Momentum Transfer Control of a Spacecraft with Two Wheels by Feedback Linearization

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Abstract: The momentum transfer control of a rigid spacecraft with two momentum wheel actuators is investigated using feedback linearization technique focusing on very small nutation angle as a performance index. The equations of motion are transformed to a general linearized form by feedback linearization, including a guarantee of internal dynamics stability. It is known that the configuration of inertia properties plays a pivotal role by analyzing spacecraft energy level. The assistance of added wheel is also analyzed analytically and numerically. The effectiveness of the proposed control law is verified through simulation results.

Keywords: feedback linearization, momentum wheel, misalignment, nutation angle

1. INTRODUCTION

It is well known that three actuators, either gas jets or momentum wheels, can be used to control the attitude of a rigid spacecraft and that arbitrary reorientation maneuvers of the spacecraft can be accomplished using smooth feedback[11–61]. On the other hand, it is also well proven that with less than three momentum wheel actuators the system becomes uncontrollable[71]. As a result, arbitrary reorientation maneuvers are analytically impossible using only two momentum wheel actuators. Nevertheless, the attitude stabilization problem using only two momentum wheel actuators have been performed by several researchers since wheels can fail to operate in space like FUSE and Hayabusa examples[77]–[81]. Krishnan et al. derived a discontinuous feedback control law that stabilized the spacecraft about any equilibrium attitude[81]. However, a series of eight maneuvers is required. Also, Krishnan et al. have considered stabilizing an under-actuated rigid spacecraft using two momentum wheels with the assumption that the initial velocity vector lies in the same plane as the two momentum wheels[91]. Tsiotras and Longuski have considered stabilizing an axially symmetric spacecraft using only two external pairs of gas jets[101]. However, the initial velocity is restricted to the control input plane.

We restrict the attitude control problem from arbitrary reorientation maneuver to the initial attitude acquisition maneuver by momentum transfer control. In this study, the feedback linearization technique is applied to know the help of added wheel. The paper is organized as follows. First, we simply formulate the dynamic equations of motion for two-wheeled spacecraft model, resulting in equations in terms of angular velocity and momentum. From generalized equations of motion with the restriction that two wheels are located in the same plane, they are simplified by placing two wheels to each spacecraft principal axis. Next, the control law using feedback linearization technique is presented. As a result, the equations of motion are easily transformed to a general linearized form including internal dynamics with a stability condition by Lyapunov function candidate. Finally, simulation results are presented to verify the effectiveness of the proposed control law.

2. EQUATIONS OF MOTION

Consider a rigid spacecraft installed with two momentum wheel actuators as shown in Fig. 1. We assume that the wheel spin axes, \( \hat{a}_1 \) and \( \hat{a}_2 \), lie on the principal axes \( \hat{b}_1 - \hat{b}_2 \) plane. Assuming that external force and torque are zero and two wheels are aligned along \( \hat{b}_1 \) and \( \hat{b}_2 \)-axis respectively, the governing equations of motion can be formulated in the body frame as

\[
\begin{align*}
I_1' \dot{\omega}_1 + (I_3 - I_2) \omega_2 \omega_3 - h_{\omega_2} \omega_3 + u_1 &= 0 \\
I_2' \dot{\omega}_2 + (I_1 - I_3) \omega_1 \omega_3 + h_{\omega_3} \omega_1 + u_2 &= 0 \\
I_3' \dot{\omega}_3 + (I_2 - I_1) \omega_1 \omega_2 + h_{\omega_1} \omega_2 - h_{\omega_2} \omega_1 &= 0 \\
I_1' \dot{h}_{\omega_1} + J_{\omega_1} (I_2 - I_3) \omega_3 + h_{\omega_3} \omega_3 - I_{u_1} &= 0 \\
I_2' \dot{h}_{\omega_2} + J_{\omega_2} (I_3 - I_1) \omega_1 + h_{\omega_1} \omega_2 - I_{u_2} &= 0 
\end{align*}
\]
The total kinetic energy of the spacecraft is written as

\[ E = \frac{1}{2} (I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2) + \omega_1 h_{\omega_1} + \omega_2 h_{\omega_2} \]
\[ + \frac{1}{2} h_{\omega_1}^2 + \frac{1}{2} h_{\omega_2}^2 \]  
\[ (2) \]

The time derivative of the total kinetic energy is given in the form

\[ \dot{E} = \Omega_1 u_1 + \Omega_2 u_2 \]  
\[ (3) \]

Therefore, the time derivative of the total kinetic energy is related to the torque input and angular speed of each wheel. As a convenient criterion for the momentum transfer, the nutation angle of the spacecraft is defined as

\[ \theta = \frac{(I_1 \omega_1 + c \omega h_{\omega_1} + c \beta h_{\omega_2})}{|h|} \]  
\[ (4) \]

where \(|h|\) denotes the magnitude of the total angular momentum. Note that \(|h|\) is constant by momentum conservation principle. \(\alpha\) and \(\beta\) are the clockwise alignment angle of each wheel about \(b_1\)-axis. Therefore, the nutation angle decreases from 90 deg, with the initial spin of the spacecraft about \(b_2\) or \(b_3\)-axis, to a smaller value as the DST maneuver takes place.

### 3. CONTROL LAW DESIGN BY FEEDBACK LINEARIZATION

#### 3.1 Initial Stability Analysis

The perturbed states from initial pure \(b_2\)-axis spin are prescribed as

\[ \omega_1 = \omega_1, \quad \omega_2 = \omega_2 + \omega_2^d, \quad \omega_3 = \omega_3^d \]
\[ h_{\omega_1} = h_{\omega_1}^d, \quad h_{\omega_2} = h_{\omega_2}^d, \quad u_1 = u_1^d, \quad u_2 = u_2^d \]  
\[ (5) \]

Then, the linearized equations of motion can be expressed as

\[ I_1 \dot{\omega}_1^d + (I_3 - I_2) \omega_2 \omega_3^d + u_1^d = 0 \]
\[ I_2 \dot{\omega}_2^d + u_2^d = 0 \]
\[ I_3 \dot{\omega}_3^d + (I_2 - I_1) \omega_1 \omega_3^d - \omega_2 h_{\omega_1} = 0 \]
\[ I_1 \dot{h}_{\omega_1}^d + J_{\omega_1} (I_3 - I_2) \omega_2 \omega_3^d - I_1 u_1^d = 0 \]
\[ I_2 \dot{h}_{\omega_2}^d - I_2 u_2^d = 0 \]  
\[ (6) \]

Therefore, the numerator of the nutation angle, namely, the angular momentum about \(b_1\)-axis can be written as

\[ I_1 \omega_1^d + h_{\omega_1}^d = \int (I_2 - I_3) \omega_2 \omega_3^d dt \]  
\[ (7) \]

A positive value of \(u_1^d\) satisfies \(\omega_2^d > 0\) condition regardless of the sign of \(\omega_2^d\) since the integral of \(u_1^d\) in time can guarantee positive sign of the right-hand side of Eq. (8). On the contrary, the nutation angle initially decreases to the negative torque value of \(u_1^d\) about the unstable moment of inertia properties satisfying \(I_1 > I_3 > I_2\). Moreover, \(u_2^d\) is not related to the initial stability condition to reduce nutation angle from Eq. (8). To verify this relationship exactly, the initial angular momentum about \(b_2\) axis can be rewritten as

\[ I_2 \omega_2^d + h_{\omega_2}^d = \text{constant} \]  
\[ (9) \]

It means that \(u_2^d\) does not change the angular momentum about \(b_2\)-axis in the initial state since the spacecraft initially spins about \(b_2\)-axis.

#### 3.2 Control Law Design by Feedback Linearization

The feedback linearization technique is used for the momentum transfer control of a rigid spacecraft with two momentum wheel actuators. Candidate output and reference functions for feedback linearization based on initial stability analysis are chosen as

\[ y_1 = I_1 \omega_1 + h_{\omega_1}, \quad y_2 = I_2 \omega_2 + h_{\omega_2} \]
\[ y_{1r} = h_T \left(1 - e^{-1/r_1}\right), \quad y_{2r} = h_T \sech(t/r_2) \]  
\[ (10) \]

Successive derivatives of output functions until they explicitly contain control input to determine the relative degree are as follows

\[ y_1 = (I_2 - I_3) \omega_2 \omega_3 + h_{\omega_2} \omega_3, \quad y_1 = S_1(x) + T_1(x) u_2 \]
\[ y_2 = (I_3 - I_1) \omega_3 \omega_1 - h_{\omega_3} \omega_1, \quad y_2 = S_2(x) + T_2(x) u_1 \]  
\[ (11) \]

To identify the internal dynamics, a variable \(\phi(x)\) is chosen such that

\[ \phi(0) = 0, \quad \frac{\partial \phi}{\partial x} C(x) = 0 \]  
\[ (12) \]

Hence, the partial differential equation can be solved by separating variables to obtain

\[ \phi(x) = I_1 \omega_1 + h_{\omega_1} + I_2 \omega_2 + h_{\omega_2} \]  
\[ (13) \]

which satisfies \(\phi(0) = 0\). The internal dynamics is equivalent to the sum of output functions \(y_1\) and \(y_2\) for the feedback linearization. Now, the equations of motion of the spacecraft are easily transformed to a general feedback linearized form by changing the variables such that

\[ \dot{h} = \xi_2 + \xi_1 \]
\[ \dot{\xi} = A_c \xi + B_c \gamma(x) [\omega - \alpha(x)] \]  
\[ (14) \]
\[ y = C_c \xi \]

The error dynamics and nonlinear feedback law are given by

\[ \dot{E} = A_c E + B_c [\gamma(x) (\omega - \alpha(x)) - R_2] = AE \]
\[ u = T(x)^{-1} (-S(x) + R_2 + v) \]  
\[ (15) \]
Therefore, $A$ can be Hurwitz by choosing a proper gain matrix $K$. In other words, there exists a positive definite matrix $Q$ satisfying

$$A^TP + PA = -Q$$

For a Lyapunov function candidate $V = \frac{1}{2}x^TPx > 0$ for $x \neq 0$. The Lyapunov equation guarantees a unique positive definite solution. Based on the Eq. (13), the internal dynamics is not globally asymptotically stable. However, since the Lyapunov stability condition of Eq. (16) guarantees that output functions track reference trajectories eventually, the internal dynamics is stable.

With the restriction that the misalignment angles of $\alpha$ and $\beta$ are small from orthogonal configuration, output functions for feedback linearization in Eq. (10) can be modified as

$$y_1 = I_1 \omega_1 + \cos \alpha h_{w1} + \cos \beta h_{w2}$$
$$y_2 = I_2 \omega_2 + \sin \alpha h_{w1} + \sin \beta h_{w2}$$

It is motivated by the idea that the momenta of two wheels about $b_1$ and $b_2$-axes depend on the alignment angles $\alpha$ and $\beta$. The rotor misalignment causes desired nominal spin to deviate from a pure $b_1$-axis spin, since the angular momentum about $b_2$-axis is produced by alignment angle $\alpha$. However, the added wheel aligned with $b_2$-axis can remove this term, even though the added wheel also has a misalignment angle about $b_2$-axis.

### 4. STABILITY ANALYSIS

A linear stability analysis is considered. A perturbed form of the nominal motion is established by defining variables as follows:

$$\omega_1 = \omega_p + \omega_1^d, \quad \omega_2 = \omega_2^d, \quad \omega_3 = \omega_3^d$$
$$h_{w1} = h_p + h_{w1}^d, \quad h_{w2} = h_{w2}^d$$

The equations of motion are simplified into

$$\dot{\omega}_1^d + \lambda_1 \omega_1^d = 0$$
$$\dot{\omega}_2^d + \lambda_2 \omega_2^d + \delta h_{w2}^d = 0$$
$$\dot{h}_{w2}^d + J_{w2} \lambda_2 \omega_2^d = 0$$

where

$$\lambda_1 = \frac{(I_1 - I_3) \omega_p + h_p}{I_2}, \quad \lambda_2 = \frac{(I_2 - I_3) \omega_p - h_p}{I_3}, \quad \delta = \frac{\omega_p}{I_3}$$

Then the determinant of coefficients by taking Laplace transforms of Eq. (19) gives the characteristic equation such as

$$s^2 + \lambda_1 (J_{w2} \delta - \lambda_2) = 0$$

### Table 1: Stability conditions for a rigid spacecraft with two momentum wheels

<table>
<thead>
<tr>
<th>Con1</th>
<th>$(I_1 \omega_p + h_p) - I_3 \omega_p &gt; 0$</th>
<th>$(I_1 \omega_p + h_p) - I_3^2 \omega_p &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol1</td>
<td>If $\omega_p = 0$, then $h_p &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>Sol2</td>
<td>If $h_p = 0$, then $I_1 - I_3 &gt; 0$ and $I_1 - I_3^2 &gt; 0$</td>
<td>$I_1$ has to be the maximum moment of inertia</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Con2</th>
<th>$(I_1 \omega_p + h_p) - I_3 \omega_p &lt; 0$</th>
<th>$(I_1 \omega_p + h_p) - I_3^2 \omega_p &lt; 0$</th>
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Requiring the coefficient of Eq. (21) to be positive yields the two necessary stability conditions to guarantee non divergent solutions for $\omega_2$ and $\omega_3$ as shown in Table 1. Solution 1 indicates that the addition of a spinning rotor within the spacecraft could contain enough angular momentum to stabilize a spacecraft’s spin axis, even for minor or intermediate axes. Therefore, this result does not specify any inertia relationships. However, solution 2 shows that the rotation about either the major or minor inertia axes is stable, and is similar to the rigid body stability condition. Therefore, the real spacecraft is only stable when spinning about the major axis in this state, since no spacecraft is ever a perfectly rigid body. These stability conditions are open loop analysis results without control inputs by linearization about an equilibrium point in (18), and the closed loop stability by a feedback control law is already verified in section 3.2. The closed loop stability analysis guarantees the asymptotic stability by feedback linearization technique, but there is no relationship between stability conditions and spacecraft dynamics. On the contrary, based on Table 1, the above stability conditions contain moment of inertia properties and angular momentum of the spacecraft. Therefore, we can understand the physical meaning of stability conditions, even though they cannot satisfy asymptotic stability conditions.

### 5. SIMULATION

Numerical simulations are conducted to verify the performance of proposed control law. Time constants $\tau_1$ and $\tau_2$ determining reference trajectory shapes are set to 800 and 1200, respectively. A feedback gain matrix $K$ is designed, $\alpha_0 = 0.1, \alpha_1 = 0.001, \beta_0 = 0.1, \beta_1 = 0.001$, to assign the eigenvalues of $A_c - B_c K$ at a desired location in the left-half complex plane. To prevent unusually excessive control commands, torque commands are implemented with a limiter such that $-N_1 \leq u_1(t), u_2(t) \leq N_1$, where the maximum torque input $N_1$ is set to 0.005 Nm. Moreover, to prevent unusually excessive wheel speed, the wheel speed is implemented with a limiter such...
that \(-N_2 \leq \Omega_1(t), \Omega_2(t) \leq N_2\), where the maximum wheel speed \(N_2\) is set to 5000 rpm for each wheel. In other words, the maximum angular momentum of each wheel is set to 26 Nms. The spacecraft initially spins about \(b_2\)-axis with a spin rate of 0.1771 rad/sec. The spacecraft inertia property is crucial for the successful momentum transfer maneuver. Therefore, according to the simulation cases, the initial spin axis \(b_2\) of the spacecraft is chosen as the maximum or minimum moment of inertia axis. Nevertheless, the proposed control law produces a proper tracking control torque command to execute successful momentum transfer maneuver. The performance index is the final nutation angle that needs to be made small enough.

First, a simulation is performed by using the proposed control law in Eq. (15). The moment of inertia data for the spacecraft model and two momentum wheels are \([I_1, I_2, I_3] = [85.12, 113.59, 86.24] \text{ kgm}^2\) and \([I_{w1}, I_{w2}] = [0.05, 0.05] \text{ kgm}^2\), respectively. Therefore, the initial spin axis \(b_2\) of the spacecraft is chosen as the maximum moment of inertia axis, whereas one wheel from two installed in the \(b_1\)-axis for the nominal spin is aligned along the minimum moment of inertia axis in opposition to that of the platform. The simulation results are shown in Figs. 2-3. The control input \(u_1(t)\) regardless of \(u_2(t)\) has positive value in the initial state based on the stability analysis in section 3.1, and they gradually converge to zero after appropriate torque profile generation by feedback linearization. Therefore, the nominal spin axis \(b_1\) seems to absorb the initial angular momentum of the spacecraft effectively. In other words, output functions track reference trajectories with a small error bound. As a result, the final nutation angle converges to a very small value, 0.6 deg, over the maneuver time with negligible residual oscillation. The total kinetic energy decreases from maximum value 220 J since the wheel angular momentum is smaller than 1 Nms based on Eq. (2). Even though the simulation result satisfies the open loop stability conditions in condition 2 of Table 1, since one of the two conditions is almost zero, it does not meet the stability requirement of characteristic equation in Eq. (21). It means that the spacecraft is likely to be unstable about the small disturbance torque or force at the steady-state in Fig. 2 without continuous active control. Moreover, two wheels are not saturated to the maximum speed as can be seen in the Fig. 3.

Secondly, the proposed control law is used again for the case of unstable moment of inertia configuration, namely, the initial spin axis \(b_2\) of the spacecraft is chosen as the minimum moment of inertia axis, whereas one wheel from two installed in the \(b_1\)-axis for the nominal spin is aligned along the maximum moment of inertia axis based on the stability analysis.
axis in opposition to that of the platform. Therefore, the moment of inertia data for the spacecraft model are chosen as 

\[ I_1, I_2, I_3 = [113.59, 85.12, 86.24] \text{ kgm}^2 \]

The simulation results are shown in Figs. 4-5. The control input \( u_1(t) \) regardless of \( u_2(t) \) has negative value in the initial state in opposition to the first simulation result, and they also gradually converge to zero after appropriate torque profile generation by feedback linearization. Therefore, the nominal spin axis \( h_1 \) seems to absorb initial angular momentum of the spacecraft effectively. As a result, the final nutation angle converges to a very small value, 0.6 deg, over the maneuver time with negligible residual oscillation. The wheel angular momentum \( h_{w_1}(t) \) becomes finally negative value larger than 1 Nms to satisfy the constant total angular momentum requirement. Therefore, the total kinetic energy level maintains the first maximum value 225 J through the simulation time in opposition to the first simulation result. Even though the simulation result satisfies the open loop stability conditions in condition 1 of Table 1, since one of the two conditions is almost zero like the previous simulation result, it does not meet the stability requirement of characteristic equation in Eq. (21). Therefore, the spacecraft is likely to be unstable about the small disturbance torque or force at the steady-state in Fig. 4 without continuous active control.

Finally, Figures 6-7 present the simulation results under the existence of rotor misalignment in both wheels, namely, the alignment angles of two wheels are \( \alpha = 2 \text{ deg} \) and \( \beta = 88 \text{ deg} \), respectively. Nevertheless, the proposed control law produces two proper tracking control torque commands \( u_1(t) \) and \( u_2(t) \) to execute a successful momentum transfer maneuver. As a result, the final nutation angle converges to a very small value, 0.6 deg, over the maneuver time. However, the new control law should be designed to the large misalignment angle since the proposed one is derived from two wheels configuration orthogonal to the principal axes.

6. CONCLUSIONS

In this paper, the momentum transfer control problem was proposed for a rigid spacecraft with two momentum wheel actuators. The feedback linearization technique was used for two wheels configuration orthogonal to the principal axes including the verification of asymptotic stability. As a result, the proposed control law was able to achieve satisfactory attitude resulting in a very small nutation angle. It was also analyzed that the added wheel did not contribute to the initial attitude acquisition maneuver in the initial state when it is installed in the initial spin axis. However, the added wheel overcomes the rotor
misalignment problem effectively by the proposed control law. The two-wheel configuration is related close to the attitude reorientation maneuver problem based on the previous studies since wheels can fail to operate in space. Therefore, this issue is suggested for further study.

REFERENCES


