Nonlinear Trajectory Tracking using Vectorial Backstepping Approach

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Abstract: This paper discussed a nonlinear tracking problem for stratospheric airship platform and a novel approach of the nonlinear control scheme is applied. Target airship model is the 200m stratospheric airship platform capable of flying upto 20 km altitude. Full 6-DOF nonlinear dynamics of unmanned airship are defined according to the target airship’s configuration including the moving wind field. Based on this airship model, a backstepping design formulation for the trajectory tracking control is described. The tracking control strategy of vectorial backstepping is applied to derive the tracking control law and global asymptotically stability is proved by Lyapunov stability analysis. Finally, numerical simulations have been carried out to assess the performance of the proposed tracking controller.

Keywords: Lighter-Than-Air Vehicle, Stratospheric Airship Platform, Nonlinear Control, Trajectory Tracking, Backstepping Control.

1. INTRODUCTION

The last decade has seen substantial advances in nonlinear control due to theoretical achievements. Considering the airship’s nonlinear characteristics, the nonlinear control theory must be appreciated as the efficient solution in designing a flight control system or tracking control for Stratospheric Airship Platform (SAP). Backstepping approach is a theoretically established and widely used in controlling nonlinear systems, and known as a nonlinear recursive design method based on a Control Lyapunov Function (CLF) [1-3]. It can provide the flexibility to treat nonlinear terms and guarantees the stability of a closed-loop system. In addition, it has an advantage to allow the robustness in the presence of the matched uncertainties [4].

This paper proposes a backstepping controller for trajectory tracking control of the 200m high-altitude unmanned airship. Global asymptotically stability of tracking control law is proven by applying Lyapunov stability analysis. For the convenience of application, vectorial formulation for 6-DOF nonlinear dynamics is introduced and derived. Finally, the proposed control law is simulated on full 6-DOF nonlinear airship dynamic equations to show the performance.

2. TARGET AIRSHIP (VIA-200)

Especially, the stratospheric airship platform (SAP) is being considered as a new platform which will provide satellite-like missions such as telecommunication, broadcasting relays, environmental observations, and surveillance at the stratospheric altitude. As a result of the conceptual sizing, the target SAP’s length is 200 m, its volume is 265,746 m$^3$ with its weight 21,726 kg, and the maximum available power for thrust is 100 kW. The buoyancy to weight ratio is 0.97 and its pressure altitude is 24 km [5]. VIA-200’s shape and specification is presented in Fig. 1 and Table 1.

![Fig. 1 Isometric view of SAP (VIA-200)](image)

Table 1 Specification of VIA-200

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, m</td>
<td>200</td>
</tr>
<tr>
<td>Envelope volume, m$^3$</td>
<td>265,746</td>
</tr>
<tr>
<td>Hull Area, m$^2$</td>
<td>25,564</td>
</tr>
<tr>
<td>Buoyancy, kg</td>
<td>21,074</td>
</tr>
<tr>
<td>Maximum take-off weight</td>
<td>21,726</td>
</tr>
<tr>
<td>Available power of thrust, kW</td>
<td>100</td>
</tr>
</tbody>
</table>

3. NONLINEAR DYNAMIC MODEL

The development of the 6-DOF nonlinear equations of motion for an airship follows a similar way of the fixed-wing aircraft. The major differences are due to its typical features such as buoyancy, added mass and added inertia terms [6-8]. These are commonly neglected in formulation of the aircraft dynamics and lead to more nonlinear characteristics in the airship dynamics.

3.1 Kinematics

For airship’s equations of motion, following vector notations are defined: 1) state vector $\mathbf{v} = \left[ v_x \ v_y \ v_z \right]^T \in \mathbb{R}^3$ is composed of velocity vector $\mathbf{v} = [u \ v \ w]^T$, and angular rate vector $\mathbf{\omega} = \left[ \rho \ \varphi \ \vartheta \right]^T$. 2) $\mathbf{\eta} = \left[ \mathbf{p}^T \ \mathbf{\theta}^T \right]^T \in \mathbb{R}^6$ consists of position vector $\mathbf{p} = [x \ y \ z]^T$ and attitude vector...
The position of airship is expressed in the earth-fixed inertial frame while the velocities are expressed in the body-fixed frame. Hence, kinematic equations can be defined as Eq. (1).

\[ \mathbf{\eta} = \mathbf{R}(\mathbf{\theta})\mathbf{v} \]  

(1)

\( \mathbf{\eta} \) is the transformation matrix from the body-fixed frame to the earth-fixed reference frame and can be written as Eq. (2).

\[ \mathbf{\eta} = \begin{bmatrix} \mathbf{R}_x(\mathbf{\theta}) & 0_{3 \times 3} \\ 0_{3 \times 3} & \mathbf{R}_s(\mathbf{\theta}) \end{bmatrix} \in \mathbb{R}^{6 \times 6} \]  

(2)

where,

\[ \mathbf{R}_x(\mathbf{\theta}) = \begin{bmatrix} c\theta c\psi c\theta + s\phi s\theta c\psi & c\theta s\psi + s\phi c\theta c\psi & s\phi s\theta c\psi + c\phi s\theta c\psi \\ c\theta s\psi c\theta + s\phi s\theta s\psi & c\theta c\psi c\theta + s\phi c\theta s\psi & s\phi s\theta s\psi + c\phi c\theta c\psi \\ s\phi \sec \theta c\theta & s\phi \sec \theta c\psi & c\phi \sec \theta \end{bmatrix} \]

\[ \mathbf{R}_s(\mathbf{\theta}) = \begin{bmatrix} 1 & s\phi \tan \theta & c\phi \tan \theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi \sec \theta & c\phi \sec \theta \end{bmatrix} \]  

(3)

\( s() \) and \( c() \) designate the sine and cosine function.

### 3.2 Nonlinear Dynamic Equations of Motion

For modeling the flight dynamics of the airship, we made assumptions similar to those for an aircraft; i.e. the vehicle is assumed to be perfectly rigid body and flying at a mean reference flight speed. However, the center of volume (CV), not the center of gravity (CG), is chosen as the origin of the airship body axes [8]. The buoyancy force, added mass, and inertia terms are significantly coupled and exists in equations compared to the aircraft nonlinear equations of motion. The added mass and added inertia effects arise due to the airship’s mass being of the same order of magnitude as the mass of displaced air, and are described as force and moment with respect to the linear and angular accelerations [7,9]. The Newtonian approach is summarized in Ref. 6 and 7, and VIA-200’s dynamic equations of motion are defined based on typical geometric and aerodynamic data of VIA-200. This component-type formulation is useful for conventional controller design and dynamic inversion approach. However, it is more efficient to use the state space form in order to apply the vectorial backstepping approach [6,10]. Finally, writing the equations of motions of airship in state space form, we get

\[ \mathbf{M}\ddot{\mathbf{\eta}} + \mathbf{C}(\mathbf{\eta})\dot{\mathbf{\eta}} + \mathbf{G}(\mathbf{\eta})[\mathbf{R}_s(\mathbf{\theta})] = \mathbf{U} \]  

(4)

The inertia matrix \( \mathbf{M} \) is sum of the inertia of the rigid body (\( \mathbf{M}_s \)) and the buoyancy air (\( \mathbf{M}_a \)). Diagonal terms \( \mathbf{M}_s \) and \( \mathbf{J}_s \) matrices designate the added mass and added inertia, respectively.

\[ \mathbf{M} = \mathbf{M}_s + \mathbf{M}_a = \begin{bmatrix} \mathbf{M}_s & -m[\mathbf{r}_s^a] \\ m[\mathbf{r}_s^a] & \mathbf{J}_s \end{bmatrix} \in \mathbb{R}^{6 \times 6} \]  

(5)

where,

\[ \mathbf{M}_s = \begin{bmatrix} m - X_s & 0 & 0 \\ 0 & m - Y_s & 0 \\ 0 & 0 & m - Z_s \end{bmatrix}, \quad \mathbf{J}_s = \begin{bmatrix} J_x & 0 & -J_z \\ 0 & J_y & 0 \\ -J_z & 0 & J_z \end{bmatrix} \]

The added inertia matrix \( \mathbf{M}_a \) exists in front of the time derivative of \( \mathbf{\eta} \).

\[ \mathbf{r}_s^a \times = \begin{bmatrix} 0 & -a_z & 0 \\ a_z & 0 & -a_y \\ 0 & a_y & 0 \end{bmatrix} \]  

(6)

\( m \) is the mass of the airship and the cross product matrix \( \mathbf{r}_s^a \times \) has distance variables \( (a_x, a_y, a_z) \) form CV to CG in Fig. 2.

![Fig. 2 Geometry relationship between CG, CB and CV](image)

Coriolis matrix \( \mathbf{C}(\mathbf{\eta}) \) is given in Eq. (6), which contains the nonlinear forces and moments due to centrifugal and Coriolis forces. The matrix \( \mathbf{G}(\mathbf{\eta}) \) contains the restoring forces and moments formed by the buoyancy and gravity.

\[ \mathbf{C}(\mathbf{\eta}) = \begin{bmatrix} (\omega \times)\mathbf{M}_s & -m(\omega \times)[\mathbf{r}_s^a] \\ m[\mathbf{r}_s^a](\omega \times) & \omega \times \mathbf{J}_s \end{bmatrix} \in \mathbb{R}^{6 \times 6} \]  

(7)

\[ \mathbf{G}(\mathbf{\eta}) = -[m - m_a]I_{3 \times 3} \]  

(8)

The cross product matrix \( \mathbf{r}_s^a \times \) has distance variables \( (h_x, h_y) \) form CV to CB in Fig. 2. Finally, terms of the right hand side of the Eq. (4) are external forces \( \mathbf{F}_e \) and moments \( \mathbf{T}_e \) and defined in Eq. (9) : i.e. aerodynamic, gravitational, buoyant, and propulsive forces and moments, respectively.

\[ \mathbf{U} = \begin{bmatrix} \mathbf{F}_e \delta, \mathbf{\delta}, \mathbf{\delta}, \mathbf{\delta}, \mathbf{\alpha}, \mathbf{\beta} \\ \mathbf{T}_e \delta, \mathbf{\delta}, \mathbf{\delta}, \mathbf{\delta}, \mathbf{\alpha}, \mathbf{\beta} \end{bmatrix} \]  

(9)

In summary, kinematics of Eq. (1) and dynamics of Eq. (4) is utilized for the design of the tracking controller in next section. The state vector \( \mathbf{v} = [\mathbf{\eta} \quad \omega \times \mathbf{\eta}] \in \mathbb{R}^7 \) is composed of six state variables and the real control inputs are composed of thrust \( \delta_t \), tilting angle \( \delta_\phi \), elevator \( \delta_e \) and rudder \( \delta_r \).

Especially, aerodynamic forces and moments can be calculated from the aerodynamic database, which was constructed by the wind-tunnel test by using the 1/25 scaled model. In addition, added mass and moment of inertia were obtained from the results of the CFD analysis [5]. One of major nonlinearities of Eq. (4) is the inertia matrix, \( \mathbf{M} \) exists in front of the time.
derivatives of the state vector due to the added mass and moment of inertia terms.

4. BACKSTEPPING DESIGN FOR TRAJECTORY TRACKING CONTROL

Backstepping approach is recursive procedure, which allows deriving control law for a nonlinear system, based on appropriate Lyapunov function candidate. In this section, the tracking control strategy of vectorial backstepping is applied based on the idea of Refs. 11 and 12. The vectorial backstepping gives an advantage over the integrator backstepping in case of applying to the complicated MIMO system since the control law is derived in few steps. The tracking control law is derived in a vectorial setting through two steps. Global asymptotically stability of the tracking control law is proven by Lyapunov stability analysis.

4.1 Step 1
Let’s define the first backstepping variable \( z_1 \), and desired state \( \eta \), then the tracking error variable can be
\[
z_t = \eta - \eta_i
\]  
(10)
The idea of backstepping is to choose one vector as virtual control. We suggest the virtual control by introducing the second backstepping variable \( z_2 \),
\[
\eta = R(0) v \tilde{v} + z_2 + a_i
\]  
(11)
where \( a_i \) is a stabilizing function. Time differentiation of Eq. (11) gives
\[
\tilde{z}_t = \dot{z}_t = \dot{\eta} - \dot{\eta}_i = R(0) v \tilde{\eta} + \dot{z}_2 + a_i - \dot{\eta}_i
\]  
(12)
Let us consider the Lyapunov function candidate \( V_i \) for the first step.
\[
V_i = \frac{1}{2} z_i^T P_i z_i
\]  
(13)
where \( P_i \) is a positive definite design matrix. To make \( \dot{V}_i \) negative definite with respect to \( z_i \), the stabilizing function \( a_i \) can be selected as
\[
a_i = \eta_i - \Lambda_i z_i
\]  
(14)
where \( \Lambda_i = \Lambda_i^T > 0 \) is design matrix. Substitution of Eq. (14) into the time derivative of Eq. (13) yields
\[
\dot{V}_i = z_i^T P_i \dot{z}_i - z_i^T P_i A_i z_i
\]  
(15)
Therefore, Eqs. (11) and (12) are rearranged as follows:
\[
z_t = \dot{z}_2 - \Lambda_i z_i
\]  
(16)
\[
z_2 = R(0) v - \dot{\eta}_i + \Lambda_i z_i
\]  
(17)

4.2 Step 2
Let us consider the second Lyapunov function candidate of Eq. (18)
\[
V_{1f} = V_i + \frac{1}{2} z_2^T P_2 z_2
\]  
(18)
where \( P_2 = P_2^T > 0 \) is a design matrix. After some manipulations using Eqs. (4), (14), and (17), the time derivative of \( z_2 \) is written as
\[
\dot{z}_2 = R(0) v - R(0) \Theta^T
\]
\[
\left[ C(v) v + G(\eta) \left[ R_i (0) \right]^T g + U - \dot{\eta}_i + \Lambda_i \dot{z}_i \right]
\]  
(19)
Then the derivative of the Lyapunov function \( V_{1f} \) is given by
\[
\dot{V}_{1f} = z_i^T P_i z_i - z_i^T P_i A_i z_i + z_2^T P_2 z_2
\]
\[
\left( R(0) v - R(0) \Theta^T \left[ C(v) v + G(\eta) \left[ R_i (0) \right]^T g + U \right] - \dot{\eta}_i + \Lambda_i \dot{z}_i \right)
\]  
(20)
To make \( \dot{V}_{1f} \) negative definite in \( z_i \) and \( z_2 \), the bracketed term in (20) must be equals to \( -P_2^T P_i z_i - \Lambda_i z_i \) and \( \Lambda_i = \Lambda_i^T > 0 \). Finally, the control law \( U \) can be obtained as Eq. (21).
\[
U = M R(0)^T \left[ -\Lambda_i z_i - P_i P_i z_i - R(0) \dot{v} + \dot{\eta}_i \right]
\]
\[
-\Lambda_i z_i + \Lambda_i \dot{z}_i + \left[ C(v) v + G(\eta) \left[ R_i (0) \right]^T g \right]
\]  
(21)
Substitution of Eq. (21) into the time derivative of Eq. (21) yields
\[
z_t = -P_i P_i z_i - \Lambda_i z_i
\]  
(22)
and the time derivative of second Lyapunov candidate function can be represented as
\[
\dot{V}_{1f} = -z_i^T P_i A_i z_i - z_2^T P_2 A_i z_2
\]  
(23)
It is clear that \( \dot{V}_{1f} \) can be made negative definite by choosing the positive definite weight matrices \( P_i, P_2, \Lambda_i, \) and \( \Lambda_i \). Thus, according to Lyapunov stability theory, the airship system is globally asymptotically stable [4].

4.3 Control allocation
It is necessary to find the real airship control inputs since the obtained control law of Eq. (21) regards six control forces and moments to achieve the desired tracking performance. In VIA-50A, total four real control inputs are used: thrust \( \delta_t \), tilting angle \( \delta_i \), elevator deflection \( \delta_e \), and rudder deflection \( \delta_r \). Consequently, the number of real control inputs reduces by three since VIA-200 does not use the tilting angle in flight phase except take-off and landing mode. Thus, VIA-200 belongs to an under-actuated system. Control allocation can be used to distribute the total control effort among the actuators and thrust in case that the number of controlled variables exceeds the number of actuators. Notice that the aerodynamic derivatives are loaded from the aerodynamic database. So, it is necessary to approximate them as a function of control surface inputs \( (\delta_t, \delta_i, \delta_e, \delta_r) \), angle-of-attack \( \alpha \), and sideslip angle \( \beta \) as following.
\[
C_{X}, C_{Y}, C_{Z}, C_{L}, C_{M}, C_{N} = f(\alpha, \beta, \delta_t, \delta_i)
\]
\[
C_{m}, C_{n}, C_{a}, C_{b}, C_{c}, C_{d}, C_{e}, C_{f}, C_{g}, C_{h} = f(\alpha)
\]  
(24)
The control allocation problem with aerodynamic forces and moments can be formulated according to the control law of Eq. (21).
here, \( \forall \) is a volume of an airship, and \( P_g \) is a dynamic pressure of the freestream. Solving Eq. (25) with respect to both control command \( \mathbf{U} \) and real control inputs \( (\delta_x, \delta_v, \delta_z) \) is not trivial because it has many solutions. Let us define the real control input vector \( \mathbf{U} = [\delta_x, \delta_v, \delta_z]^T \), then above equation can be rewritten as
\[
\mathbf{U} = \mathbf{B} \mathbf{U}
\]
where \( \mathbf{B} \) contains the terms directly related with \( \mathbf{U} \).

The rest terms are transposed into the right-hand side of Eq. (26) and yield \( \dot{\mathbf{U}} \) after subtracted from control command \( \mathbf{U} \) of Eq. (21). The least-square solution is simply given by Eq. (27).
\[
\mathbf{U} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \dot{\mathbf{U}}
\]

\section{5. Trajectory Tracking Results}

In this section, numerical simulations are conducted to verify the performance of the proposed backstepping controller for tracking trajectories. Numerical simulation codes are based on MATLAB/Simulink environment. The initial conditions for the simulation are given in Table 2, and the reference trajectories are generated from the kinematics of Eq. (1) cooperated with 6-DOF nonlinear dynamic equations of Eq. (4). Arbitrary control inputs scenario are scheduled to construct a helix-shape climb trajectory until SAP could reach the stratospheric altitude of 20 km. Total simulation time is 5.56 hrs (20,000 sec) and no wind conditions are considered.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial value</th>
<th>Variable</th>
<th>Initial value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_f )</td>
<td>8.71 m/s</td>
<td>( \psi )</td>
<td>0 deg</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>1.47 deg</td>
<td>( x_i )</td>
<td>0 m</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0 deg</td>
<td>( y_i )</td>
<td>0 m</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0 deg/s</td>
<td>( h )</td>
<td>0 m</td>
</tr>
<tr>
<td>( q )</td>
<td>0 deg/s</td>
<td>( \delta_x )</td>
<td>0.5031 deg</td>
</tr>
<tr>
<td>( r )</td>
<td>0 deg/s</td>
<td>( \delta_z )</td>
<td>0 deg</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0 deg</td>
<td>( \delta_y )</td>
<td>11,478 N</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0 deg</td>
<td>( \delta_v )</td>
<td>0 deg</td>
</tr>
</tbody>
</table>

The reference variables of trajectories are composed of positions and attitude of the refined trajectory results, \( \mathbf{q}_p = [x_{i, r} \ y_{i, r} \ z_{i, r} \ \phi_{i, r} \ \theta_{i, r} \ \psi_{i, r}]^T \). The proposed backstepping control law is conducted with scheduling design matrices \( \mathbf{P}_1, \mathbf{P}_2, \mathbf{A}_1, \) and \( \mathbf{A}_2 \).

Three-dimensional trajectory tracking result are shown in Fig. 3 and position tracking results in Fig. 4. The solid line represents the tracking result of the proposed controller, and the dotted line represents the reference trajectory. Referring to Figs. 1 and 2, the proposed backstepping controller follows the reference positions successfully.

\[ \text{Fig. 3 Three dimensional trajectory tracking results vs. reference trajectory} \]

\[ \text{Fig. 4 Tracking results of position variables vs. references} \]

Another tracking results of attitudes are shown in Fig. 5 and proposed backstepping controller presents accurate attitude following result. Other state variables are shown in Figs. 5 - 7, respectively. They show such a good performance of the proposed tracking controller.
Real control inputs of thrust $\delta_T$, tilting angle $\delta_\mu$, elevator deflection $\delta_e$, and rudder deflection $\delta_r$ are represented in Fig. 8. Notice that VIA-200 has no control means for rolling motion because it has not aileron.

Fig. 5 Tracking results of attitude variables vs. references

Fig. 6 Tracking results of flight speed, angle-of-attack, and sideslip angel vs. references

Fig. 7 Tracking results of angular rate vs. references

Fig. 8 Tracking results of real control inputs vs. references

In Fig. 10, some peak values are exists around $t = 3.3$ hr and 5.0 hr. These are caused by a discontinuous thrust input as shown in Fig. 8.

The root mean square (rms) error is used for compare the tracking error with the reference value. The rms error of position vector $\mathbf{p} = [x, y, z]^T$ is $0.0054$ m, $0.0204$ m, and $4.059 \times 10^{-4}$ m, respectively. One of the main source of the position errors are resulted from the attitude errors due to the transformation relationship of Eq. (1). For the attitude vector $\mathbf{\theta} = [\phi, \theta, \psi]^T$, the rms error is $0.001$ deg, $4.307 \times 10^{4}$ deg, and $1.637 \times 10^{4}$ deg, respectively. Other rms errors of state variables are summarized in Table 3.
These tracking errors are mainly affected by the pseudo-inverse error as well as the attitude tracking error of itself. Unlike the stabilization problem, tracking errors can not converge to zero because the airship’s reference trajectories are continuously changing along the flight time.

6. CONCLUSIONS AND REMARKS

This paper is concerned with the design of nonlinear tracking controller for the large-scaled unmanned airship. The vectorial backstepping approach was applied and the stability analysis for the proposed control law was carried out by using the Lyapunov theory. The nonlinear dynamics and kinematics modeling of the airship was re-derived and rearranged in the form of state space representation. Necessary aerodynamic, propulsive, and geometric data are based on the results of wind-tunnel test and CFD analysis. Numerical simulation has been carried out and demonstrates the performance of the proposed tracking controller. It keeps track of the given reference trajectories and state variables. In addition, it is shown that the tracking errors are small enough to present its good performance. Originally, the reference trajectories should be generated from the solution of the optimal trajectory problem. Although this paper is not dealing with the optimal trajectories, further study will be extended to that topic.

REFERENCES