Adaptive LMMSE receivers for CDMA systems over time-varying multipath fading channels

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Abstract

Adaptive linear minimum mean square error (LMMSE) receivers for frequency selective fading channels are developed by modifying the constrained LMMSE receiver in [6]. In particular, additional constraints regarding interpath interference are imposed on the constrained optimization in [6], and simple adaptation rules are derived by converting the constrained MMSE problem into an unconstrained optimization. It is shown that the instantaneous SINR of the proposed receiver is always higher than that of the RAKE receiver. Computer simulation results indicate that the proposed receivers can perform better than the existing RAKE and adaptive LMMSE receivers in multipath fading environments.

Index Terms -- LMMSE receiver, CDMA, adaptive algorithm, blind algorithm, multipath fading

1. Introduction

Various adaptive LMMSE receivers have been proposed for detection of DS-CDMA systems. For AWGN channels, adaptive LMMSE receivers were developed based on the standard MSE cost function [1]-[3]. In the case of flat fading channels, a channel estimator was employed and, in an attempt to improve the tracking capability, the MSE cost was modified [4]-[6]. Furthermore, in [6] a constraint regarding filter coefficients was imposed on the MMSE problem; it was shown that such a constraint improved the receiver performance. For frequency selective fading channels, use of an adaptive filter for each resolvable transmission path has been suggested, and the receivers in [4]-[6] may be applied to each path. In this case, however, the receiver performance is degraded due to interpath interference (IPI).

The objective of this paper is to develop LMMSE receivers for frequency selective fading channels. The proposed receiver employs an adaptive filter for each resolvable transmission path, and the adaptive filters compensate for IPI as well as the multiple access interference (MAI). The proposed adaptive filter is an extension of the one in [6]: its cost function and channel estimator remain the same, but multiple constraints are employed to consider IPI. Simple adaptive rules are developed by converting the constrained MMSE problem into an unconstrained optimization. The efficiency of the proposed receivers is demonstrated by computer simulation.

2. System model

Let us consider the impulse response of a multipath fading channel at time \( t \) as

\[
h(t) = \sum_{m=1}^{M} h(t), \delta(t - \tau_m),
\]

where \( h(t), m \) and \( \tau_m \) are the time-varying fading factor and propagation delay of the \( m \)th multipath, respectively, and \( \delta(t) \) represents the Dirac-delta function. For the sake of simplicity, let \( \tau_m = (m-1)T_c \), where \( T_c \) is the chip duration. It is assumed that the fading factors are not changed for a symbol duration, that is, \( h(t), m = h_m(n) \) for \( nT \leq t < (n+1)T \), where \( T \) is the symbol duration. Denote by \( y(l) \) the output of the matched filter of the received signal. The normalized spreading code for the desired user is denoted by \( c = [c_1, c_2, \ldots, c_N]^T \), where \( N = T/T_c \) is the processing gain. Then, the \((N + M - 1) \times 1\) signal vector from \( y(l) \) is written as

\[
y(n) = [y(nN), y(nN+1), \ldots, y(nN+N+M-2)]^T
\]

\[
y(n) = \sum_{m=1}^{M} h_m(n) \begin{bmatrix} 0_{m-1}^T \ \ c \ \ 0_{M-m}^T \end{bmatrix} \Delta_{=c_m} \ d(n) + u(n), \quad (1)
\]

where \( d(n) \) is the data symbol of the desired user and \( u(n) \) of size \((N + M - 1) \times 1\) is the sum of interfering signal vectors and background noise vector. Let \( \mathbf{C} = [c_1, c_2, \ldots, c_M]^T \). Then, \( y(n) \) can be rewritten as

\[
y(n) = \mathbf{C} \mathbf{h}(n) d(n) + u(n), \quad (2)
\]

where \( \mathbf{h}(n) = [h_1(n), h_2(n), \ldots, h_M(n)]^T \).
3. LMMSE receivers over time-varying multipath fading channels

In this section, we first analyze the LMMSE receiver in [6] and then the analytical results are extended to the proposed receiver.

A. Analysis of the receiver in [6]

The receiver in [6] has been derived by solving the following optimization problem: for the $m$-th resolvable path

$$ w_{c,m} = \arg \min_{w_m} E[|w_m^H y(n) - h_m(n)d(n)|^2] $$

subject to $w_m = c_m + z_m$ and $z_m \perp c_m$. \hspace{1cm} (3)

where $w_m$ is the filter coefficients vector which is being decomposed into $c_m$ and $z_m$. The vector $z_m$ represents the adaptive part of the weight vector $w_m$. The orthogonality between $z_m$ and $c_m$ guarantees the unbiasedness of the filter for flat fading channels. In order to find the optimal weight vector in (3), we need the following result.

**Lemma 1** Let $||c||^2 = 1$. Then, it follows that

$$ w = c + z \quad \text{and} \quad c \perp z \quad \text{if and only if} \quad w^H c = 1. \quad (4) $$

Then, the constrained optimization problem in (3) can be solved by the Lagrangian multiplier.

**Lemma 2** If $E[h_m(n)h_{m'}(n)] = 0$ for $m \neq m'$, the optimal weight vector $w_{c,m}$ is written as

$$ w_{c,m} = \frac{1}{c_m^H R_y^{-1} c_m} R_y^{-1} c_m, \quad (5) $$

where $R_y = E[y(n)y^H(n)]$.

Next, it is shown that the constrained optimization problem in (3) can be converted into an unconstrained optimization problem to derive adaptive algorithms straightforwardly.

**Lemma 3** Let $P_{c,m} = I - c_m c_m^H$. The optimal weight vectors $w_{c,m}$ are written as

$$ w_{c,m} = c_m + z_{c,m}, \quad m = 1, 2, \ldots, M. \quad (6) $$

Here, $z_{c,m} = P_{c,m} \tilde{z}_{c,m}$ and the vectors $\tilde{z}_{c,m}$ are the solution vector of the following unconstrained optimization problem:

$$ \tilde{z}_{c,m} = \arg \min_{\tilde{z}_c} E[|\tilde{z}_m^H \tilde{y}_m(n) - (h_m(n)d(n) - c_m^H y(n))|^2], \quad (7) $$

where $\tilde{y}_m(n) = P_{c,m} y(n)$.

Once the optimal weight vectors are found, the decision variable which is obtained from the maximal ratio combining (MRC) is written as

$$ \hat{d}_c(n) = \sum_{m=1}^M h_m^*(n) w_{c,m}^H y(n). \quad (8) $$

The signal $\hat{d}_c(n)$ contains IPI components because the orthogonality condition in (3), which has been derived for flat fading channels, does not guarantee zero IPI. In the following, we shall show that IPI can be removed by putting additional orthogonality conditions in (3).

B. Derivation of the proposed receiver

Again referring to (3), IPI components can be suppressed if $z_{c,m}$ is orthogonal to all code vectors that correspond to multipaths of the desired user. This observation leads to the following optimization: for the $m$-th path,

$$ w_{co,m} = \arg \min_{w_m} E[|w_m^H y(n) - h_m(n)d(n)|^2] $$

subject to $w_m = c_m + z_m$ and $z_m \perp \text{Range}(C)$. \hspace{1cm} (9)

Under the constraint that $z_m$ is orthogonal to the $\text{Range}(C)$, the $m$-th branch filter output is written as

$$ w_m^H y(n) = (c_m + z_m)^H \left( \sum_{m'\neq m} c_{m'} h_{m'} d(n) + u(n) \right) = h_m(n)d(n) + (c_m + z_m)^H u(n). \quad (10) $$

Note that IPI components do not appear in (10). In deriving the solution of (9), it is convenient to consider the singular value decomposition (SVD) of the code matrix $C$. Let $s_m$ be the left singular vectors of $C$ and $S = [s_1 \ s_2 \ \cdots \ s_M]$. Then $\text{Range}(C) = \text{Range}(S)$ and

$$ \text{Ch}(n) = Sv(n), \quad (11) $$

where $v(n)$ is an $M \times 1$ vector given by

$$ v(n) = S^H Ch(n). \quad (12) $$

Using in (12) in (2), the received vector $y(n)$ is written as

$$ y(n) = S v(n)d(n) + u(n) \quad (13) $$

Now the optimization in (9) is rewritten as

$$ w_{co,m} = \arg \min_{w_m} E[|w_m^H y(n) - v_m(n)d(n)|^2] $$

subject to $w_m = s_m + z_m$ and $z_m \perp \text{Range}(S)$. \hspace{1cm} (14)

The solution of (14) is derived through the following lemmas.

**Lemma 4** Let $P_S = I - SS^H$. The weight vector $w_{co,m}$ is given by

$$ w_{co,m} = s_m + z_{co,m}, \quad m = 1, 2, \ldots, M. \quad (15) $$

Here, $z_{co,m} = P_S^\perp \tilde{z}_{co,m}$ and the vector $\tilde{z}_{co,m}$ are the solution vector of the following unconstrained optimization problem:

$$ \tilde{z}_{co,m} = \arg \min_{\tilde{z}_c} E[|\tilde{z}_m^H \tilde{y}_m(n) - (v_m(n)d(n) - s_m^H y(n))|^2], \quad (16) $$

where $\tilde{y}_m(n) = P_S y(n)$. 

Suppose that
\[ \gamma_{\text{co}}(n) = \frac{|v^H(n)v(n)|^2}{v^H(n)Q_{11}v(n)} \]  
(22)

\[ \gamma_{\text{RAKE}}(n) = \frac{|v^H(n)v(n)|^2}{v^H(n)Q_{11}v(n)} \]  
(23)

where \( \gamma_{\text{RAKE}}(n) \) is the instantaneous SINR of the RAKE receiver whose decision variable is given by
\[ \hat{d}_{\text{RAKE}}(n) = h^H(n)C^H y(n). \]  
(24)

Note that from Lemma 3,
\[ \gamma_{\text{co}}(n) = \frac{v^H(n)v(n)}{\alpha_n^2}, \]  
(25)

if \( R_u = \sigma_n^2 I \). Hence, the constrained LMMSE receiver with \( \hat{w}_{\text{co},m} \) can provide the same performance as the rake receiver which is the optimum in AWGN channels.

It is noteworthy that although the interferers from other users depend on their time-varying channel coefficients, the effect of the channel variation is smoothed out, because \( u(n) \) is the sum of all those interferers.

4. Adaptive algorithm for the proposed receiver

Suppose that \( \hat{z}_{\text{co},m}(n) \) in (16) is recursively obtained. Let \( \hat{z}_{\text{co},m}(n) \) denote the vector after the \( n \)-th iteration. If \( \hat{z}_{\text{co},m}(0) \in \text{Null}(c_m) \), then
\[ \hat{z}_{\text{co},m}(n) = \text{Null}(c_m), \quad n \geq 1, \]  
and the weight vector is expressed as
\[ \hat{w}_{\text{co},m}(n) = s_m + \hat{z}_{\text{co},m}(n), \quad m = 1, 2, \cdots, M. \]  

Hence, hereafter, we assume that the initial vectors are properly chosen and, thereby, the difference between the vectors \( \hat{z}_{\text{co},m}(n) \) and \( \hat{z}_{\text{co},m}(n) \) [and \( \hat{z}_{\text{co},m}(n) \) and \( \hat{z}_{\text{co},m}(n) \)] is ignored. Using the steepest recursion in (16), we get
\[ \hat{z}_{\text{co},m}(n) = \hat{z}_{\text{co},m}(n-1) + \mu E[(a_{\text{co},m}(n) - \hat{z}_{\text{co},m}(n-1)\hat{y}(n))^* \hat{y}(n)] \]  
(26)

If we let \( a_{\text{co},m}(n) = v_m(n)d(n) - s_m^H y(n) \), then
\[ E[\hat{y}(n)a_{\text{co},m}(n)^*] = -P_S^R y s_m, \]  
and (26) is rewritten as
\[ \hat{z}_{\text{co},m}(n) = \hat{z}_{\text{co},m}(n-1) - \mu (P_S^R y s_m) - \mu E[\hat{y}(n)^* \hat{y}(n)]\hat{z}_{\text{co},m}(n-1). \]  
(27)

Note that
\[ E[\hat{y}(n)^* \hat{y}(n)] = P_S^R y s_m. \]  
(28)

Using (28) in (27) and dropping \( E[\cdot] \), the following recursion is obtained:
\[ \hat{z}_{\text{co},m}(n) = \hat{z}_{\text{co},m}(n-1) - \mu (P_S^R y s_m) + \hat{y}(n)^* \hat{y}(n)\hat{z}_{\text{co},m}(n-1). \]  
(29)

This recursion algorithm is a blind algorithm that does not need any training sequence, and thus it should be useful for practical applications.
5. Simulation results

An asynchronous CDMA system with QPSK modulation was considered. Gold code with length 31 was used for spreading. A frequency selective Rayleigh fading channel with bandwidth 3.968MHz was assumed and the carrier frequency was 2.0 GHz. Transmitted powers of all active users were set to be equal. The pilot symbol assisted technique was employed and one pilot symbol was inserted for every eight data symbols. Both pilot and data symbols were used for adaptation. The proposed adaptive LMMSE receiver was compared with the one in [6] and the RAKE receiver. For the adaptation LMMSE receivers, the channel was estimated by using 10 pilot symbols; but the channel was assumed to be known for the RAKE receiver. Figures 1 and 2 show the BER performances corresponding to 3 users and 9 multipaths. It is seen that the adaptive LMMSE receivers performed much better than the RAKE receiver. The proposed receiver outperformed the one in [6]. The performance gain was minor when the mobile speed was 10km/h (Figure 1), but it became impressive for the speed of 100km/h (Figure 2): about 5dB gain was achieved at the BER of $10^{-3}$. The proposed receiver should be a useful alternative to the RAKE receiver in [6] for frequency-selective channels.

A Proof of Lemma 5

Note that

$$v_m(n)d(n) - s_m^H y(n) = -s_m^H u(n).$$

Hence,

$$E \left[ |P_m \tilde{y}_m(n) + s_m^H u(n)|^2 \right] = E \left[ |P_m \tilde{y}_m(n) + s_m^H u(n)|^2 \right].$$

Since $P_S^\perp S = 0$, we can show that

$$E \left[ |P_m \tilde{y}_m(n) + s_m^H u(n)|^2 \right] = \tilde{z}_m^H P_S^\perp R_u P_S^\perp \tilde{z}_m + 2\text{Re}(\tilde{z}_m^H P_S R_u s_m) + s_m^H R_u s_m.$$  

Hence, a vector $z_m$ which minimizes $E \left[ |P_m \tilde{y}_m(n) + s_m^H u(n)|^2 \right]$ should satisfy

$$P_S^\perp R_u P_S^\perp \tilde{z}_m + P_S^\perp R_u s_m = 0.$$ 

Clearly, a solution vector is written as

$$\tilde{z}_{m,0} = - (P_S R_u P_S^\perp)^{\dagger} P_S^\perp R_u s_m.$$ 

It completes the proof.

References