Thermal stability of superconductors with large surface barriers to flux entry: Superconducting power-line conductors

S. Kim, R. E. Howard, and M. R. Beasley

Edward L. Ginzton Laboratory, W. W. Hansen Laboratories of Physics, Stanford University, Stanford, California 94305
(Received 5 August 1977; accepted for publication 14 September 1977)

The effect of surface barriers to flux entry on the stability of superconductors at low fields is considered theoretically in the context of the usual critical-state model. The presence of such a barrier is found to greatly reduce overall stability unless a thin normal-metal overcoat is used to stabilize the superconductor. The relevance of these results to superconducting power transmission lines is discussed.

PACS numbers: 74.60.Jg, 84.40.Nm, 74.30.Ek

I. INTRODUCTION

It has recently been found that the A-15 superconductors Nb$_3$Sn and Nb$_3$Ge can be made to exhibit very low ac losses in ac fields exceeding the estimated values of $H_{c1}$. As a result, the A-15 superconductors (in particular Nb$_3$Sn) are increasingly favored materials for low-field high-power ac applications, as the most prominent of which is, at the present, superconducting ac power transmission. Such lines are currently under active development.

The observed low ac losses in the A-15 superconductors have been obtained by two means. In one case, very strong flux pinning was obtained in a finely layered composite (i.e., a superconductor containing a regular array of plane pinning barriers) that had very high critical-current density $J_c$ and hence low hysteretic ac losses. However, in the materials currently being used in prototype power lines, the bulk critical-current densities are only moderate and the low losses are believed to result from the presence of a substantial surface barrier to flux penetration of the sort first identified by Bean and Livingston. (An alternate interpretation in terms of an anomalously high $H_{c1}$ has been suggested, however.) These surface barriers reduce the ac losses by delaying the onset of flux penetration and the concomitant hysteretic ac losses to applied fields in excess of $H_{c1}$. Barrier fields as high as 1700 Oe have been observed in Nb$_3$Sn, and the theoretical maximum is believed to be $\sim H_{c1}$, the bulk thermodynamic critical field. Surface barriers to flux entry not only reduce ac losses but usually also provide a substantial fraction of the total critical current of the conductor. Consequently, it is important to consider their role in flux-jump stability and, in particular, in the fault capacity and stability of superconducting power-line conductors.

A considerable amount of theoretical and experimental work has gone into understanding the mechanism of flux-jump instability in simple superconductors without a surface barrier, but little or no work has been carried out to determine the effect of a surface barrier within these models. In this paper, we extend the usual models of adiabatic stability to include the presence of a surface barrier and examine its effect on the stability of the conductor. Calculations based on these models indicate that the surface barrier has a potentially large destabilizing effect. However, the presence of even a small amount of normal metal on the surface should be quite effective in stabilizing the conductor.

More specifically, our results show that the surface-barrier region is prone to instability due to its high current density and the large electric fields present there during flux motion. Taken together, these two factors lead to a very high density of heat generation in the surface region during a flux jump. Consequently, the theoretical total maximum stable linear current (i.e., current per unit width of conductor) is smaller for a superconductor with a large surface current than for the same conductor without a surface barrier. By coating a high-conductivity normal metal on the high-field side of the conductor, however, the thin surface-barrier region can be fully stabilized. In this case, the total maximum stable current can be increased considerably by taking advantage of a large surface current. The eddy-current loss from the normal metal necessary to achieve stability in this model is small and entirely negligible in the case of practical superconducting power-line conductors.

It should be noted, though, that the calculations presented here are based upon extensions of the usual stability theories, which themselves have not been well tested in the low-field regime relevant for superconducting power transmission line conductors or for the A-15 superconductors with or without surface barriers.

II. PREVIOUS STABILITY CALCULATIONS

If a superconductor is in the critical state in an applied field, $H_{sat}$, at temperature $T$, and if a slight fluctuation in temperature occurs, the corresponding decrease in $J_c$ will cause a redistribution of flux, as indicated in Fig. 1(a). If the energy released by this flux motion causes a larger temperature rise than the
initial fluctuation in temperature, the process will continue leading to thermal runaway.

Wipf has carried out a careful analysis of this process. He assumed that when there is a small perturbation in field, both the Lorentz force and the pinning force change. If the Lorentz force is smaller than the pinning force after the flux redistribution, then the superconductor is stable. Moreover, since the magnetic diffusivity of the superconductor is usually much higher than the thermal diffusivity, he noted that the superconductor is locally adiabatic, and that this stability condition must be fulfilled simultaneously over the entire superconductor. Using this model and a critical-current density of the form \( J_c = \alpha(B + B_0) \), where \( B_0 \) is a temperature-independent constant, he arrived at the stability condition

\[
\sigma < \frac{\pi c^2}{16} C \left[ J_c \left( -\frac{\partial J_c}{\partial T} \right)^{-1} \right]^{1/3},
\]

where \( C \) is the specific heat of the superconductor and \( \sigma \) is the total linear current density. Note that this stability condition does not depend on \( B_0 \), i.e., \( J_c = \alpha/B \) and \( J_c = \text{const} \). This would presumably not be the case if \( B_0 \) were temperature dependent, as has in fact been observed.

Wilson et al. presented an approximate theoretical estimate of the stability condition which can be arrived at rather simply. Instead of treating the superconductor as locally adiabatic, these authors assumed that the bulk of the superconductor is isothermal, but there is no heat exchange between the superconductor and its surroundings. With this much simpler model, they derived the stability condition

\[
\sigma < \frac{3c^2}{4\pi C} \left[ J_c \left( -\frac{\partial J_c}{\partial T} \right)^{-1} \right]^{1/3} = \sigma_m.
\]

The difference between the approximate and more rigorous treatments is seen to be only about 10%.

Even though this result is in reasonable qualitative agreement with the measurements of Wipf and of Wilson et al., the stability criterion for superconductors with a surface barrier is expected to be more complicated. In addition, at low temperatures \( \sigma_m \) is not large enough to ensure that a conductor can be fully stable at the fault current levels anticipated in superconducting power transmission lines. The most common way to stabilize such a superconductor is by using a normal-metal coating to increase both the magnetic diffusion time and heat capacity of the composite conductor. In Secs. III and IV we extend the simple approach of Wilson et al. to include both surface-barrier currents and the effect of a normal-metal stabilization coating.

**III. UNSTABILIZED CONDUCTORS WITH SURFACE BARRIERS**

Figure 1 shows the field distribution inside the simple Bean model superconductor with and without a surface barrier. Figure 1(a) shows a conductor with no surface barrier. Stability in this case is governed by Eqs. (1) and (2). In the case of Fig. 1(b), we have a conductor with a simple surface barrier. The stability criterion for this case differs from that with no barrier and can be obtained as follows. For simplicity, we assume that the surface current is uniform over the thickness of the surface-barrier region and that the critical-current density \( J_c \) of the bulk region is independent of field. We characterize the surface current region by means of a thickness \( \Delta_b \) and a maximum linear current density \( \sigma_s \).

Now consider the case where the total linear current density \( \sigma \) is larger than \( \sigma_s \). Note that this corresponds to a total surface field \( B = (4\pi/c) \sigma \) in cgs units. Then,

\[
\sigma = \sigma_s + J_c(x - \Delta_b)
\]

\[
\equiv \sigma_s + \sigma_b,
\]

where \( x \) is the total depth of flux penetration and \( \sigma_b \) is the total bulk linear current density. An increase \( \Delta T \) in the temperature causes a decrease \( \delta J_c \) in \( J_c \) and \( \sigma_b \) in \( \sigma_s \). The decrease in \( J_c \) and \( \sigma_b \) causes, in turn, an increase \( \Delta x \) in the depth of flux penetration. The resultant movement of flux generates an amount of heat \( 5Q \) per unit area. The portion of this heat produced in the surface region \( 0 < x < \Delta_b \) is given by

\[
5Q_s = J_s \int_0^{\Delta_b} J_s \int_{\text{initial}}^{\text{final}} E(x') dx' dx,
\]

\[
= (1/c) \int_0^{\Delta_b} J_s \Delta \phi(x') dx',
\]

where \( J_s = \sigma_s/\Delta_b \) is the current density in the surface.

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region when \( \sigma = \sigma_s \) and \( \Delta \phi(x') \) is the change of flux per unit length in the region inside of the point \( x' \) which induces the local electric field \( E(x') \). For the case illustrated in Fig. 1(b), this flux change is given by

\[
\Delta \phi(x') = \left( 4\pi/2\sigma_s \right) \int_{J_e}^{J_{s}} (x - \Delta_s) \Delta x + (x - \Delta_s)^2 \delta J_e \bigg|_{J_e}^{J_{s}} + \left( 4\pi/2\sigma_s \right) \Delta J_e \bigg|_{x'}^{x'} \quad \Delta s < x' < x, \quad (4)
\]

where

\[
\Delta x = \frac{\delta J_e}{\delta c} (x - \Delta_s) + \frac{\delta J_e}{\delta c} \Delta_s,
\]

\[5J_e = - \frac{\partial J_e}{\partial T} \delta T,
\]

and

\[5J_s = - \frac{\partial J_s}{\partial T} \delta T.
\]

Substituting Eq. (4) into Eq. (3), we obtain for the heat released in the surface region

\[
\delta Q_s = \left( 4\pi/2\sigma_s \right) \int_{J_s}^{x} J_e \Delta \phi(x') \, dx' + \frac{1}{2} (x - \Delta_s)^2 \delta J_e.
\]

Similarly, the heat in the bulk region is

\[
\delta Q_b = \left( 1/c \right) \int_{J_s}^{x} J_e \Delta \phi(x') \, dx' = \left( 4\pi/2\sigma_s \right) \int_{J_s}^{x} J_e \Delta \phi(x') \, dx' + \frac{1}{2} (x - \Delta_s)^2 \delta J_e.
\]

From these values of the heat generated during the flux motion, the stability of the critical state can be calculated either by assuming that the entire sample is isothermal or that each region, the surface-barrier region and bulk current region, is individually isothermal with no heat transfer between them. We shall refer to this as the quasiadiabatic approximation, since it is similar in spirit to the locally adiabatic approach taken by Wipf.\(^{19}\) Assuming the heat is uniformly distributed (equivalent to the Wilson et al. approach with no surface barrier), the temperature rise is

\[
\delta T' = \frac{5 \delta Q_s + 5 \delta Q_b}{C_x}.
\]

where \( C \) is the specific heat of the sample. The stability condition to avoid thermal runaway then is

\[
\frac{\delta T'}{\delta T} = \frac{4\pi}{2\sigma_s} \frac{1}{c} \int \left[ \frac{1}{2} (x - \Delta_s)^2 \sigma_s \left( \frac{\partial J_e}{\partial T} \right) + \frac{1}{2} (x - \Delta_s)^3 \sigma_s \left( \frac{\partial^2 \sigma_s}{\partial T^2} \right) \right] \leq 1.
\]

The temperature dependences of \( J_e \) and \( \sigma_s \) are, in general, different; but for simplicity, we shall assume they have the same dependence. We expect that qualitatively the results will be the same. Rearranging Eq. (8), the stability condition becomes

\[
3 \left( \frac{\sigma_s}{\sigma_m} \right)^3 + 3 \left( \frac{\sigma_s}{\sigma_m} \right)^2 \left( \frac{\sigma_s}{\sigma_m} \right) + \frac{\sigma_s}{\sigma_m} - 1 \leq 0,
\]

where \( \alpha = \Delta_s (\sigma_s/\sigma_m)^{-1} \) and \( \sigma_m \) is defined in Eq. (2). Here, \( \alpha \) is the ratio of the thickness of the surface barrier to the maximum stable depth of flux penetration in the critical state for an equivalent superconductor which has no surface barrier. For real conductors, we estimate \( \alpha \approx 0.01 - 1.0 \).

If we assume that the temperature rises independently in the surface barrier and bulk regions, the temperature increase is

\[
\delta T'_s = \frac{5 \delta Q_s}{C \Delta_s},
\]

in the surface region, and

\[
\delta T'_b = \frac{5 \delta Q_b}{C (x - \Delta_s)}.
\]

![Fig. 2. Instability boundaries for a superconductor with a surface-barrier current \( \sigma_s \) of relative thickness \( \alpha \). The upper graph (a) is based on the approximation that the entire sample is isothermal, while the lower (b) is for the surface and bulk region being individually isothermal with no heat transfer between them.](image)

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in the bulk region. The stability condition for the surface region becomes

$$\frac{\delta T'}{\delta t} = \frac{4n}{c^2} \frac{1}{C} \frac{\sigma_s}{\Delta_s} \left[ \frac{2}{v - \Delta_s} \left( \frac{\partial \Delta_s}{\partial T} \right) + (x - 3\Delta_s) \left( \frac{\partial \sigma_s}{\partial T} \right) \right] \leq 1.$$  

(10)

This reduces to

$$\frac{3}{2} \left( \frac{\sigma_s}{\sigma_m} \right)^2 + \alpha \left( \frac{\sigma_s}{\sigma_m} \right)^2 \left( \frac{\sigma_s}{\sigma_m} \right) - \alpha \left( \frac{\sigma_s}{\sigma_m} \right)^2 - 1 \leq 0,$$  

(11)

where we have again assumed $\sigma_s$ and $J_c$ have the same temperature dependence. Similarly, the stability condition for the bulk region is

$$\left( \frac{\sigma_s}{\sigma_m} \right)^2 + \frac{3}{2} \left( \frac{\sigma_s}{\sigma_m} \right) - 1 \leq 0.$$  

(12)

The stability criterion for the first case with the surface barrier and the bulk at the same temperature [Eq. (9)] is plotted in Fig. 2(a). The second case with its two regions separately adiabatic is plotted in Fig. 2(b) [Eqs. (11) and (12)]. For a given $\sigma_s$, which is a property of each sample, the above equation gives a boundary between the stable and unstable regions. For example, from Fig. 2(a) we see that a sample with $\sigma_s = 0.01$ and $\sigma_m = 0.3\sigma_m$ is stable only for $\sigma_c \leq 0.5\sigma_c$. Therefore, the total maximum stable linear current density in this approximation is $\sigma_c = 0.83\sigma_m$, which is smaller than $\sigma_m$, the maximum stable current for a superconductor without a surface current. The situation is even worse for the second approximation. From Fig. 2(b), we see that the same sample as above is stable for $\sigma_c \leq 0.03\sigma_m$. Thus, the total maximum stable current in this second approximation is $\sigma_c = 0.33\sigma_m$.

We also note that in Fig. 2(a), as $\sigma$ goes to zero (i.e., the surface-barrier region becomes very thin), the stability boundary approaches a finite limit in which the maximum stable surface current is $\sigma_m/\sqrt{3}$. However, in Fig. 2(b) it can be seen that the surface current region is very unstable in the quasilocalladiabatic approximation as the thickness of the barrier region becomes smaller. The decreased stability in this case arises from the fact that for a given temperature fluctuation the amount of heat produced in the surface region is finite, but the volume of the surface region, and thus its total heat capacity, goes to zero proportional to $\sigma$. The situation in real conductors should lie between these two extreme cases. From the above results, we see that the existence of the surface current density reduces the maximum stable linear current density, and that as the thickness of the surface barrier becomes smaller, the stability condition becomes considerably more severe.

IV. STABILIZATION BY A NORMAL-METAL OVERCOAT

The instabilities discussed above, particularly that in the surface barrier region, can be alleviated substantially by coating the superconductor with normal metal on the high-field side, as is discussed below. Stabilization can also be achieved by means of a thick undercoat of normal metal on the low-field side. This situation is not analyzed here, however.

The heat which is produced during the first quick process is the important heat which produces the instability and is given by

$$\delta Q_I = \frac{(\pi/3c^2)}{x^2} J_c \delta J_c.$$  

(13)

This is one-fourth of the heat which is generated in the superconductor without normal-metal coating. If we assume that the normal metal shares the heat with the superconductor, the stability requirement is
\[ \sigma < 2\sigma_m \left(1 + \frac{d_s C_s}{d_s C_a}\right)^{1/2} \]

\[ = 2\sigma_m, \quad (14) \]

where \( C_a \) is the specific heat of the normal metal, \( C_s \) is the specific heat of the superconductor, and \( d_s \) is the thickness of the superconductor. More exactly, in Eq. (14) \( d_s \) should be replaced by the penetration depth \( x \), but for simplicity we use \( x = d_s \). The factor 2 in Eq. (14) is the principal gain in stability produced by the normal-metal coating on the high-field side and is a consequence of the magnetic damping. The additional factor reflects the added heat capacity of the copper and leads to a smaller enhancement in this case, since in practice we expect \( d_s > d_s \).

We return now to the more interesting situation in which the conductor has a surface current and is stabilized by a normal-metal coating. This situation is illustrated in Fig. 3(b), beginning with a temperature perturbation at time \( t = 0 \). We can obtain the stability requirement from a calculation similar to those above. Without heat sharing of the copper, the stability conditions are

\[ 3\left(\frac{\sigma_k}{\sigma_m}\right)^2 + 4\left(\frac{\sigma_x}{\sigma_m}\right)^2 - 4\left(\frac{\sigma_k}{\sigma_m}\right)^2 - 4 \left(\frac{\sigma_x}{\sigma_m}\right)^2 \leq 0, \quad (15) \]

for the surface region, and

![Stabilized Surface Barrier](image)

**FIG. 4.** Instability boundaries for a superconductor with a surface barrier and a normal-metal overcoat which does not share heat with the surface-barrier region.

![Stabilized Surface Barrier](image)

**FIG. 5.** Enhanced surface stability obtained with a normal-metal overcoat which completely shares heat with the surface-barrier region as well as contributing to the magnetic damping of flux motions.

\[ \left(\frac{\sigma_k}{\sigma_m}\right)^3 - 4\left(\frac{\sigma_k}{\sigma_m}\right) + \alpha \left(4\left(\frac{\sigma_k}{\sigma_m}\right)^2 + 3\left(\frac{\sigma_k}{\sigma_m}\right) - 4\right) \leq 0, \quad (16) \]

for the bulk region. These conditions are plotted in Fig. 4. Even if the copper does not share the heat, the stability is considerably improved. For example, a sample with \( \alpha = 0.01 \) and \( \sigma_x = 0.3\sigma_m \) is stable for \( \sigma_k > 2\sigma_m \). The total maximum stable linear current density is thus \( \sigma_k = 2.3\sigma_m \). This value of \( \sigma_k \) is much higher than that found previously for the superconductor without a normal-metal coating (0.33\sigma_m < \sigma_k < 0.83\sigma_m).

If we now allow the copper to fully share the heat generated in the surface region, the specific heat \( C_s \) of the surface region must be replaced by \( C_s(1 + d_s C_a/\Delta C_s) \), and the stability becomes

\[ 3\left(\frac{\sigma_k}{\sigma_m}\right)^2 + 4\left(\frac{\sigma_x}{\sigma_m}\right)^2 - 4\left(1 + \frac{d_s C_a}{\Delta C_s} \frac{\sigma_k}{\sigma_m}\right) \]

\[ + \alpha \left(\frac{\sigma_k}{\sigma_m}\right)^2 - 4 \left(1 + \frac{d_s C_a}{\Delta C_s} \frac{\sigma_k}{\sigma_m}\right) \leq 0. \quad (17) \]

The stability condition in the bulk remains the same as before.

Figure 5 shows a plot of these results for fixed \( \alpha = 0.01 \) and various thicknesses of the normal-metal overcoat. The left-hand curve shows the case of no heat sharing while the right-hand curve shows the result for full heat sharing, where a typical value of 10 for
\( C_A d_s / C_A \Delta s \) was used. Figure 5 shows that the maximum stable surface current can be increased by a factor of 3 when the normal metal shares the heat.

The remaining question is to establish the actual thickness of normal metal needed to stabilize the superconductor. The thermal-diffusion time of the superconductor should be the same or shorter than the magnetic diffusion time of the normal metal such that some fraction of the heat which is generated in the second slow process can be dissipated.

\[ \tau_{\text{th}}^{(n)} \leq \tau_{\text{m}}^{(s)} \]

This means

\[ \frac{d_s^2}{D_{\text{th}}} \leq \frac{d_s^2}{D_{\text{m}}} \]

or

\[ d_s \leq d_s (D_{\text{m}} / D_{\text{th}})^{1/2} \]  \hspace{1cm} (18)

From the above argument the proper thickness of Cu with a resistance ratio of 270 to stabilize a typical SPTL conductor (NbSn thickness \( \sim 5 \mu m \)) is \( d_s \geq 8 \mu m \). The eddy-current loss for this copper is about 0.05 \( \mu W/cm^2 \) for \( \sigma = 500 \) A/cm rms, which is negligible compared to the 10 \( \mu W/cm^2 \) considered acceptable in SPTL applications.


\(^{13}\)L. Wipf, Phys. Rev. 161, 404 (1967).


\(^{17}\)An alternative way of writing this critical current is in terms of a zero-field critical-current density \( J^* \). If \( J_s = \sigma (B + B_s)^{-1} \), then one can also write \( J_s = \sigma (B + B_s)^{-1} \). This form has the advantage of being easily related to both the \( J_s = \text{const} \) and \( J_s = J^*(B_s/B) \leq D^* \) limiting cases. Temperature dependence of both \( J^* \) and \( B_s \) is reported in Ref. 4. it can be seen from these data that at least in the one NbSn sample studied at several temperatures \( J^*/B_s \) is independent of temperature. This implies that \( \sigma / B_s^2 \) is temperature independent, i.e., \( B_s \) has a much smaller temperature dependence than \( \sigma \). Thus, replacing \( \alpha / \partial T \) with \( \alpha / \partial T \), as is usually done [see Eq. (1)], is a reasonable approximation, but not strictly true, for NbSn.