Reorientational magnetic transition in high-density arrays of single-domain dots

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A theoretical study on the reorientational transition from in-plane to out-of-plane magnetized state is performed for two-dimensional magnetic dot arrays coupled by magnetostatic interaction. The square lattice of nanoscale cylindrical dots is considered with the assumptions that the dots are magnetically soft and they have uniform magnetization. The present study predicts that the interdot magnetostatic coupling determines the reorientation transition for close-packed arrays of such magnetic dots. Recent experimental results on the nanometer-scale single-domain dot arrays are discussed in light of the present calculation.

Research of two-dimensional arrays of nanoscale magnetic dots has attracted much attention during the past years. The dot shape is usually a rectangular prism or a circular cylinder. Besides promising technological applications, periodic arrays of magnetic dots are the model systems for studying magnetic interactions and switching behavior. In this case interdot exchange coupling is absent, and therefore magnetic properties of the array are mainly governed by magnetostatic coupling and in-dot anisotropy. These properties depend strongly on geometry of the dot arrays due to long-range character of magnetostatic coupling. For the application to high-density data storage the dots in the perpendicularly magnetized state are the most desirable. When the interdot spacing is small in comparison with the dot radius, interdot magnetostatic interaction becomes important in high-density arrays of magnetic dots. This interaction will destabilize and even destroy the out-of-plane state. On the other hand, reducing the dot height-to-radius ratio is also desirable. But this favors the in-plane magnetized state. Magnetization reversal transition from the in-plane to the perpendicular magnetized state in coupled dot arrays is important not only for applications to the field of data storage but also for studying basic questions of magnetic ordering in patterned magnetic films stabilized by magnetostatic interaction. In the present work we have investigated the reorientational transition as functions of the dot size and the dot lattice period. We report theoretical limitations on the existence of the perpendicularly magnetized state in the square array of cylindrical dots.

The calculation of the magnetostatic coupling energy is carried out for typical two-dimensional dot arrays with the square lattice of nanoscale single-domain dots. The circular cylindrical magnetic dots with the saturation magnetization \( M_s \), dot radius \( R \), and height \( L \) are considered. The dots are three-dimensional particles and both in-dot and interdot magnetostatic interactions are taken into account. The dipole approximation used in Ref. 9 is insufficient for the close-packed dot arrays and a more correct approach for magnetostatic energy calculation is necessary (see Refs. 10, 11, and the references therein). We neglect the in-dot magnetocrystalline anisotropy. This assumption is reasonable for the dots composed of soft magnetic materials (for instance, FeNi and Ni) or for the polycrystalline dots. The interdot magnetostatic coupling energy, according to the estimation of Ref. 4, is on the order of \( 10^6 \) erg/cm\(^2\) and thus, it is believed to play a major role for the soft magnetic dot arrays.

The interdot magnetostatic coupling is essential during remagnetization of the dot array. Magnetization reversal of the array of magnetostatically coupled dots will be more complicated in comparison with the one of the isolated single-domain dot. The ground state of the lattice of the strongly coupled single-domain dots will be a good starting point for calculation of switching the dot arrays. The noncollinear in-plane configurations of the dot magnetizations existing for the square dot lattice are considered in Ref. 11.

We use the following general expression for the magnetic self-energy density of magnetic film with inhomogeneous magnetization \( \mathbf{M}(\mathbf{r}) \) and volume \( V \) through magnetostatic scalar potential \( \varphi_m(\mathbf{r}) \)

\[
W_m[\mathbf{M}(\mathbf{r})] = \frac{1}{2V} \int d^3 \mathbf{r} \mathbf{M}(\mathbf{r}) \cdot \nabla \varphi_m(\mathbf{r}),
\]

\[
\varphi_m(\mathbf{r}) = \int d^3 \mathbf{r'} \mathbf{M}(\mathbf{r'}) \cdot \nabla \frac{1}{|\mathbf{r} - \mathbf{r'}|}.
\]

Since the typical dot density is \( 10^8 \sim 10^{10} \) dots/cm\(^2\) and dot array size is about 1 cm\(^2\), the edge effects of array can be neglected. To calculate \( W_m \) we consider a periodical arrangement of the dots in the film plane with the reciprocal lattice vector \( \mathbf{k} = (k_x, k_y) \). For the square lattice \( (k_x, k_y) = 2\pi(n_1, n_2)/T \). Here, \( T \) is the lattice period and \( n_1, n_2 \) are integers. We assume that the dot diameter and thickness are smaller than critical values of the existence of the single-domain state and the magnetization distribution \( \mathbf{M}(\mathbf{r}) \) within the dot is uniform or nearly uniform. Thus, \( \mathbf{M}(\mathbf{r}) = \mathbf{M}_s \) within the dot and \( \mathbf{M}(\mathbf{r}) = 0 \) outside the dot. To avoid the singularity in Eq. (1), the integration over \( \mathbf{r'} \) is first accomplished in cylindrical coordinates for each dot (the cylinder axis \( 0z \) is...
perpendicular to the film plane), followed by summing over the whole dot array in the square lattice. Let us suppose that the dot magnetization lies perpendicularly to the film plane. In this case the magnetostatic energy in Eq. (1) is simplified as follows:

$$W_m = 2\pi \sum_{k} f(kL)|M_k^i|^2,$$

(2)

where $$f(x) = [1 - \exp(-x)]x$$, 

$$M_k^i = S^{-1} f_x d^2 \rho M_x^i(\rho) \times \exp(ik\rho),$$

$$S$$ is the square of unit cell of the dot lattice, and $$\rho$$ is the radius vector in the $$x$$–$$y$$ plane.

The dot magnetizations, in general, may not be parallel, so that some noncollinear configurations can exist and provide the minimum of magnetostatic energy $$W_m$$ for in-plane magnetized dots. To calculate $$W_m$$ for perpendicular dot magnetizations, the method proposed by Guslienko 11 is adopted. The magnetization direction of the $$n$$ dot and its position within the unit cell are characterized by the polar angle $$\theta_n (n = 1, \ldots , N)$$ and the vector $$p_n$$, respectively. The magnetostatic energy is expressed in the following form:

$$W_m(\theta_n) = \sum_{k} F(k)|s(k, \theta_n)|^2,$$

(3)

$$F(k) = (2 \pi)^3 \left( \frac{R^2}{S} \right)^2 \frac{M_k^2}{k^2} J_1^2(kR) - f(kL),$$

where the structure factor is given by

$$s(k, \theta_n) = \sum_{n} \cos \theta_n exp(i k p_n).$$

Here, $$J_1(x)$$ is the Bessel function, $$k = 2\pi n_a / N^{1/2}(2R + d)$$, $$S = N(2R + d)^2$$, $$\alpha = x, y$$. $$N$$ is the number of the magnetic dots per unit cell and $$d$$ is the inter-dot distance. Then, the magnetostatic energy for the N dot unit cell can be expressed in the following form:

$$W_m(\theta_n) = W_m^0 + \sum_{n<n'} A_{nn'} \cos \theta_n \cos \theta_n',$$

(4)

$$W_m^0 = 2\pi (M_{k=0}^i)^2 + N \sum_{k=0} F(k),$$

$$A_{nn'} = \sum_{k=0} F(k) \cos k(p_n - p_{n'}).$$

The contribution to Eq. (4) from the coefficients $$A_{nn'}$$ has the usual exchange-like Ising form. Let us consider a simple case for the unit cell consisting of four dots ($$N = 4$$). We have only two coupling parameters $$A_{12} = A_{14} = A_{23} = A_{34} = a$$ (along the dot lattice axes), $$A_{13} = A_{24} = a_d$$ (along diagonal of the unit cell) due to symmetry of the square lattice. The coefficients $$a$$ and $$a_d$$ are negative for the typical dot diameter and the dot lattice period, and $$|a_d| > |a|$$.

We now compare the magnetostatic energies of different perpendicular configurations ($$\theta_n, n = 1, \ldots , 4$$) of the dot magnetizations, which correspond to the critical values of the function $$W_m(\theta_n)$$. For the ferromagnetic ($$F$$) configuration of $$\theta_n = 0$$, the magnetic moments of all dots are parallel. From Eq. (4) it follows that $$W_m(F) = W_m^0(F) + 4a + 2a_d$$. For the antiferromagnetic configuration AF1 with the parametrization $$\theta_1 = \theta_3 = 0$$ and $$\theta_2 = \theta_4 = \pi$$, the magnetostatic energy $$W_m(\text{AF1}) = W_m^0(\text{AF}) - 2a_d$$ is not minimal. This energy is minimal for the antiferromagnetic configuration AF2 with $$\theta_1 = \theta_3 = 0$$ and $$\theta_2 = \theta_4 = \pi$$, where $$W_m(\text{AF2}) = W_m^0(\text{AF}) - 4a + 2a_d$$. The energetically nearest configuration to the AF2 ground state is the AF1 configuration. These energies are practically different only for small $$d$$ and large dot aspect ratio $$B = L/R$$. The ferromagnetic configuration has high energy due to large contribution of $$\frac{M_{k=0}^i}{2}(M^i)^2$$, which is absent for the AF configurations. The magnetostatic energies of all dot magnetization configurations calculated above are positive due to the finite dot sizes. The “checkerboard” ground state AF2 of perpendicularly magnetized dots was tested by numerical simulations by Aign et al. 5 and Pardavi-Horvath 10 for the rectangular dots.

Let us compare the ground state configuration energy of the perpendicularly magnetized dots with one of the in-plane magnetized dots. The in-plane configuration calculated in Ref. 11 has the total magnetostatic energy as follows:

$$W_m(\|) = 4 \sum_{k} F_1(k) \cos^2 \varphi_k - 2 \sum_{k} F_1(k)(-1)^{n_1 + n_2},$$

$$+ 4 \sum_{k} F_1(k) \cos 2 \varphi_k (-1)^{n_1},$$

(5)

where function $$F_1(k)$$ differs from $$F(k)$$ by substitution $$f(x) \to -f(x)$$ and $$\varphi_k$$ is the azimuthal angle of the vector $$k$$.

We define the uniaxial anisotropy constant $$K_\| = W_m(\|) - W_m(\perp)$$ as the difference between the magnetostatic energies of the lowest in-plane and perpendicular to the film plane magnetization configurations. Thus, the equation $$K_\| (\beta, d) = 0$$ will describe the magnetization reorientational transition. This condition can be rewritten as the dependence of the critical dot aspect ratio $$\beta_c$$ on the interdot distance $$d$$ (Fig. 1). The dependence of $$\beta_c (d)$$ decreases rapidly and goes to the finite limiting value for $$d/R \rightarrow \infty$$, which corresponds to $$\beta_c$$ of noninteracting dots. The phase diagram could be interpreted that the interdot magnetostatic interaction encourages in-plane orientation of the dot magnetizations. This effect is especially strong for high-density dot arrays ($$d/R < 1$$), when dipole–dipole approximation is insufficient. 11 Consideration of the first-order uniaxial in-dot (or interface) anisotropy constant $$K_u$$ will change the transition equation to $$K_\| (\beta, d) + K_u = 0$$. The reorientation transition described above is of the first order due to neglecting all high-order anisotropy constants. In principle, this transition should reveal a hysteresis due to the overlap of stability regions of the in-plane and out-ofplane magnetic configurations as well as nucleation within the dots. This hysteresis study is in progress.

To consider the limitation of isolated cylindrical dots ($$d/R \rightarrow \infty$$) we use the effective demagnetizing factors introduced in Ref. 13, $$N_{eff} = W_m(M|\|)/\langle M^2 \rangle_v$$. It is well known that in nonellipsoidal magnetic bodies the demagnetizing field is not uniform. For a uniformly magnetized cylinder we get

$$N_{\|zz}(\beta) = 8 \pi \int_0^\infty dt f(t) \frac{J_2^2(t)}{t},$$

$$N_{\|xx}(\beta) = N_{\|yy}(\beta) = \frac{1}{2} \left[ 4 \pi - N_{\|zz}(\beta) \right].$$

(6)

From Eq. (6) we can get the critical value of $$\beta_c = 1.813$$ for the transition from the perpendicular to in-plane magnetization state in accordance with Ref. 14. Note that this value is larger than the mean field one ($$L/R = 1.54$$) calculated for the uniformly magnetized cylindrical dot by Scheinfein et al. 15 The demagnetizing factors $$N_{\|xx}$$ reach the
values $N_{aa}^0(\beta)$ of isolated dots determined from Eq. (6) at $d/R \to \infty$. For close packed dots ($d/R \to 0$), the critical value $\beta_c$ is 5.62.

The reorientational transition described above can be also observed experimentally for fixed dot sizes with changing temperature, if we take into account another contributions $K_\alpha$ to the total anisotropy constant. The critical line of $\beta_c(d)$ will be shifted according to the value of this additional anisotropy constant $K_\alpha$. The shift of $\beta_c(d)$ will be upward for $K_\alpha<0$ and downward for $K_\alpha>0$. Therefore, we like to stress that the present approach can be generalized by including the uniaxial anisotropy constant $K_\alpha$. This transition was numerically calculated in Ref. 16 for the small $N \times N(N = 5 - 8)$ dot arrays at fixed geometric dot parameters within the point-dipole approximation.

Experimental results have been reported by several investigators for weakly coupled dots ($d/R \approx 1$). These dot arrays are in a perpendicularly magnetized state for $\beta = 2.77, 9 \beta = 2.86, 17$ and $\beta = 5.3^{18}$ because these aspect ratios are greater than $\beta_c = 1.81$. While the dot array with small $\beta = 0.8^{17}$ reveals the in-plane state, the out-of-plane magnetized state observed by Meier et al. at $d/R = 1.33, \beta = 1.25$ contradicts the present calculation. The reason for this disagreement is the noncylindrical shape of the dots (truncated cones). The magnetization is also perpendicular to the film plane for the Ni dot (pillars) array with $\beta = 3.97, d/R = 1.45$ according to Fig. 1. But one is in-plane for the close-packed Ni dot array, with $\beta = 3.30$ and $d/R = 0.22$, in spite of the high dot aspect ratio due to strong interdot coupling.

Magnetostatic coupling determines the properties of array of the Ni pillars, which agree with the present calculation (Fig. 1). It is noteworthy that the reorientation transition was detected in nanowires array by Strijkers et al. When wires with high wire length ($L/R \approx 1$) are well separated, the easy magnetization axis should be along the wire length. But, in Ref. 20 it is perpendicular to the wire length. For the 320 nm distance between the wires of 100 nm diameter, we can neglect the surrounding magnetostatic field. The reorientation can not occur due to this field as concluded in Ref. 20.

We speculate on the detected reorientational transition caused by competition between the magnetocrystalline anisotropy of the hcp Co fraction and the magnetostatic shape anisotropy with changing the wire aspect ratio.

The magnetostatic energy of the dot lattices depends on the dot size, interdot distance, and dot lattice symmetry. This energy increases with increasing $L$ up to the transition of the dots to the perpendicularly magnetized state. The type of arrangement of the dot magnetizations in the ground state of the square dot lattice with zero net magnetization is mostly determined by the long-range character of the dipole–dipole interaction.

In summary, the magnetostatic energies of the two-dimensional periodic configurations of the dot magnetic moments placed at the sites of the square lattice were investigated. The magnetostatic coupling between the dots plays an essential role and determines the magnetization directions of the dot lattices. The reorientational transition from in-plane to perpendicularly state of the square dot lattice of the uniformly magnetized cylindrical dots is driven by the relative dot size ($L/R$), interdot distance, and temperature. This transition takes place from the perpendicular “checkerboard” to the in-plane noncollinear antiferromagnetic ground state. The critical dot aspect ratio $\beta_c(d)$ for the transition between these ground state configurations depends strongly on the interdot distance due to magnetostatic coupling. This ratio decreases with increasing $d$ from its maximal value of 5.62 to the isolated dot value of 1.81.

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