Wave Packet Dynamics in Stabilization of Ionization in a Circularly Polarized Laser Field

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We present the results of a quantum dynamical study of ionization stabilization of a two-dimensional hydrogen atom in an intense circularly polarized laser field. We show that stabilization can be explained by the formation of a nonspreading electron wave packet rotating with the laser frequency. The radius of the circular orbit is found to be greater than the radius of the classical circular motion of a free electron under the same laser field. We also investigate the classical nonlinear dynamics of the electron and examine the relationship between nonlinear resonance islands and the wave packet dynamics.

In recent years, many non-perturbative phenomena, such as multiphoton ionization, above threshold ionization, high harmonic generation, and stabilization of ionization, have been discovered in the interaction of an intense laser with atoms [1–3]. The stabilization of ionization, the subject of this paper, refers to the phenomenon of the ionization rate decreasing or saturating with increased laser intensity.

Adiabatic stabilization has been predicted in theoretical calculations [4,5] and observed in experiments [6,7]. For a linearly polarized high-frequency laser, one-dimensional calculations of the ionization rate have been performed using the approximated Floquet theory [4], and it was found that stabilization occurred by a ‘dichotomy’ of the electron wave function in the KH(Kramers-Henneberger) frame [8]. The wave function was found to consist of two large peaks separated by $2\alpha_0$, where $\alpha_0$ is the maximal displacement of the classical oscillatory motion of an electron under the incident laser field. In the K-H frame, as the field strength is increased with time at the beginning stage of the interaction, the electron wave packet, which is centered initially at the core, evolves adiabatically toward a double-peaked packet. The two peaks are centered at the two minima, separated by $2\alpha_0$, of the double-well potential which is the time-averaged Coulomb potential in the KH frame. For a circularly polarized high-frequency laser, Pont and Gavrila [5] showed that stabilization from the atomic ground state may occur. Calculations of the ionization rate from Rydberg states were also carried out [9] because it was realized that it would be difficult to meet experimental conditions for observing stabilization from the ground state [6,7]. The dependence of the ionization and the stabilization on the polarization of the laser was investigated in a series of experiments [10] and calculations [11]. Protopapas et al. [12] found stronger evidence of stabilization in a two-dimensional soft-core model atom under a circularly polarized laser rather than a linearly polarized laser. Different degrees of stabilization and electron localization patterns were found for various ellipticities [13], and were interpreted well by using the adiabatic dichotomy feature in the K-H frame. Chism et al. [14] provided the classical origin of the localized wave packet [12] and observed that the wave packet evolve without spreading in the form of a ‘Trojan wave packet’ [15–18] for the case of a two-dimensional soft-core model atom in a circularly polarized laser field.

In dynamic stabilization, quantum mechanical interference between ionization probability amplitudes from the coherent wave packet of states plays the main role [19]. Experiments have been carried out to observe this type of stabilization [20]. Population trapping against the ionization has been studied [21]. It has been shown that spatially extended wave packets are created through virtual transitions from the initial state via high-lying Rydberg and continuum states to a superposition of Rydberg states with a small initial overlap with the atomic core [22]. The pulse shape effect has been studied [23,24]. The relation between the adiabatic and dynamic stabilization has also been an important issue. Adiabatic stabilization can be considered as a limiting case of dynamic stabilization [23,25].

In this paper, we investigate the ionization and the stabilization of a two-dimensional hydrogen model atom initially in its ground state in the presence of a circularly polarized laser. We present numerical evidence that the stabilization is characterized by the existence of a nonspreading wave packet localized in the region away from

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the atomic core. We also provide the classical origin of the nonspreading wave packet.

The wave packet evolution is obtained from the Schrödinger equation for the electron in the two-dimensional hydrogen model atom driven by a circularly polarized laser field, which is given by

\[ i \frac{\partial}{\partial t} |\psi\rangle = \left( \frac{1}{2} \frac{\partial^2}{\partial r^2} - \frac{1}{r} + F(t)(x \cos \omega t + y \sin \omega t) \right) |\psi\rangle \quad (1) \]

in atomic units, where we use the dipole approximation and treat the laser field classically [2]. For multiphoton ionization, stabilization occurs in the region where the ponderomotive energy \( F^2/2\omega^2 \) is larger than the photon energy \( h\omega \) and where the photon energy is larger than the energy \( |E_0| \) needed to ionize the atom from the initial state [2]. Thus, we set the frequency of the laser \( \omega = 2.2 \) (a.u.) so that the photon energy is greater than the ground state ionization energy of the two-dimensional hydrogen atom, \( |E_0| = 2 \) (a.u.) = 54.4 eV [26]. The field strength is varied over the range \( 0 < F < 30.0\) (a.u.), which corresponds, in terms of the laser intensity \( I \), to \( 0 < I < 2.853 \times 10^{19} \) W/cm². The initial wave packet is taken to be that of the ground state of the two-dimensional hydrogen atom without an external field. The eigenfunctions of the two-dimensional hydrogen atom are given by [26]

\[ \psi_{n,l} = \beta_n \frac{1}{\sqrt{2\pi}} \sqrt{\frac{(n-l-1)!}{(2n-1)(n+l-1)!}} e^{-\frac{1}{2}r^2} (\beta_n r)^l L_{n-l-1}^l (\beta_n r) e^{im\phi}, \quad (2) \]

where \( L \) denotes the associated Laguerre polynomial, \( \beta_n = 2/(n-1/2) \), \( l = |m| \), \( n = 1, 2, 3, \ldots \), and \( 0 \leq l < n \). We assume that the laser field is turned on according to the relation

\[ F(t) = F \sin^2(\pi t/2T_{\text{switch}}), \quad (3) \]

for \( 0 \leq t \leq T_{\text{switch}} \). After the switch time \( T_{\text{switch}} \), a constant amplitude \( F \) is maintained. The Schrödinger equation is solved by using the two-dimensional Crank-Nicolson method [27] with an absorbing boundary condition [28]. Fig. 1 shows the computed ionization probability after 8 laser cycles \( t = 8 \cdot 2\pi/\omega \). We choose the switch time of the laser, \( T_{\text{switch}} \), as 4 laser cycles, which is sufficiently long to suppress the population of very low energy continuum states due to shake-off [13]. The ionization probability is determined by subtracting from 1 the probability that the wave packet remains inside the absorbing boundary. The ionization probability is seen to increase as the field strength is increased to \( F \approx 12.0 \), but it decreases when \( F \) increases above \( F \approx 12.0 \).

In order to gain insight into the stabilization mechanism, we study the wave packet dynamics when stabilization occurs. The wave packet centered initially at the core spreads smoothly during the turn on of the laser and develops into a double-peaked structure. One peak is located at a negative value of \( x \), and the other at a positive value of \( x \) along the \( y \) axis, as shown in Fig. 2(a). The two peaks are separated by about 12 a.u. which is much greater than twice the maximum displacement, \( 2a_0 = 2 \cdot F/\omega^2 = 7 \) (a.u.), in the K-H frame. The double-peaked wave packet does not spread for a long time indicating that stabilization has occurred. Below, we present a classical analysis to find the origin of this localized and nondispersive wave packet.

The Hamiltonian given in the velocity gauge by

\[ H = \frac{1}{2} \frac{\partial^2}{\partial r^2} - \frac{1}{r} - \frac{F}{\omega} (p_x \sin \omega t - p_y \cos \omega t) \quad (4) \]

can be transformed into a time-independent two-dimensional Hamiltonian

\[ H = \frac{1}{2} \frac{\partial^2}{\partial r^2} - \frac{1}{r} - \omega L_z + \frac{F}{\omega} p_y \quad (5) \]
The resulting Hamiltonian takes the form
\[ H = \frac{1}{2}(p_x^2 + p_y^2) - E(u^2 + v^2) - \frac{1}{2}\omega(u^2 + v^2)(up_x - vp_y) + \frac{F}{\omega}(up_x + vp_y) = 2. \]  

By going to a frame rotating with the laser frequency, a coordinate transformation into semi-parabolic coordinates \( \{x = (u^2 - v^2)/2, y = uv\} \) removes the \( 1/r \) singularity. The resulting Hamiltonian takes the form
\[ \frac{H}{\hbar} = \frac{1}{2}(p_x^2 + p_y^2) - \omega(u^2 + v^2)(vp_x - up_y) + Fvp_x = 2. \]  

In Fig. 3, we show the Poincaré surfaces of the section in the \((v, p_v)\) space for different values of the quasienergy \(E\), where \(u = 0\) and \(p_u\) is chosen as the greater of the two roots of Eq. (6) \(p_u \geq - (F/\omega)(v + \sqrt{v^2})\). Fig. 3(a) shows the electron motion at a low quasienergy of \(E = -8.0\) (a.u.) and \(F = 2.5\) (a.u.), \(\omega = 2.2\) (a.u.), \(\omega = -7.0\) (a.u.), \(F = 2.5\) (a.u.), \(E = -1.0\) (a.u.), \(F = 17.0\) (a.u.), \(E = -6.4\) (a.u.), and \(F = 17.0\) (a.u.), \(E = -4.5\) (a.u.).

Fig. 4. ZVS curves as a function of \(x\) at \(y = 0\) for the case (a) \(F = 2.5\) (a.u.), \(\omega = 2.2\) (a.u.) and (b) \(F = 25.0\) (a.u.), \(\omega = 2.2\) (a.u.).
y = 0.0). However, the classical initial points representing the ground state $\psi_{100}$ evolves to cover, at $t = T_{\text{switch}}$, the range of quasienergy $E = -4.5 \pm 2.0$ (a.u.) at which the classical motions are mostly unstable as shown in Fig. 3(d).

In conclusion, we have presented numerical evidence that stabilization of ionization in a circularly polarized laser field can be explained by the formation of a non-spreading wave packet. The electron wave packet is localized at a circular orbit with a radius larger than $\alpha_0$ and does not pass through the core. When the initial state is the excited state $\psi_{32}$, the localized wave packet originates from stable classical orbits generated by nonlinear resonance of the atom with the intense laser. However when the initial state is the ground state $\psi_{100}$, the localized wave packet is due to quantum localization, the suppression of classically chaotic diffusion.

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