Numerical simulation of the flow behind a rotary oscillating circular cylinder

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A numerical study was made of flow behind a circular cylinder in a uniform flow, where the cylinder was rotationally oscillated in time. The temporal behavior of vortex formation was scrutinized over broad ranges of the two externally specified parameters, i.e., the dimensionless rotary oscillating frequency ($0.110 \leq \omega_f \leq 0.220$) and the maximum angular amplitude of rotation ($\theta_{\text{max}} = 15^\circ, 30^\circ,$ and $60^\circ$). The Reynolds number ($Re = U_cD/\nu$) was fixed at $Re = 110$. A fractional-step method was utilized to solve the Navier–Stokes equations with a generalized coordinate system. The main emphasis was placed on the initial vortex formations by varying $S_f$ and $\theta_{\text{max}}$. Instantaneous streamlines and pressure distributions were displayed to show the vortex formation patterns. The oscillatory forcing was in the vicinity of the lock-on range, which can be applied to flow feedback control afterwards. The vortex formation modes and relevant phase changes were characterized by measuring the lift coefficient ($C_L$) and the time of negative maximum $C_L (t_{\text{max}})$ with variable forcing conditions. © 1998 American Institute of Physics. [S1070-6631(98)00904-0]

I. INTRODUCTION

The formation and shedding of vortices from a circular cylinder have been a long subject in the study of unsteady flow separation.1 The vortices emanating from the separation points are shed to the rear side of the cylinder and they persist for some distance downstream in the wakes. The presence of vortex shedding, together with vortex wakes, gives rise to increased unsteadiness, pressure fluctuation, structural vibration, and noise. Also it tends to enhance heat and mass transfer and to augment mixing. It is necessary to find methods to control the unsteady separated flows in the wake.

A literature survey reveals that there have been many attempts to control or lessen the unfavorable behaviors associated with the formation and shedding of vortices from a circular cylinder. Several methodologies have been applied by many researchers. In general, simple geometrical configurations have been employed, e.g., splitter plate, second cylinder, etc.2–4 Direct disturbed methods were also applied: inhomogeneous inlet flow,5 oscillatory inlet flow,6,7 localized surface excitation by suction and blowing,8 and vibrating cylinder.9–12 Among the techniques in the literature, the introduction of a rotary oscillation has been contemplated by some researchers.13–16 Due to the rotary oscillation of a cylinder, the action of the body to the fluid and the reaction of the fluid to the body are mutually interacted between the moving body and the fluid. The basic rationale is that, in the case of the counterclockwise rotation, the flow of the upper cylinder is decelerated and easily separated. On the other hand, the flow of the lower cylinder is accelerated and the separation can be delayed or suppressed. It was demonstrated that, by means of the rotary oscillation, overall characteristics of the vortex shedding and wake patterns were altered significantly.

A relatively small number of researchers have investigated the effects of rotary oscillation of the cylinder. Furthermore, most of the prior studies were made experimentally. Visual observations were made by Taneda13 in the range of $30 \leq Re \leq 330$. He showed that the vortex shedding process as well as the stagnant-fluid region behind the cylinder could be nearly eliminated at very high oscillation frequencies. Tokumaru and Dimotakis14 examined the efficacy of the forced rotary oscillations at a moderate Reynolds number of $1.4 \times 10^5$ for actively controlling the cylinder wake. The forcing conditions where the drag is reduced were found. Filler et al.15 investigated the response of the shear layers separating from a circular cylinder to small-amplitude rotational oscillations ($250 \leq Re \leq 1200$). When the cylinder was oscillated at or near the natural Karman frequency, large velocity fluctuations were observed in the shear layers, producing the large response peak. These experimental findings present a promising ground for extending studies into the feasibility of active control by utilizing the rotational oscillation. It should be noted here that, since the rotation rates are easily manipulated by a simple electronic device and mechanical means, the present method is practically applicable.

Based on the preceding experimental observations, a direct numerical simulation is made in the present study to portray the unsteady dynamics of wake flows. The emphasis of this study is to analyze the vortex formation behind a circular cylinder for different rotary oscillation conditions. Two main parameters are selected for comparison, i.e., the frequency of rotation ($S_f = f U_c/D$) and the maximum angular displacement ($\theta_{\text{max}}$). The Reynolds number based on the diameter $D$ is fixed at $Re = 110$, in which the vortex shed-
ding flow is assumed to be two dimensional. The primary advantage of the numerical simulation is that wide ranges of the relevant flow variables can be encompassed. In the present study, these two parameters are varied in a range 0.110 ≤ St ≤ 0.220 and θ max = 15°, 30°, and 60°.

The viability of the present simulation procedure is ascertained by repeating calculations of the nonoscillating cylinder flow situations at first. Assessments are made of the interrelationships between the vortex formation and the lift coefficient (C l), where the simulation is performed in the vicinity of lock-on. This is because lock-on is well-defined and can be directly controlled by the input rotational oscillation frequency. These numerical data will provide a baseline source, to which an active control can be applied.

II. NUMERICAL METHOD

A. Governing equation

The nondimensional governing equations for an unsteady incompressible flow are

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} u_i u_j = - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_j} u_i, \quad (1)$$

$$\frac{\partial u_i}{\partial x_i} = 0, \quad (2)$$

where x i are the Cartesian coordinates and u i are the velocity components in each direction. The free-stream velocity U ∞ and the cylinder diameter D are used for nondimensionalization. The Reynolds number is defined as Re = U ∞ D/ν, where ν is the kinematic viscosity.

To simulate wake flows behind a cylinder, it is better to transform the governing equations (1) and (2) into the generalized coordinates y i . The velocity components u i are transformed into the volume fluxes across the faces of the cell q i . Formulation of the problem in terms of the contravariant velocity components, weighted with the Jacobian J, in conjunction with the staggered variable configuration leads to discretized equations. The resulting pressure Poisson equation is solved, where the discretized mass conservation is satisfied. The transformed governing equations are rewritten as

$$\frac{\partial q_i}{\partial t} + N^i(q) = -G^i(p) + L_1^i(q) + L_2^i(q), \quad (3)$$

$$D^i q^i = \frac{1}{J} \left[ \frac{\partial q_i}{\partial y_i} + \frac{\partial q_j}{\partial y_i} \right] = 0, \quad (4)$$

where N i is the convection term, G i (p) is the pressure gradient term, L 1 i and L 2 i are the diffusion terms without and with cross derivatives and D i is the divergence operator, respectively. The terms in Eq. (3) are rewritten in the following form

$$N^i(q) = \frac{1}{J} \gamma_m \frac{\partial}{\partial y_j} \frac{1}{J} c_m^i q^i, \quad (5)$$

$$G^i(p) = \alpha^{ij} \frac{\partial p}{\partial y_j}, \quad (6)$$

B. Numerical procedure

A fully implicit, fractional-step method composed by four-step time advancement is used to solve the governing equations. The fractional step, or time-split method, is in general a method of approximation of the evolution equations based on decomposition of the operators they contain. In application of this method to the Navier–Stokes equations, one can interpret the role of pressure in the momentum equations as a projection operator which projects an arbitrary vector field into a divergence-free vector. In the Cartesian coordinate, these four steps are

$$\frac{\hat{u}_i - u_i^n}{\Delta t} + \frac{1}{2} \frac{\partial}{\partial x_y} \left( \hat{u}_j \hat{u}_y + u_y^n u_y^n \right)$$

$$= - \frac{\partial p^n}{\partial x_i} + \frac{1}{2Re} \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_j} \left( \hat{u}_i + u_i^n \right), \quad (9)$$

$$\frac{\partial p^n}{\partial x_i} = \frac{\Delta t}{\Delta x_i}, \quad (10)$$

$$\frac{\partial}{\partial x_i} \frac{\partial p^{n+1}}{\partial x_i} = \frac{1}{\Delta t} \frac{\partial u_i^*}{\partial x_i}, \quad (11)$$

$$\frac{\partial}{\partial x_i} \frac{\partial u_i^{n+1}}{\partial x_i} = \frac{\partial p^{n+1}}{\partial x_i}, \quad (12)$$

A second-order central difference scheme is used for the spatial derivatives and a Crank–Nicolson method is employed in the time advancement. The substitution of Eqs. (10) and (12) into Eqs. (9) indicates that the present scheme is second-order accurate in time. The discretized nonlinear momentum equations are solved by using a Newton-iterative method. Solving the Poisson equation for p satisfies the continuity equation. In this computation, Eqs. (9)–(12) are also transformed from the Cartesian coordinate to the generalized coordinate.

A flow configuration of the present rotary oscillation is shown in Fig. 1. A C mesh is used for the present
simulation. This type of mesh is ideally suited for simulating wake flows since better streamwise resolution can be provided in the wake region. The use of a C mesh also simplifies the application of outflow boundary conditions. The outflow boundaries are located at 20D. Uniform free-stream velocity is prescribed at the inflow and far-field boundaries, and a convective boundary condition is employed at the outflow boundary in order to smoothly convert the disturbances out of the computational domain. On the cylinder wall, the no-slip condition, constant rotation, and periodic rotary oscillation conditions are used, depending on the given conditions, respectively.

Since the cylinder is rotated sinusoidally in time \( t^* \) at a forcing rotational frequency \( f^* \), the nondimensional cylinder velocity (\( \Omega \)) is expressed by

\[
\Omega = \alpha \sin(2 \pi S_f^* t^*),
\]

where the quantities are nondimensionalized by adopting the following relations: \( t = t^* U_\infty / D \) and \( S_f = f^* D / U_\infty \). Here, the asterisk denotes the dimensional counterparts. The nondimensional maximum rotational velocity \( \alpha \) is \( \alpha = \theta_{\text{max}} D / 2 U_\infty \), where \( \theta_{\text{max}} \) is the maximum angular velocity. A simple arithmetic relation can be found between \( \theta_{\text{max}} \) and \( \alpha \), i.e., \( \theta_{\text{max}}(\alpha) = 180(\alpha) \pi^2 S_f \). Thus if one variable is fixed, the other is automatically calculated from the above relation. In the present simulation \( S_f \) and \( \theta_{\text{max}} \) are employed as the main parameters. The initial conditions are the solutions for nonoscillating flow, i.e., the case where the cylinder is stationary. When the oscillatory rotation starts, the counter-clockwise rotation is preceded in the period of 0.5T from 0T. Then, the clockwise rotation is accompanied from 0.5T to T. Accordingly, the counter-clockwise rotational velocity is maximum at 0.25T and the clockwise one is maximum at 0.75T, respectively. Here T denotes the nondimensional period.

## III. RESULTS AND DISCUSSION

First, it is important to ascertain the reliability and accuracy of the present numerical simulation. Toward this end, a two-dimensional (2-D) laminar flow behind a circular cylinder has been calculated using the present numerical code. As addressed by Williamson,\(^{17}\) a 2-D periodic laminar vortex shedding regime is found in the range 50 \( \leq \text{Re} \leq 160 \). A single \( \text{St-Re} \) curve relevant to parallel vortex shedding can be obtained in the form, \( \text{St} = -3.3265 / \text{Re} + 0.1816 + 1.600 \times 10^{-4} \text{Re} \). Several trial calculations are repeated to monitor the sensitivity of the results to grid size, where the grid points are crowded near the wall boundary. The grid convergence has been checked (Fig. 2), the outcome of these tests was found to be satisfactory for the present two-dimensional computations \((x,y) = (321 \times 101)\). The predicted value by the present simulation at \( \text{Re} = 110 \) is \( \text{St} = 0.171 \), which is in good agreement with that obtained from the formulas, i.e., \( \text{St} = 0.169 \).

Next, the wake flow past a rotating circular cylinder was also calculated for comparison. The present results for the lift coefficient \( (C_L) \) as a function of \( \alpha \) was compared with that of Ingham and Tang.\(^{21}\) As shown in Fig. 3, an excellent agreement was found. These guarantee the accuracy of the present rotary simulation.

Now, the vortex formation dynamics behind a circular cylinder is inspected when the rotary oscillation is imposed. As mentioned earlier, the present rotary oscillation simulations are made in the vicinity of lock-on. When lock-on takes place, the shedding frequency is subjected to the imposed rotational frequency (\( S_f \)). The near-wake structure is invariant from cycle to cycle of the cylinder rotation. To secure the lock-on boundary, time evolutions of the lift coefficient (\( C_L \)) are shown in Fig. 4. The lock-on criterion can be discriminated by checking the variation of \( C_L \), which is known to be sensitive to the vortex formation near the wake region. Two cases are exemplified for comparison: the lock-on case for \( S_f = 0.150 \) and \( \theta_{\text{max}} = 15^\circ \) and the non-lock-on case for \( S_f = 0.140 \), and \( \theta_{\text{max}} = 15^\circ \). As shown in Fig. 4(a), a single evolution of \( C_L \) is clearly captured for the lock-on case. However, for the non-lock-on case, \( C_L \) oscillates in a compound fashion.

The maximum values of \( C_L \) are displayed in Fig. 5, where the flows are within the lock-on range. As seen above, when lock-on takes place, the amplitude of \( C_L \) is larger than that of the nonoscillating case. The dotted line in Fig. 5 represents the nonoscillating case. This may be attributed to the fact that the rotational oscillation frequency is synchronized and resonated with the natural shedding frequency. To reduce the amplitude of \( C_L \), the much higher frequency is needed. The forcing frequency higher than a certain value makes a new mode in the shear layer characterized by smaller vortices of shorter wavelength and retards the onset of large-scale vortex formation in the downstream region of the wake. As \( \theta_{\text{max}} \) increases, the ranges of \( S_f \) in which lock-on takes place are widened in both directions. These
trends are supported by the experimental findings of Koopmann\textsuperscript{22} for the vertical oscillation of a cylinder. A closer inspection of Fig. 5 indicates that the peak of $C_{L_{\text{max}}}$ is slightly moved to the lower frequency ($S_f$), as $u_{\text{max}}$ increases.

Filler\textit{et al.}\textsuperscript{15} found that when the cylinder is rotary oscillated at or near the natural shedding frequency, large velocity fluctuations are observed in the shear layer with a large response peak. At relatively small-amplitude rotary oscillations, in which the peripheral speed of the cylinder was only 0.5%–3% of the free-stream speed, the oscillation magnitudes were as large as 30%–50% of the free-stream. They also observed the secondary instability of the separating shear layers at higher frequencies at $Re=1100$. These observations reflect that further studies are required to relate the nature of the vortex formation to the instantaneous cylinder oscillation. The idea is that the vortices behind a circular cylinder can be manipulated by the introduction of rotary oscillation. In the present study, the forcing rotary oscillations are near the natural Karman shedding frequency ($St=0.171$), while the forcing amplitude is relatively larger than that of Filler\textit{et al.}\textsuperscript{15}

At first, the case of $S_f=0.140$ and $\theta_{\text{max}}=30^\circ$ is selected, where the rotary oscillation frequency is lower than the Karman frequency ($St=0.171$). To observe the vortex formation mode in this lower frequency forcing case, the instantaneous streamlines are displayed. It is well known that the streamlines are variant with respect to a change in reference frame. Accordingly, the vortex formation can appear differently in a different frame. The instantaneous streamlines observed in a fixed frame at the cylinder are shown in Fig. 6 during a half period of the counterclockwise rotation. Just when the counterclockwise rotation is started, as seen in Fig. 6(a), a small circulation is located in the bottom of the cylinder. A large-scale vortex is also located in the near-wake region. However, as time goes by, the small counterclockwise circulation vanishes soon, Fig. 6(c). This is due to the counterclockwise fluid generated near the cylinder by its rotation. Moreover, the counterclockwise rotation helps the large-scale vortex move downward. However, since the forced rotational velocity is decelerated after the maximum, a new circulation is formed in the bottom rear-side of the cylinder Fig. 6(h). As the rotation becomes weaker, the small circulation vortex is gradually grown up, which will be supported by the clockwise rotation afterward. This circulation region will meet the upstream flow at the top of the cylinder, which gives rise to the formation of new small circulation. When the clockwise rotation becomes weaker, the small circulation vortex is gradually grown up, which will be supported by the clockwise rotation afterward.
rotation starts, the aforesaid procedure is repeated in the opposite direction. This is one period vortex formulation in the lower frequency oscillation.

As compared with the regular vortex formation without rotation,23,24 the above result looks similar on the surface. However, a closer examination of the vortex formation near the cylinder indicates that an instant of no circulation exists behind the cylinder in the above rotary oscillation mode. This means that the generated circulation vanishes soon due to the instantaneous cylinder rotation. However, for the regular vortex formation case, the circulation does not vanish until it is shed afterward.

Instantaneous pressure distributions are shown in Fig. 7 for the same condition (\(S_f = 0.140\) and \(\theta_{\text{max}} = 30^\circ\)). The pressure is normalized by \(\rho U_c^2\) and the pressure contour interval is 0.04. The pressure in the top region of the cylinder is lower than that in the bottom in Fig. 7(a), which means the lift force action in the positive vertical direction. As time proceeds, the pressure at the top increases and the pressure at the bottom decreases, Figs. 7(b)–7(d). If we compare Fig. 7(e) with Fig. 7(d), the top pressure decreases and the bottom pressure increases, i.e., the maximum \(C_L\) in the negative vertical direction exists between Figs. 7(d) and 7(e). In general, once a vortex is made, a local lowest pressure region appears to balance its centrifugal force. At the same time, a closed streamline region can be observed in Fig. 6(j).

To look into the transient changes of vortex formation by increasing the rotary oscillation frequency gradually, the case of \(S_f = 0.170\) and \(\theta_{\text{max}} = 30^\circ\) is shown in Fig. 8. The instantaneous streamlines are displayed during the counterclockwise half period. Close to the cylinder, Fig. 8(a), a clockwise vortex exists and is convected downstream with the aid of the counterclockwise rotation of the cylinder. This continues until Fig. 8(j) and a small counterclockwise circulation vortex in the right bottom of the cylinder is formed after Fig. 8(j). However, it is formed much later in time, as compared with the former lower frequency case (\(S_f = 0.140\) and \(\theta_{\text{max}} = 30^\circ\)). The reverse circulation region formed by the vortex and the flow from the other side is negligible.

The case of \(S_f = 0.200\) and \(\theta_{\text{max}} = 30^\circ\), which is higher than the Karman frequency, is shown in Fig. 9. The instantaneous streamlines are displayed during the counterclockwise half period. As the cylinder starts to rotate in the counterclockwise direction, a reactive clockwise circulation is generated in the \(+45^\circ\) direction, Fig. 9(b). The accelerated flow during the last half period and the flow due to the cylinder rotation are confronted at the \(+45^\circ\) direction, which brings forth a circulation region. As the angular velocity increases, the circulation becomes larger and it is convected downstream with the aid of the accelerated flow at the lower cylinder. This continues until the rotation is reversed. After this reverse, the flow in the lower cylinder is repeated by generating the counterclockwise circulation in the \(-45^\circ\) direction. The pressure distribution is shown in Fig. 10. The pressure contour interval is 0.06. In the upper region of the cylinder, the pressure decreases from 0.05T to 0.25T and increases from 0.25T to 0.45T. In the lower cylinder, the opposite is made. This leads to the maximum positive \(C_L\) between 0.15T and 0.35T.

The switch in phase of the initially formed vortex can be observed in the \(C_L\) distribution with respect to the cylinder rotation velocity (\(\Omega\)). It is evident that whether the phase change occurs or not depends on the forcing rotation frequency (\(S_f\)). To quantitatively characterize the phase of the vortex shedding relative to the cylinder rotation (\(\Omega\)), the unsteady \(C_L\) distributions are shown in Fig. 11. As seen in Figs.
6, 7, 9 and 10, the vortex is formed in the local lowest pressure region. If the vortex is formed in the lower side of the cylinder, the cylinder is forced downward, i.e., \( C_L \) becomes negative. On the contrary, when the vortex formation is made on the upper cylinder, the cylinder is forced upward and \( C_L \) is positive. When \( S_f \) is lower than the Karman frequency, i.e., \( S_f < 0.140 \), \( C_L \) is out of phase with respect to \( V \). This means that, as shown in Fig. 12, the vortex is formed on the lower side of the cylinder when the cylinder is rotated in the counterclockwise direction, Fig. 12(a), and on the upper side of the cylinder for the clockwise rotation, Fig. 12(c). However, when \( S_f \) is higher than the Karman frequency (\( S_f = 0.171 \)), \( C_L \) is in phase with \( V \). Consequently, the vortex is formed on the upper side of the cylinder with the opposite direction when the cylinder is rotated in the counterclockwise direction, Fig. 12(b) and vice versa, Fig. 12(d).

To clarify the change of vortex formation depending on the forcing condition, the streamlines of the same forcing phase are shown in Fig. 12. When the forcing frequency is lower than the Karman frequency, the vortex rolls up in the same direction as the corresponding rotation Fig. 12(a). But, the switching phenomena occurs in the neighborhood of the natural Karman frequency, the vortex formed side is changed with respect to the cylinder rotation and the vortex rolls up in the opposite direction Fig. 12(b). This switching of the vortex formation side is also seen for the forced vibration of the cylinder perpendicular to the free-stream. In vertical oscillation, the moving down has the same effect in the rear of the cylinder with the counterclockwise rotation. At a lower frequency than the natural frequency, the flow is not accelerated enough nor separated to make a vortex. But, at the higher frequency, the vortex is formed on the lower side of the cylinder when the cylinder is rotated in the counterclockwise direction, Fig. 12(a), and on the upper side of the cylinder for the clockwise rotation, Fig. 12(c). However, when \( S_f \) is higher than the Karman frequency (\( S_f = 0.171 \)), \( C_L \) is in phase with \( V \). Consequently, the vortex is formed on the upper side of the cylinder with the opposite direction when the cylinder is rotated in the counterclockwise direction, Fig. 12(b) and vice versa, Fig. 12(d).
frequency, the adequately accelerated flow during the last half period makes a vortex with the rotating cylinder of the opposite direction.

Since $C_L$ is closely interrelated with the initial vortex formation and the relevant phase changes, it is important to consider the time of negative maximum $C_L$, i.e., $t - C_L_{max}$. It is recalled that the counterclockwise rotation is preceded in the half period ($0 \leq t < 0.5T$) and the clockwise rotation is followed from 0.5T to T. The $t - C_L$ distribution is displayed in Fig. 13 over a wide range of $S_f$ and $\theta_{max}$. When $S_f = 0.140$ and $\theta_{max} = 30^\circ$, the value of $t - C_L_{max}$ is represented as $t - C_L_{max} = 0.421T$, i.e., the cylinder is still in the counterclockwise rotation. As seen in Fig. 6, the vortex is formed in the bottom rear-side of the cylinder. Accordingly, the cylinder receives the maximum lift force in the negative vertical direction. The pressure distribution in Fig. 7 also indicates that the vortex is formed and shed at this instant in the local lowest pressure region. For $S_f = 0.200$ and $\theta_{max} = 30^\circ$, $t - C_L_{max} = 0.747T$. This means that, before and after a half period (0.5T), the time of maximum positive $C_L$ corresponds to $tC_L_{max} = 0.247T$. This coincides with the time of developing the circulating region in Fig. 9. The vortex forming and shedding in the local lowest pressure region can be also detected in Fig. 10.

The influence of $\theta_{max}$ on $t - C_L$ is also examined. A closer inspection of Fig. 13 discloses that, as $\theta_{max}$ increases, the change rate of $t - C_L_{max}$ decreases. For example, at a fixed $S_f$ in the lower frequency region ($S_f < 0.170$), the value of $t - C_L_{max}$ at $\theta_{max} = 15^\circ$ is smaller than that at $\theta_{max} = 60^\circ$. However, the reverse appears in the higher frequency region ($S_f > 0.170$). As $\theta_{max}$ is very small, the change of $t - C_L_{max}$ will increase significantly. This is consistent with the $\pi$ order phase change of Filler et al.,15 where small-amplitude rotational oscillations were imposed. It is evident that the phase change occurs across the natural shedding frequency ($S_f = 0.170$). Another important finding in Fig. 13 is in the following. If the same values of $t - C_L_{max}$ are obtained by varying $S_f$ and $\theta_{max}$, the corresponding vortex formation modes are nearly the same, whereas some quantitative differences exist. For example, the vortex formation modes of $S_f = 0.150$ and $\theta_{max} = 15^\circ$ and of $S_f = 0.120$ and $\theta_{max} = 60^\circ$ are similar to those of $S_f = 0.140$ and $\theta_{max} = 30^\circ$. It was also found that the vortex formation modes of $S_f = 0.180$ and $\theta_{max} = 15^\circ$ and of $S_f = 0.210$ and $\theta_{max} = 60^\circ$ are similar to those of $S_f = 0.200$ and $\theta_{max} = 30^\circ$.

IV. CONCLUSIONS

A fractional-step method was utilized to analyze the vortex formation modes behind a cylinder, where the cylinder was rotary oscillated in time. The present simulation procedure was proven to be useful to characterize the unsteady vortex formation and shedding accurately. The simulation was made within the lock-on range. A salient vortex formation mode change was observed when the forcing oscillation frequency ($S_f$) approximates the natural shedding frequency. When $S_f$ is lower than the natural shedding frequency, the initial vortex of the same direction is formed. This means that the first vortex is formed on the lower side of the cylinder when the cylinder is rotated in the counterclockwise direction, and on the upper side of the cylinder for the clockwise direction due to insufficient acceleration. However, when $S_f$ is higher than the Karman frequency ($S_f = 0.171$), a reactive vortex is formed with respect to the cylinder rotation and the the vortex is rolled-up in the opposite direction to the corresponding rotation. Accordingly, the vortex is formed on the upper side of the cylinder with the opposite direction when the cylinder is rotated in the counterclockwise direction and vice versa. This vortex formation and relevant phase changes were closely related with the $t - C_L_{max}$ distribution. It was found that, as $\theta_{max}$ decreases, the lock-on range decreases, but the change rate of $t - C_L_{max}$ increases significantly. The vortex formation modes are similar when the values of $t - C_L_{max}$ have the same values by varying $S_f$ and $\theta_{max}$. It is evident that the phase change occurs across the natural shedding frequency.