Spatial simulation of the instability of channel flow with local suction/blowing

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A direct numerical simulation was made of instability in a spatially evolving channel flow. A local surface suction/blowing was imposed at the upper wall (x/h = 20). A Tollmien–Schlichting (TS) wave was superimposed on the laminar channel flow at the inflow. At the outflow, the buffer domain technique was applied to suppress the reflection of outgoing waves. The influence of the local suction/blowing on the linear and nonlinear instabilities of the flow was examined. It was found that the local suction/blowing increases the disturbance energy significantly in the interaction zone for subcritical (Re=5000) and supercritical (Re=10 000) cases. The effects of the blowing strength (0 ≤ A_S ≤ 0.1) and the initial TS wave amplitude (0 ≤ A_{TS} ≤ 2.0) on the subcritical channel flow were scrutinized. Two regimes of the wave/flow interaction were found by varying A_S, i.e., "monotonic" and "vortex splitting" regimes. © 1997 American Institute of Physics. [S1070-6631(97)01511-0]

I. INTRODUCTION

The process of laminar-turbulent transition is usually initiated by the amplification of unstable instability waves, called Tollmien–Schlichting (TS) waves, through receptivity to the environmental disturbances. The stability of parallel laminar flow to two-dimensional small disturbances is described by the primary linear stability theory. When the primary unstable wave is amplified to a critical threshold, it becomes unstable to three-dimensional disturbances as a result of secondary instability. At later stages of the transition, these new mechanisms produce three-dimensional vortex structures, which lead to K-type or N-type breakdowns, depending on the amplitude of the initial disturbances and the background noise characteristics. In the transition flow, linear secondary growth of the primary wave evolves rapidly to transition through nonlinear interactions. Once they are initiated, nonlinear processes in the transition become violent and turbulent. As a consequence, it becomes very difficult to control the flow field effectively. Hence, it is necessary to understand the physics of linear instability and find possible ways to control the flow instability in the (near-) linear range.

A literature survey reveals that there have been many attempts to control or lessen the unfavorable behavior associated with laminar-turbulent transition. Several methods have been employed, e.g., near-wall fluid suction and blowing, cooling and heating, surface modifications in the form of hump and dip, flow streamlining by favorable pressure gradient (for more details, see Morkovin and Reshotko, and references therein). Among others, the method of local surface suction/blowing has attracted considerable interest from researchers. This is a well-known and much-explored method of controlling the transition of wall-bounded shear flows because it provides a powerful and simple means to perturb laminar-turbulent transition locally. In addition, it is relatively easy to impose the various strengths of surface suction/blowing. It was demonstrated that the suction/blowing influences the instability of laminar shear flows through a local change of mean velocity profiles, which in turn affects both TS wave amplification and dispersion properties, as well as the flow receptivity to external or upstream disturbances.

For theoretical investigations of flow transition due to traveling TS wave instabilities, the plane Poiseuille flow can be regarded as a prototype case. The base flow is strictly parallel and has an exact solution of the Navier–Stokes equations unlike the Blasius boundary layer flow. It is known that plane channel flow is linearly stable up to Re_{cr}. Since the possible growth rate of a TS wave in a plane channel flow is small by magnitude (O=10^{-2}), its sign is very sensitive to the geometric perturbations and mean velocity distributions. It is highly desirable to make a full three-dimensional spatial simulation. However, due to the demand for enormous computation time, most works were done in the temporal simulation. For periodic disturbances in the streamwise direction, such as grooved channels, baffled channels, and a periodic array of cylindrical objects, several temporal stability studies have been made by using the periodic boundary condition in the mean flow direction.

In the present study, a direct numerical simulation has been made to scrutinize the influence of local surface suction/blowing on the linear and nonlinear instability of spatially evolving channel flow. As transition evolves in the streamwise direction with strong upstream influence, a
proper spatially evolving approach was required to allow the
disturbance to modify the mean flow field. The main aim of
this work is to depict the TS wave interaction with the planar
channel flow perturbed by the local steady suction/blowing.
The influence of the blowing strength and the TS wave am-
plitude was examined.

II. NUMERICAL METHOD

A. Governing equations

The incompressible Navier–Stokes equations in primitive
variables may be written as follows:

\[ \frac{\partial u_j}{\partial t} + \frac{\partial}{\partial x_j} (u_i u_j) = -\frac{\partial p}{\partial x_j} + \frac{1}{Re} \frac{\partial^2 u_j}{\partial x_i \partial x_j}, \]  

(1)

\[ \frac{\partial u_i}{\partial x_i} = 0, \]

where \( u_i \) is the velocity and \( p \) the pressure. The velocity is
decomposed into \( u_i = U_i + u_i' \), where \( U_i \) represents the mean
velocity and \( u_i' \) the disturbance velocity, respectively. All
quantities are nondimensionalized using the center-line ve-
clocity \( U_c \) and the channel half-height \( h \). The Reynolds
number is defined as \( Re = U_c h / \nu \), where \( \nu \) is the kinematic viscosity.

As mentioned earlier, a local steady surface suction/blowing
was applied through a port located at the upper wall
to perturb the transitional channel flow. The effect of the
surface suction/blowing was represented by the boundary
condition at the upper wall,

\[ u = 0, \quad v = -A_S H(x) \quad \text{at} \quad y = 2h, \]

(2)

\[ H(x) = e^{-(x-x_S)^2/\sigma^2}, \]

(3)

where \( x_S \) is the streamwise location of the center of the
suction/blowing port at the upper wall. \( H(x) \) represents the
velocity profile across the port, being chosen as a Gaussian
distribution in the present study. The small parameter \( A_S \)
corresponds to the maximum velocity of the surface suction/
blowing and \( A_S \gg 1 \) for blowing. The shape of the wall ve-
clocity distribution was controlled by a parameter \( \sigma \). In these
calculations, \( \sigma \) was set as \( \sigma = 1 \) and the location of the port
was fixed at \( x_S = 20h \). For the present configuration, the con-
sequent volume flux through the port is \( \sqrt{\pi \sigma A_S} \). Note that
the total flow rate at inflow is 4/3 based on the nondimen-
sionalization adopted here.

For the instability computation, a two-dimensional TS
wave was superimposed on the laminar parabolic profile at
the inflow boundary:

\[ u_i'(x=0,y,t) = A_{TS} \Re \left[ \hat{u}_i(y) e^{-i\omega_R t} \right], \]

(4)

where \( A_{TS} \) is the amplitude of the inflow perturbations and
\( \hat{u}_i(y) \) is the complex velocity vector calculated from the spa-
tial eigenfunctions of the Orr–Sommerfeld equation corre-
sponding to the real frequency \( \omega_R \). \( \Re \) represents the real part
of a complex number with \( i = \sqrt{-1} \).

The governing equation for the linear stability of parallel
shear flow can be obtained by assuming a wavelike solution
of the form

\[ u_i(x,y,t) = R[\hat{u}_i(y) e^{ax-i\omega_R t}], \]

(5)

In the above, \( \alpha \) is the complex wave number defined as
\( \alpha = \alpha_R + i \alpha_I \). After linearization with respect to \((U,V)\), the
resulting Orr–Sommerfeld equation can be written as:

\[ \left[(U_0 - \omega_R / \alpha) \left( \frac{d^2}{dy^2} - \alpha^2 \right) - \frac{d^2}{dy^2} U_0 \right. \]

\[ + \left. \frac{i}{\alpha} \left( \frac{d^2}{dy^2} - \alpha^2 \right) \right] \hat{v} = 0. \]

(6)

Here, \( U_0 \) is the base flow of the form \( U_0(y) = y(2-y) \). \( \alpha_R \)
and \( \omega_R \) are related by the relation, \( c = \omega_R / \alpha_R \). After obtaining
\( \hat{v} \) from Eq. (6), \( \hat{u} \) can be easily calculated from the con-
tinuity equation, \( \hat{u} = i \hat{v} / \alpha \).

The perturbation field \((u',v')\) can be extracted by sub-
tracting the initial unperturbed flow from the perturbed one:

\[ u_i' = u_i - U_i. \]

(7)

For the linear stability studies, the linear perturbation equa-
tions are obtained by substituting Eq. (7) into the flow equa-
tions [Eq. (1)], assuming that the perturbation is so small that
the higher powers of the perturbation can be neglected, and
using the fact that \((U,V)\) is also a solution of the Navier–
Stokes equations. Unlike the linear stability theory, where
the equations for an infinitesimal perturbation are solved in-
dependently (e.g., Ghaddar et al.12), the present study is not
limited to linear perturbations. Large nonlinear perturbations
can also be studied. The mean flow \( U_i \) in Eq. (7), which is a function
of \( x \) and \( y \) for local suction/blowing, can be determined by time
averaging as below. Here, the period \( T \) is much larger than the
period of the TS wave (\( T_{TS} \)):

\[ U_i(x,y) = \frac{1}{T} \int_{t_0}^{t_0+T} u_i(x,y) dt, \quad T \gg T_{TS}. \]

(8)

At the outflow, the buffer domain technique of Streett
and Macaraeg17 was used to suppress the reflection of out-
going waves. In the buffer domain, the governing equations
must be modified to have strictly outgoing waves. For this
purpose, the streamwise viscous terms are reduced to zero
using a smooth coefficient function. Since the solution within
the buffer domain is no longer physically meaningful, the
extent of the buffer domain should be kept short as long as it
plays the original role of suppression. In the present study,
we made many preliminary calculations to establish the suit-
ability of the buffer domain technique, and find an optimal
value for the buffer domain size.

B. Solution procedure

A fractional-step method was employed to solve the un-
steady Navier–Stokes equations. This is based on a time-
splitting method in conjunction with the approximate factor-
ization technique.18,19 Following Le and Moin,19 a substep
time advancement scheme for the governing equations can
be written as

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\[
\frac{u_i^k - u_i^{k-1}}{\Delta t} = (\alpha_k + \beta_k)(\hat{u}^k_i) + \beta_k L(u_i^k - u_i^{k-1}) - \gamma_k N(u_i^{k-1}) - \xi_k N(u_i^{k-2}) - (\alpha_k + \beta_k) \frac{\partial p^{k-1}}{\partial x_i},
\]

where \( k \) denotes the substep number (\( k = 1,2,3 \)). The coefficients \( \alpha_k, \beta_k, \gamma_k, \) and \( \xi_k \) are

\[
\gamma_1 = 8/15, \quad \gamma_2 = 2/12, \quad \gamma_3 = 3/4, \\
\xi_1 = 0, \quad \xi_2 = -17/60, \quad \xi_3 = -5/12, \\
\alpha_1 = \beta_1 = 4/15, \\
\alpha_2 = \beta_2 = 1/15, \\
\alpha_3 = \beta_3 = 1/6.
\]

\( L(u) \) and \( N(u) \) represent the viscous and convective terms, respectively:

\[
L(u_i) = \frac{1}{\text{Re}} \frac{\partial^2}{\partial x_j \partial x_j}, \quad N(u_i) = \frac{\partial}{\partial x_j} (u_i u_j).
\]

For each substep, appropriate boundary conditions for the intermediate velocity field \( \hat{u}^k_i \) are necessary to avoid numerical errors. Details regarding the numerical procedures can be found in Le and Moin. Based on the time-splitting method, the solution procedure consists of semi-implicit approach with a three-step Runge–Kutta method for the nonlinear convective terms and a Crank–Nicholson method for the viscous terms. This method is second-order accurate in time for the viscous terms and third-order accurate in time for the convective terms. The overall accuracy is second order in time. The resulting Poisson equation for the pressure correction in the streamwise direction is the Neumann condition. For each substep, a cosine transform was employed.

Numerical stability is limited by the explicit treatment of the convective terms. With central differences for the convective terms, the stability limit CFL number is \( v^3 / \text{Re} \) based on the total time step \( \Delta t \). The length of the time step \( \Delta t \) was chosen to be adequate not only to retain numerical stability of the method (based on the CFL number), but also to represent the instability characteristics of the flow properly. A constant time step, equal to 1/256 of one TS period (\( T_{\text{TS}} \)), was adopted for the three-step Runge–Kutta method, which never exceeds one half of the limiting value allowed by the numerical method.

All numerical computations were carried out on a CRAY-C90 of SERI supercomputer center. Several trial calculations were repeated to monitor the sensitivity of the results to the grid size, and the outcome of these tests was satisfactory (1024×129). For this grid system, the difference between the eigenvalues obtained from the simulation and from the linear Orr–Sommerfeld solution was less than 1%.

### III. RESULTS AND DISCUSSION

#### A. Numerical parameters

In the present study, we consider a two-dimensional laminar channel flow between two parallel plates located at \( y = 0 \) and \( y = 2h \), with a local steady suction/blowing through a port (\( x/h = 20 \)) at the upper wall. A configuration of the flow and relevant coordinate definitions are illustrated in Fig. 1. The streamwise distance of the computational domain is 120\( h \) from the inflow, which is equivalent to approximately 20 times the TS wave length at \( \text{Re} = 5000 \). At outlet, the buffer domain holds 10\( h \) (110 \( \leq x/h \leq 120 \)).

Before proceeding further, it is important to ascertain the reliability and accuracy of the present simulation. Toward this end, the plane Poiseuille flow was selected as a test case. This is because the instability characteristics of the planar channel flow are well understood and documented in the literature. Two Reynolds numbers were tested, which are subcritical (\( \text{Re} = 5000 \)) and supercritical (\( \text{Re} = 10000 \)). In the subcritical case, the frequency for the least stable eigenmode is \( \omega_{R} = 0.330 \). The Orr–Sommerfeld solution gives \( \alpha = 1.1557 + i0.0106 \). In the supercritical case, the eigenvalue is \( \alpha = 1.0069 + i0.0109 \) for \( \omega_{R} = 0.237 \). The frequency \( \omega_{R} \), which corresponds to the least stable mode at each Reynolds number, was chosen to ensure the rapid decay of all other frequencies.

A typical comparison was made for the normal perturbation velocity \( u' \) for both cases with a small value of \( A_{\text{TS}} \). The disturbance amplitude was defined as the rms magnitude of the \( u \) component of the TS wave, which was set as \( A_{\text{TS}} = 10^{-4} \) to exclude the possibility of the subcritical transition due to the high amplitude of initial TS wave (see Sec. III D). Figure 2 shows the disturbance velocity at the centerline along the streamwise direction. As shown in Fig. 2, the results from the present simulation and the linear Orr–Sommerfeld solution are in excellent agreement for both cases. The disturbance kinetic energy was found to decay exponentially with a growth rate \( \alpha_i = 0.0106 \) at \( \text{Re} = 5000 \). The wave number of the perturbation (\( \alpha_k \)) was also found to be very close to the accurate value from the Orr–Sommerfeld solution (\( \alpha_k = 1.1557 \)). The values of \( \alpha_k \) and \( \alpha_i \) have been checked at various \( y \) locations including the critical layer. Excellent agreement was obtained at the criti-
cal layer. These observations indicate that the present transition simulations are adequate for the spatially evolving transition flows.

B. Effect of local suction/blowing on TS wave stability

In order to demonstrate the effect of the small-intensity suction/blowing on the streamwise development of a TS wave, computations were performed for subcritical ($Re = 5000$) and supercritical ($Re = 10000$) channel flows. As mentioned earlier, the local suction/blowing was applied at the upper wall ($x_S = 20h$). Note that since some extra amount of fluid ($\sqrt{\pi \sigma A_S}$) is supplied (taken out) through the blowing (suction) port, the consequent effective Reynolds number is increased (decreased).

The disturbance streamfunction contours of TS waves with blowing ($A_s = 0.01$) and suction ($A_s = -0.01$) are illustrated in Figs. 3 and 4 for subcritical and supercritical cases, respectively. Quasi-steady, time-periodic behavior of the disturbed flow has been established after a transient period of $32 \tau_{TS}$, which corresponds to approximately 8000 time steps. A closer inspection of Figs. 3 and 4 near the port reveals that the TS wave grows in the streamwise direction for both cases, exhibiting considerable distortions. Due to the suction/blowing, the TS waves are observed to be no longer symmetric with respect to the channel centerline ($y = h$). The wave near the blowing port leads the wave of the center line, while the wave in the opposite side of the blowing port lags. On the contrary, a reversed trend is observed in the suction case. In the supercritical case, the phase difference is more significant compared to the subcritical case.

Variations of the $u_{rms}$ profiles by the local suction/blowing are exhibited in Figs. 5 and 6 for both subcritical

![FIG. 2. Comparison of the present simulation with the theoretical solution: ---, present calculation; - - - - , linear solution.](image2)

![FIG. 3. Disturbance streamfunction contours at $Re = 5000$.](image3)

![FIG. 4. Disturbance streamfunction contours at $Re = 10000$.](image4)

![FIG. 5. Variation of the $u'$ profile at $Re = 5000$.](image5)
and supercritical cases. Upstream the suction/blowing port, the typical channel flow $u'_{rms}$ profiles are displayed. However, the $u'_{rms}$ profiles are significantly changed at the downstream of the port. Two peaks in the blowing side and one peak in the opposite side are detected near the blowing port. In particular, the increase of peak value in the opposite side is appreciable. This is due to the enhanced shear near the wall by the continuity constraint.3 The third peak value observed in the blowing side increases with the distance from the suction/blowing port, and then decreases slowly, returning to the $u'_{rms}$ profile of the channel flow. The appearance of a third maximum in the $u'_{rms}$ profile is a typical feature with an inflection point in the mean flow profile.21 In the supercritical case, the afore-stated phenomena are more evident. The enhanced peak value in the opposite side decreases slowly. It is observed that the flow near the exit plane is not symmetric up to $x/h=70$ in the supercritical case, while the flow becomes symmetric for $Re=5000$. This means that the impact of the local suction/blowing on TS waves are substantial as $Re$ increases.

To describe the effect of suction/blowing quantitatively, it is useful to measure the spatial growth of the disturbances. Several different criteria for measuring the disturbance amplitude have been devised in the literature.1,22 Among them, the following disturbance amplitude $E(x)$ has been adopted here:

$$E^2(x) = \frac{1}{2} \int_0^2 \left( \bar{u}^2(x) + \bar{v}^2(x) \right) dy,$$

where $E$ stands for the integral of disturbance kinetic energy across the channel height. The total influence of suction/blowing within the whole channel is considered in Eq. (13). For more details, see Fasel and Konzelmann.22

Variations of the amplification factor $\log(E(x)/E_0)$ are displayed in Fig. 7 for both subcritical and supercritical cases. For no suction/blowing ($A_s=0$), $E(x)$ changes exponentially in the downstream direction as $e^{-\alpha_f x}$, where the amplification factor shows a straight line in a log plot with the slope of $-\alpha_f$. Inside the interaction zone ($20 \leq x/h \leq 50$), it is seen that the surface suction/blowing increases the amplification factor significantly in both cases. In the subcritical case, blowing is found to have more significant effect on the downstream disturbance behavior than suction.

C. Effect of blowing strength

The influence of the local blowing strength on the instability characteristics is investigated in the subcritical case ($Re=5000$), which is listed in Table I for $\omega=0.330$ at $A_{TS}=10^{-4}$. Figure 8 displays the instantaneous disturbance streamlines for four equally spaced time instants over one complete cycle of the flow at $A_S=0.03$. The general feature of the flow patterns is qualitatively different from that of the moderate amplitude blowing ($A_S=0.01$) of Fig. 3(a). A
small vortex near the lower wall is observed in Fig. 8(a) at three TS wavelengths downstream \((x=40h)\). The small vortex is then merged with the upstream large vortex. The resulting combined vortex is splitted into two descendant vortices: one is near the upstream lower wall, the other near the downstream upper wall, as shown in Fig. 8(b). The upstream descendant vortex shrinks in strength as it flows, while the downstream one recovers its strength across the whole channel [Fig. 8(c)]. The small upstream descendant vortex will decay as it goes downstream and be merged with the upstream one [Fig. 8(d)]. This forms one complete cycle of "vortex splitting." The disturbance streamfunctions of \(A_S=0.04\) and \(A_S=0.05\) were seen to be similar to those of \(A_S=0.03\), indicating the appearance of a new developed regime of the wave/flow interaction, termed "vortex splitting" regime.

Streamwise variations of the amplification factor \(\log(E(x)/E_0)\) at various blowing strengths are presented in Fig. 9. The amplification factor decreases exponentially before the blowing port \((x/h=20)\). As the instability wave passes the blowing port, it grows rapidly to a maximum in the interaction zone. The size of the interaction zone also increases as the blowing strength increases. This means that the extent of the interaction between the instability wave and the flow is significant at higher blowing strengths. As seen in the disturbance streamfunction (Fig. 8), the characteristics of the disturbance field is not homogeneous across the channel height.

In an effort to see the local changes of the disturbance field, the disturbance energy at a certain location is considered, instead of the full integration value across the channel height. Here, the disturbance energy is defined by the centerline value of \(v'\) amplitude, i.e., \(E(x)=v_{rms}^c\). Streamwise variations of the fundamental wave component of \(v_{rms}^c\) are presented in Fig. 10. A threshold value of the blowing is clearly found, above which the wave development changes its behavior. As can be seen, the response of the flow to the surface blowing is systematic up to \(A_S=0.02\) (called "monotonic" regime). Beyond this threshold value, a new "vortex splitting" regime is observed, in which spikelike peaks appear in the distribution. For example, the peak occurs at about \(x=40h\) for \(A_S=0.03\). These locations are related to the vortex splitting observed in Fig. 8. The location of the peak goes downstream as \(A_S\) increases.

As a TS wave passes the interaction zone, its amplitude distribution across the channel has been redistributed from the initial least stable mode to other (higher) TS modes with large damping rates. In a distance of two or three TS wavelengths, almost all of their energy returned to the mean flow.

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**TABLE I. Summary of the computation cases.**

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<th>(A_{TS}(%))</th>
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**FIG. 8.** Instantaneous disturbance streamlines for four equally spaced time instants over one period at \(A_S=0.03\) and \(A_{TS}=10^{-4}\).

**FIG. 9.** Streamwise variations of the amplification factor \(\log(E(x)/E_0)\) at various blowing strengths.

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Thus, there is a competition between the total energy growth in the interaction zone and the final energy content in the least stable TS mode. This means that, at the given (least stable) frequency of excitation, the final disturbance energy could be either enhanced or suppressed depending on the blowing parameters. These observations potentially open a possibility to control the disturbance level in a channel and, hence, promote or inhibit the laminar-turbulent transition.

Figure 11 represents the growth rates of the fundamental wave of $v_{rms}$ at various blowing strengths, which were calculated from Fig. 10. For large blowing strengths ($A_S > 0.03$), the value $\alpha_I$ decreases dramatically to a negative extreme and then abruptly changes the sign, which decays to the unperturbed value without suction/blowing. Note that the locations of the peaks and the abrupt change of the sign are consistent with those of the vortex splitting and the spikelike peaks observed in Figs. 8 and 10. Figure 12 shows the disturbance energy at $x = 100h$, calculated from Fig. 9. In all the cases investigated, the outgoing flow is recovered to near-linear one according to almost parallel behavior of $\log(E(x)/E_0)$ lines of the fundamental wave. The disturbance energy has a maximum at $A_S = 0.015$ and then decreases as $A_S$ increases.

D. Effect of initial TS wave amplitude

Next, the value of $A_{TS}$ is increased to 0.02 to see the effect of the initial TS wave amplitude on the downstream development of the disturbances in the planar channel (cases 11–22 in Table I). The blowing amplitude was set as $A_S = 0.03$ to depict the flow/wave interaction in the ‘‘vortex splitting’’ regime. Streamwise variations of the amplification factor $\log(E(x)/E_0)$ are shown in Fig. 13. Since the disturbance amplitudes are normalized by the initial TS wave amplitude, all the curves should merge into a single curve if the
flow remains in a linear regime. As clearly seen, the curves deviate each other downstream the blowing port. This is a clear evidence of the existence of the nonlinearity in this simulation.

Streamwise variations of the fundamental and second harmonic components of $v'_{rms}$ are shown in Figs. 14 and 15, respectively. The fundamental and second harmonic components of $v'_{rms}$ was subtracted from the total disturbance energy by using Fourier transform. The amplification factor is normalized by its inflow value of the fundamental wave at each TS wave amplitude. For $A_{TS} \leq 10^{-3}$, the change (increase) in the fundamental wave is almost negligible. The distribution for $A_{TS}=10^{-4}$ is exactly the same as that for $A_{S}=0.03$ in Fig. 10, which validates the proper choice for the size of the computational domain and grid resolution used here. As $A_{TS}$ further increases, the fundamental wave follows a monotonic increase beginning from a distance of two TS wavelengths downstream the blowing port ($x=30h$), while the second harmonic component increases in the whole channel.

Figure 16 shows the fundamental and second harmonic components of $v'_{rms}$ at $x=36h$, respectively. This location corresponds to the location of the peak for $A_{S}=0.03$, which is shown in Fig. 14. The fundamental wave increases linearly to $A_{TS}=2 \times 10^{-3}$. Beyond this value, the response is roughly proportional to $A_{TS}^3$, while the second harmonic is proportional to $A_{TS}^2$. It is evident that the nonlinearity is inherent in this flow, which is not available in any linear calculations. Based on our observations, the proportionality of the fundamental wave component beyond $A_{TS}=2 \times 10^{-3}$ is found to be a function of $x$ and $y$: at some downstream locations ($x/h=50$), the fundamental wave is proportional to $A_{TS}^2$. In general, the behavior of the fundamental wave for moderate amplitudes of TS wave ($A_{TS}=2 \times 10^{-3}$) lies between $A_{TS}^3$ and $A_{TS}^2$. Regardless of the downstream location, however, the second harmonic shows almost perfect square proportionality. Although the initial amplitude of the second harmonic is very small compared to that of fundamental wave,
at certain downstream locations, the former exceeds the latter.

IV. CONCLUDING REMARKS

Far-ranging numerical simulation has been presented to capture the effect of the local surface suction/blowing on the downstream development of a TS wave in a spatially evolving channel flow. The computations were performed for subcritical (Re=5000) and supercritical (Re=10 000) channel flows. By imposing the local suction/blowing, the TS wave was no longer symmetric with respect to the channel centerline. A third maximum of the $u''_{rms}$ profile was observed near the blowing port due to the increased boundary layer. The appearance of a third maximum in the $u''_{rms}$ profile was a typical feature of flows with an inflection point in the mean flow profile. In the opposite side of the blowing port, the disturbance amplitude was increased due to the enhanced shear near the wall.

A threshold value of the blowing was found, above which the wave development changed its behavior, i.e., the spikes and the corresponding vortex splitting appeared in the streamwise amplitude distribution. A new developed regime of the wave/flow interaction was observed, which is termed “vortex splitting” regime. The locations of the peaks and the abrupt change of the sign of $\alpha_1$ were consistent with those of the vortex splitting and the spikelike peaks. Some nonlinear phenomena in the interaction zone were found, which is a competition between the total energy growth in the interaction zone and the final energy content in the least stable TS mode. As $A_3$ increases, the strength of the second harmonic increased monotonously in the interaction zone (20 $\leq x/h \leq$ 50).

The value of $A_{TS}$ was increased to 0.02 to see the effect of the initial TS wave amplitude on the downstream development of the disturbances. The fundamental wave was increased linearly to $A_{TS}=2 \times 10^{-3}$. When $A_{TS} \approx 2 \times 10^{-3}$, the response was roughly proportional to $A_{TS}^3$. Beyond this value ($A_{TS}=2 \times 10^{-3}$), the proportionality was a function of the downstream location. Regardless of the downstream location, however, the second harmonic showed almost perfect square proportionality.