Nonlinear Momentum Transfer Control of a Gyrostat with a Discrete Damper by Using Neural Networks

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Abstract: An adaptive feedback linearization technique combined with neural networks is addressed for the momentum transfer control of a torque-free gyrostat with an attached spring-mass-dashpot damper. The input normalization neural network is used to adaptively compensate for the model error uncertainties of a nutation damper as an internal dynamics and avoid the unnecessary assumptions for stability analysis. The whole spacecraft angular momentum component of the wheel spin axis is selected as the output function of the feedback linearization. Thus, a desired output function is predefined that the total angular momentum of the spacecraft is absorbed into the wheel spin direction at the steady state so that the nutation angle converges to zero. The ultimate boundedness of the tracking error is simply proved by the Lyapunov stability theory. We also investigate the effect of rotor misalignment on the steady spin of the spacecraft. The effectiveness of the proposed control law is verified through simulation results.

Keywords: gyrostat, feedback linearization, neural network, rotor misalignment

1. INTRODUCTION

Basically, by the principle of gyroscopic effect, the dual-spin turn (DST) maneuver of gyrostats results in attitude change of spacecraft and momentum transfer between main platform and spinning wheel. An easy strategy for the momentum transfer maneuver is to apply constant torque to the wheel. The wheel accelerates and the torque command is continued until the wheel angular momentum level becomes that of the whole system. However, the perfect alignment of wheel spin axis to the spacecraft’s angular momentum vector may not be achieved by constant torque input. The residual nutation error is usually produced and it can be removed by incorporating a passive energy dissipation mechanism in the platform.

In a recent research, the feedback linearization technique is applied to the nonlinear momentum transfer control of a spacecraft with single momentum wheel in Ref. 1. The whole spacecraft angular momentum component of the wheel spin axis is selected as the output function for this technique. Thus, the desired output function is predefined that the total angular momentum of the spacecraft is absorbed into the wheel spin direction at the steady state so that the nutation angle converges to zero. The performance in terms of final nutation error is enhanced compared to the previous works referred in this paper.

In this study, the feedback linearization technique is used again for a torque-free gyrostat with an attached spring-mass-dashpot damper. However, the dynamics of proposed spacecraft model is too complex to apply this technique directly. Thus, the uncertain model error regarded as an internal nutation damper is handled with an identification or on-line adaptation method by neural networks. The feedback linearization technique is limited to the full knowledge of the nonlinear system model and applicable only when the system is feedback linearizable. If the exact model is not known or only uncertain system information is available, the boundedness of the error and the stability of the closed loop system are not guaranteed. From this observation, an adaptive feedback linearization technique combined with the neural network to compensate for the uncertain model error of a nutation damper is proposed. The ultimate boundedness of the tracking error is simply proved by the Lyapunov stability theory. We also investigate the effect of rotor misalignment on the steady spin of the spacecraft. The effectiveness of the proposed control law is verified through simulation results.

2. EQUATION OF MOTION

2.1 Gyrostat with a single momentum wheel

The gyrostat model we study is shown in Fig. 1. The system consists of a rigid body \( \mathbf{B} \), containing a rigid axisymmetric rotor \( \mathbf{R} \), and a spring-mass-dashpot damper with mass particle \( \mathbf{P} \), which is constrained to move along a line defined by the unit vector \( \mathbf{n} \) fixed in \( \mathbf{B} \). The rest of the damper is considered massless. The rotor spins about its axis of symmetry defined by the unit vector \( \mathbf{a} \). \( \mathbf{b} \) and \( \mathbf{c} \) are the rest position of damper mass from the origin and damper displacement in \( \mathbf{n} \) respectively. First, let’s consider the equations of motion in case damper is not included. The equations of motion are developed by Hughes’ using a Newton-Euler approach. Assuming that external force and torque is zero, the governing equations of motion can be expressed as

\[
\begin{align*}
I_1' \dot{\theta}_1 + (I_2 - I_3) \omega_2 \omega_3 + u &= 0 \quad (1) \\
I_2 \dot{\theta}_2 + (I_1 - I_3) \omega_1 \omega_3 + \omega_2 h_c &= 0 \quad (2) \\
I_3 \dot{\theta}_3 + (I_1 - I_2) \omega_1 \omega_2 - \omega_3 h_c &= 0 \quad (3) \\
I_1' \dot{h}_c + J_\omega (I_1 - I_3) \omega_1 \omega_2 - I_1 u &= 0 \quad (4)
\end{align*}
\]

where \( I_i' = I_i - J_{\omega} \). Now, the preceding equations can be put into a general nonlinear state equation form as

\[
\dot{x} = f(x) + g(x)u
\]

As a convenient criterion for the momentum transfer,
the nutation angle of the system is defined as

$$\theta = \cos^{-1} \left( \frac{\mathbf{b}_1 \cdot \mathbf{h}}{||\mathbf{h}||} \right)$$

(6)

The nutation angle decreases from 90 degree, with initial spin of the spacecraft about \( \mathbf{b}_1 \) axis, to a smaller value as the DST maneuvers take place. The smaller the nutation angle is, the more successful maneuver performance is.\(^6\)

![Fig. 1 Single-rotor gyrostat with discrete damper.](image)

**2.2 Single-rotor gyrostat with discrete damper**

Assuming that external force and torque is zero, the equations of motion of the spacecraft containing internal damper are expressed as\(^7\)

$$I_1 \dot{\alpha}_1 + (I_3 - I_1) \omega_2 \omega_3 + u + \delta f_1(x) = 0$$

(7)

$$I_3 \dot{\alpha}_3 + (I_3 - I_1) \omega_2 \omega_3 + \delta f_3(x) = 0$$

where

$$\delta f_1 = -2m_h \omega_2 \omega_3$$

$$\delta f_3 = -m_h \omega_2 \omega_3$$

and the preceding equations can be put into a general nonlinear state equation form as

$$\dot{x} = f(x) + g(x)u$$

(8)

where the equations of motion can be decoupled into the nominal terms, \( \hat{\ }, \) regarded as best estimates of the system and modeling errors, \( \delta, \) including internal motion variables as uncertainties.

### 3. Adaptive Controller Design

#### 3.1 Feedback linearization

The feedback linearization technique is applied to control the DST maneuver of a gyrostat expressed as Eq. (7). Since the internal motion variables due to the damper system are difficult to identify exactly, they are treated as uncertainties to be controlled by neural networks. Thus, the spacecraft model represented by Eqs.(1) \( \sim \) (4) is selected as the best estimates for the feedback linearization. First, the output and reference function are chosen as

$$y = I_1 \omega_1 + h_0$$

(9)

$$y_r = H_1 [1 - \exp(-t/\tau)]$$

(10)

Then, the closed-loop system is expressed as\(^1\)

$$\dot{\xi} = A_1 \xi + B_1 \gamma(x) [u - \alpha(x)]$$

(11)

$$y = C_1 \xi$$

(12)

The error dynamics is also derived through the change of variables from Eq.(5) given by

$$\dot{E} = A_1 E + B_1 \gamma(x) [u - \alpha(x)] - y_r(x)$$

(16)

Finally, the feedback nonlinear control law is given as

$$u = \alpha(x) + \beta(x) [v + y_r(x)]$$

(17)

where \( \alpha(x) \) is an adaptive control term.

#### 3.2 Neural network application

A general three-layered neural network architecture is used. The output can be expressed in a simpler form as

$$y = W^T \sigma(V^T x)$$

(18)

Now, the INNN can be easily implemented to the spacecraft model by defining the normalized input vector as

$$z = s \frac{x}{||x||}$$

(19)

where \( s \) is a positive scaling parameter. The 2-norm of the normalized input vector is given by

$$||z||_2 = s$$

(21)

By inserting the feedback control law in Eq.(17) into Eq.(16), the reference model tracking error dynamics can be reformulated as

$$\dot{E} = AE + B_1 [\nu_{ad} + \Delta(x, u(x))]$$

(22)

where \( A = A_1 - B_1 K \). The pseudo control input is given.
3.3 Lyapunov analysis

Let us consider a Lyapunov function candidate for the proposed control law as

\[ V_L = \frac{1}{2} E^T PE + \frac{1}{2} \operatorname{tr}(W^T M^{-1} W) + \frac{1}{2} \operatorname{tr}(\hat{W}^T L^{-1} \hat{W}) \]  

(27)

The variables are selected as the state errors of the system and the weights differences to compensate for model uncertainties.

Consequently, by abbreviation of the calculation process, the time derivative of \( V_L \) is simplified into

\[ \dot{V}_L \leq -\lambda_{\min}(Q_L) \| \hat{L} \|^2 + \alpha_1 \| \hat{L} \|^2 \]  

(28)

where

\[ Q_L = \begin{bmatrix} \lambda_{\min}(Q) & -a_q \alpha_s \\ -a_q \alpha_s & k \end{bmatrix}, \quad L = \begin{bmatrix} \| E \|_2 & \| \hat{Z} \|_2 \end{bmatrix}^T \]  

(29)

Choosing

\[ \| \hat{L} \|^2 > \alpha_1 / \lambda_{\min}(Q_L) \]  

(30)

ensures that \( \hat{V} \) is negative definite. According to the Lyapunov stability theory, this result verifies the uniform ultimate boundedness of \( \| \hat{L} \| \). Also the following condition is required for the matrix\( (Q_L) \) to be a symmetric positive definite matrix.

\[ \lambda_{\min}(Q_L) k > a_q^2 \alpha_s^2 \]  

(31)

4. Simulation

Numerical simulations are conducted to verify the performance of the proposed control law. In order to compare the control performance, identical model data and simulation conditions are chosen\(^{1,12}\). The moment of inertia data for the spacecraft model and momentum wheel are \( I_L, I_z, I_1 = [85.12, 86.24, 113.59] \text{kg-m}^2 \) and \( I_1, I_2, J_3 = [0.05, 0.025, 0.025] \text{kg-m}^2 \), \( m_d = 10 \text{kg} \), \( m = 1000 \text{kg} \), \( b = 1 \text{m} \), \( k_d = 2 \text{N-m/s} \), and \( c_y = 2 \text{N-s/m} \), and initial flat spin about its major \( b \) axis with a spin rate of 0.1771 rad/sec is selected. To prevent unnecessarily excessive control command, a limiter is used as

\[ u(t) = \begin{cases} N & \text{if } u(t) > N \\ u & \text{if } -N < u(t) < N \\ -N & \text{if } u(t) < -N \end{cases} \]  

(101)

where the maximum torque input \( N \) is set to the magnitude of 0.005N-m. The spacecraft initially spins about \( b \) axis with the spin rate of 0.1771 rad/sec. Its spin axis has the maximum moment of inertia, whereas the rotor is aligned along the minimum moment of inertia axis. The spacecraft inertia property for the rotor spin axis is important for the successful momentum transfer maneuver. The proposed control law produces tracking control torque command to perform a successful momentum transfer maneuver. Consequently, the spacecraft finally spins about \( b \) axis which is the same as wheel spin axis. The smaller final nutation angle is used for the performance index.

Two kinds of control laws are introduced in the previous section by using feedback linearization and neural networks. These control laws are compared through the simulation results to verify the performance improvement.

A simulation is performed first about the single gyrostat with platform damper in Eq.(7) by using feedback linearization in Eq.(15). The simulation results are shown in Fig. 3. In order to examine the performance more exactly, both uncertainties and parameter errors are added to the system together with small leaking control torque. A major disadvantage of the feedback linearization control law is that it requires accurate system parameters. Consequently, the simulation results show that the performance is degraded due to the system model error and uncertainties. As shown in Fig. 3, steady state errors are produced by the unwanted leaking torque being seemingly identical to disturbances. The disturbances seem to perturb the desired command torque. Therefore, even though there are some inherent advantages, this control law seems to be subject to the performance degradation.
Fig. 3 feedback linearization to the single rotor gyrost at with a discrete damper.

from the modeling errors, uncertainties and disturbances.

Finally, a simulation is performed by using the neural network-augmented adaptive control law in Eq.(25). The simulation results are shown in Fig. 4. Compared to the simulation results only by the feedback linearization control law in Fig. 3, the proposed control law absorbs the initial angular momentum of the spacecraft with modeling uncertainties more efficiently. Therefore, the neural network plays a central role for the compensation of model error uncertainties. Residual nonzero angular velocity components about \( b_2 \) and \( b_3 \) axes are eliminated and thus, the final nutation angle approaches around zero. Fig. 4 also shows the time history of the neural network weights. They are updated by the on-line learning algorithm in Eq.(72). The weights trend
matches with the nutation angle response. As expected, they decrease from a value toward zero gradually.

The effect of rotor misalignment on the pure $b_r$ axis steady spin is investigated. Lyapunov’s indirect method is considered to analyze the effect. Unfortunately, a pure $b_r$ axis spin is impossible since it’s not an equilibrium point. Thus, we assume that $b_r$ axis spin is very small in consideration of the orbital rate of geostationary orbit. All eigenvalues of Jacobian matrix are examined by using identical spacecraft data with previous simulation. The simulations about zero or constant small value of $b_r$ axis spin are presented in Figs. 5, 6. The results show all the eigenvalues in case the angles of vector vary from 0 deg to 90 deg with 1 deg interval. From the result, it is easily seen that, for zero speed, all the roots are in the stable region. However, for small value of 0.1771 rad/sec, the equilibrium points are sometimes unstable.

4. Conclusions

The proposed control law was able to achieve satisfactory DST maneuver resulting in a very small nutation angle under the presence of model error uncertainties. The neural network was used to adaptively compensate for the model error uncertainties of nutation damper. The ultimate boundedness of the tracking error was shown through the Lyapunov stability theory. The stability of rotor misalignment was analyzed by Lyapunov’s indirect method analytically and numerically.

References


Fig. 5 Pure $b_r$ axis spin with zero speed.

Fig. 6 Pure $b_r$ axis spin with 0.1771 rad/sec.