contributed with the followings. First, we analyzed the differences between two areas and suggested a two-stage quantity discount model, which integrates two types of motivation on the quantity discount. Second, we suggested discount rate for the implementation of the two-stage quantity discount model. These discount rates necessary at each stage of two-stage quantity discount were derived. Finally, we suggested that with a two-stage quantity discount model, supplier and buyer are able to maximize the channel profit.

Some limitations in this paper also exist. We assumed a simple channel that consists of a supplier and a buyer having a monopolistic position in market. This assumption was necessary to clarify the concept of two-stage quantity discount model and to analyze conveniently. The relaxation of this assumption on the simple channel will enable the further study.

Reference