Performance Evaluation of a Variable Processing Gain
DS/CDMA System

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SUMMARY In this paper, we analyze the multiple access interference of a variable processing gain DS/CDMA system and define discrete partial crosscorrelation functions. We also evaluate the bit error rate of the system using Gaussian approximation and bounding technique. Three kinds of spreading codes (long, short, and random codes) are considered in the analysis of the system. It is shown that the bit error rate of a user is not relevant to the processing gain of interfering users: it is relevant only to the processing gain of the user, transmitted powers, PN sequences, and spreading codes. The performance of short codes turns out to be better than that of long and random codes as in other systems.

key words: multiple bit rate, variable processing gain, DS/CDMA, crosscorrelation, spreading codes, average bit error rate

1. Introduction

Third generation mobile and personal communication systems are expected to support multi-media services such as speech, image, data, and so on. A DS/CDMA system has the flexibility to integrate services of various bit rates [1]. To transmit signals with a wide range of bit rates sharing a given bandwidth, the most easiest way is to alter the processing gain according to their own bit rates and spread all signals to the same bandwidth. Note that the higher the bit rate is, the lower the processing gain is.

Due to the complexity required to obtain quantitative results on the performance of a variable processing gain (VPG)-DS/CDMA system, only a few results have been investigated. In [2], the performance of a synchronous VPG-DS/CDMA system is investigated. Also, in [3], the performance of an asynchronous VPG-DS/CDMA system with random codes is considered when the processing gain is multiples of the smallest processing gain. In this paper, we analyze the multiple access interference (MAI) of an asynchronous VPG-DS/CDMA system without any restriction on processing gain, and evaluate the bit error rate (BER) of the system using Gaussian approximation (GA) [4], [5] and bounding technique [6], [7]. In the analysis, we consider three kinds of spreading codes: long, short, and random codes [6], [8].

2. The VPG-DS/CDMA System Model

We consider the asynchronous VPG-DS/CDMA system with $K$ users. If the chip waveform $\psi(t)$ is nonzero only in the interval $[0, T_c]$, is normalized to have chip energy $T_c$, the spectral-spread signal of the $k$th user can be expressed as

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_{k,j} \psi(t - jT_c),$$  

(1)

where $\{a_{k,j}\}$ is a binary sequence of period $L_k$. The data signal of the $k$th user can be expressed as

$$b_k(t) = \sum_{m=-\infty}^{\infty} b_{k,m} P_{T_k}(t - mT_k),$$  

(2)

where the sequence $\{b_{k,m}\}$ is the binary data sequence of the $k$th user with $Pr\{b_{k,m} = +1\} = Pr\{b_{k,m} = -1\} = 1/2$, and $P_{T_k}(t) = 1$ for $0 \leq t < T_k$ and $P_{T_k}(t) = 0$ otherwise. Assume that the $k$th user has processing gain $N_k$, i.e., $T_k = N_k T_c$. Thus, the transmitted signal of the $k$th user is

$$s_k(t) = \sqrt{2P_k a_k(t)} b_k(t) \cos(\omega_c t + \theta_k),$$  

(3)

where $P_k$, $\omega_c$, and $\theta_k$ are the transmitted power, carrier frequency, and carrier phase of the $k$th user, respectively. Note that $P_k$ and $T_k$ differ from user to user in this paper.

For an asynchronous system, the received signal $r(t)$ is given by

$$r(t) = \sum_{k=1}^{K} \sqrt{2P_k a_k(t - \tau_k)} b_k(t - \tau_k) \cos(\omega_c t + \phi_k) + n(t),$$  

(4)

where $n(t)$ is the white Gaussian channel noise process with two-sided spectral density $N_0/2$, $\tau_k$ is the time delay associated with the $k$th user, and $\phi_k = \theta_k - \omega_c \tau_k$. Since, we are concerned with relative phase shifts modulo $2\pi$ and relative time delay modulo $L_k T_c$, there is no loss in generality in assuming $\theta_1 = 0$ and $\tau_1 = 0$ and considering only $0 \leq \tau_k < L_k T_c$ and $0 \leq \theta_k < 2\pi$ for $2 \leq k \leq K$. 

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Without loss of generality, we can restrict our consideration to the first receiver which is assumed to be a correlation receiver. The output of the first receiver at \( t = (n + 1)T_1 \) is

\[
Z_1^{(n+1)} = \int_{-T_1}^{n+1} r(t) a_1(t) \cos \omega_c t \, dt.
\]  

(5)

By ignoring the double-frequency terms in (5), as allowed under some mild conditions \([4]\), \( Z_1^{(n)} \) can be rewritten as

\[
Z_1^{(n)} = \eta_1^{(n)} + T_1 \sqrt{\frac{P_k}{2}} \times \left[ b_{1,n} + \sum_{k=2}^{K} \sqrt{\frac{P_k}{P_1}} f_{k,1}^{(n)} \left( b_{k}^{(n)}, \tau_k, \phi_k \right) \right].
\]  

(6)

In (6), the random variable \( \eta_1^{(n)} = \int_{-T_1}^{n+1} a_1(t) \cos \omega_c t \, dt \) is Gaussian with zero-mean and variance \( N_0 T_1/4 \) and

\[
f_{k,1}^{(n)} \left( b_{k}^{(n)}, \tau_k, \phi_k \right) = \frac{\cos \phi_k}{T_1} \sum_{m=\beta_{k,1}^{(n)}}^{\beta_{k,1}^{(n+1)}} b_{k,m} R_{k,1}^{(m,n)} (\tau_k),
\]  

(7)

where \( b_{k}^{(n)} = \left\{ b_{k,0}^{(n)}, b_{k,1}^{(n)}, \ldots, b_{k,n-1}^{(n)} \right\} \) represents a set of consecutive data bits of the kth user, \( b_{k,0}^{(n)} \) is the first data bit of the kth user which interferes with \( b_{1,n} \), \( \beta_{k,1}^{(n)} = \left\lfloor \frac{N_0 x_{k,1}^{(n)} - \sqrt{1 + \left( N_0 x_{k,1}^{(n)} \right)^2}}{N_0} \right\rfloor \), \( x_{k,1}^{(n)} \) is the integer part of \( x \). The function \( R_{k,1}^{(m,n)} (\tau_k) \) denotes the continuous-time partial cross-corrrelation function between \( a_k^{(m)} (t) \) and \( a_1^{(n)} (t) \), where

\[
a_k^{(m)} (t) = \sum_{j=mT_k}^{(m+1)T_k} a_{k,j} \Psi(t - jT_k)
\]

is the spectral-spreading signal for \( b_{k,m} \). Since, \( a_k^{(m)} (t) \) and \( a_1^{(n)} (t) \) are transmitted through the time interval \([mT_k + \tau_k, (m+1)T_k + \tau_k]\) and \([nT_1, (n+1)T_1]\), respectively, \( R_{k,1}^{(m,n)} (\tau_k) \) can be expressed as

\[
R_{k,1}^{(m,n)} (\tau_k) = \begin{cases} 
\int_{\text{max}}^{\text{min}} a_k^{(m+1)}(t - \tau_k) a_1^{(n)}(t) \, dt, & \beta_{k,1}^{(n)} \leq m \leq \beta_{k,1}^{(n+1)}, \\
0, & \text{otherwise}.
\end{cases}
\]  

(8)

3. Crosscorrelation Analysis

According to the number of data bits of the kth user interfering with \( b_{1,n} \), \( R_{k,1}^{(m,n)} (\tau_k) \) in (8) can be rewritten:

first, if only one data bit of the kth user interferes with \( b_{1,n} \), i.e., if \( \beta_{k,1}^{(n+1)} = \beta_{k,1}^{(n)} \),

\[
R_{k,1}^{(m,n)} (\tau_k) = \begin{cases} 
\int_{\text{max}}^{\text{min}} a_k^{(m+1)}(t - \tau_k) a_1^{(n)}(t) \, dt, m = \beta_{k,1}^{(n)}, \\
0, & \text{otherwise}.
\end{cases}
\]  

(9)

Second, if more than or equal to two data bits of the kth user interfere with \( b_{1,n} \), i.e., if \( \beta_{k,1}^{(n+1)} > \beta_{k,1}^{(n)} \),

\[
R_{k,1}^{(m,n)} (\tau_k) = \begin{cases} 
\int_{\text{max}}^{\text{min}} a_k^{(m+1)}(t - \tau_k) a_1^{(n)}(t) \, dt, m = \beta_{k,1}^{(n)}, \\
\int_{\text{max}}^{\text{min}} a_k^{(m+1)}(t - \tau_k) a_1^{(n)}(t) \, dt, m > \beta_{k,1}^{(n)}, & \text{otherwise}.
\end{cases}
\]  

(10)

For \( N_k \leq N_1 \), \( R_{k,1}^{(m,n)} (\tau_k) \) can be expressed by only (10) because \( \beta_{k,1}^{(n+1)} - \beta_{k,1}^{(n)} \leq 1 \). If \( N_k > N_1 \), \( \{m|\beta_{k,1}^{(n)} < m < \beta_{k,1}^{(n+1)}\} \) in (10) is an empty set because \( \beta_{k,1}^{(n+1)} - \beta_{k,1}^{(n)} = 0 \) or 1.

The function \( R_{k,1}^{(m,n)} (\tau_k) \) in (9) and (10) can be expressed in terms of the discrete partial crosscorrelation functions of spreading codes and the continuous-time partial autocorrelation functions of the chip waveform. The discrete partial crosscorrelation functions \( C_{k,1}^{(m,n)} (\cdot) \) and \( C_{k,1}^{(m,n)} (\cdot) \) of the VPG-DS/CDMA system are defined as

\[
C_{k,1}^{(m,n)} (\tau_k) = \begin{cases} 
\sum_{j=mN_k + i - 1}^{(m+1)N_k} a_{k,j-i} a_{1,j}, & 0 \leq i \leq N_k \text{ and } N_k < N_1, \\
\sum_{j=mN_1}^{(m+1)N_k + i - 1} a_{k,j-i} a_{1,j}, & 0 \leq i \leq N_k \text{ and } N_k > N_1, \\
0, & \text{otherwise},
\end{cases}
\]  

(11)

\[
C_{k,1}^{(m,n)} (\tau_k) = \begin{cases} 
\sum_{j=mN_1}^{(m+1)N_k + i - 1} a_{k,j-i} a_{1,j}, & 0 \leq i \leq N_k, \\
0, & \text{otherwise},
\end{cases}
\]  

(12)
\[ C_{k,1}^{(m,n)}(i) = \begin{cases} \sum_{j=mN_{k}+1}^{(n+1)N_{k}-1} a_{k,j-i} a_{1,j}, & 0 \leq i \leq N_{k}, \\ 0, & \text{otherwise}. \end{cases} \] (13)

The continuous-time partial autocorrelation functions of the chip waveform are defined as in [6] for \( 0 \leq s \leq T_{c} \) by \( R_{\Psi}(s) = \int_{0}^{T_{c}} \Psi(t + T_{c} - s)\Psi(t) \, dt \) and \( \hat{R}_{\Psi}(s) = \int_{0}^{T_{c}} \Psi(t + T_{c} - s)\Psi(t) \, dt \); for \( s > T_{c} \) or \( s < 0 \), \( R_{\Psi}(s) = \hat{R}_{\Psi}(s) = 0. \)

The function \( R_{k,1}^{(m,n)}(\tau_{k}) \) in (9) and (10) can now be rewritten as
\[ R_{k,1}^{(m,n)}(\tau_{k}) = \begin{cases} \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}) + \hat{C}_{k,1}^{(m,n)}(l_{k} + 1)R_{\Psi}(S_{k}), & m = \beta_{k,1}^{(n)}, \\ 0, & \text{otherwise}, \end{cases} \] (14)

for \( \beta_{k,1}^{(n+1)} = \beta_{k,1}^{(n)} \), and
\[ R_{k,1}^{(m,n)}(\tau_{k}) = \begin{cases} \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}) + \hat{C}_{k,1}^{(m,n)}(l_{k} + 1)R_{\Psi}(S_{k}), & m = \beta_{k,1}^{(n)}, \\ \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}) + \hat{C}_{k,1}^{(m,n)}(l_{k} + 1)R_{\Psi}(S_{k}), & \beta_{k,1}^{(n)} < m < \beta_{k,1}^{(n+1)}, \\ \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}) + \hat{C}_{k,1}^{(m,n)}(l_{k} + 1)R_{\Psi}(S_{k}), & m = \beta_{k,1}^{(n+1)}, \\ 0, & \text{otherwise}, \end{cases} \] (15)

for \( \beta_{k,1}^{(n+1)} > \beta_{k,1}^{(n)} \), where \( S_{k} = \tau_{k} - l_{k}T_{c} \). A further simplification is also possible for short codes: we have
\[ \hat{C}_{k,1}^{(m,n)}(\tau_{k}) = \begin{cases} \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}) + \hat{C}_{k,1}^{(m,n)}(l_{k} + 1)R_{\Psi}(S_{k}), & m = \beta_{k,1}^{(n)}, \\ \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}), & \beta_{k,1}^{(n)} < m < \beta_{k,1}^{(n+1)}, \\ \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}), & m = \beta_{k,1}^{(n+1)}, \\ 0, & \text{otherwise}, \end{cases} \] (16)

for \( \beta_{k,1}^{(n+1)} = \beta_{k,1}^{(n)} \), and
\[ \hat{C}_{k,1}^{(m,n)}(\tau_{k}) = \begin{cases} \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}) + \hat{C}_{k,1}^{(m,n)}(l_{k} + 1)R_{\Psi}(S_{k}), & m = \beta_{k,1}^{(n)}, \\ \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}), & \beta_{k,1}^{(n)} < m < \beta_{k,1}^{(n+1)}, \\ \hat{C}_{k,1}^{(m,n)}(l_{k})\hat{R}_{\Psi}(S_{k}), & m = \beta_{k,1}^{(n+1)}, \\ 0, & \text{otherwise}, \end{cases} \] (17)

for \( \beta_{k,1}^{(n+1)} > \beta_{k,1}^{(n)} \), where \( ((x))_{N_{1}} \) is \( x \) modulo \( N_{1} \) and \( \alpha_{k,1}^{(n)} = nN_{1} - (l_{k} + 1) - \beta_{k,1}^{(n)}N_{k} \in [0, \ldots, N_{k} - 1] \). A derivation of (14) and (16) is described in Appendix: (15) and (17) can be similarly derived. In (16) and (17), the discrete partial crosscorrelation functions \( \hat{C}_{k,1}(\cdot) \), \( \tilde{C}_{k,1}(\cdot) \), and \( \tilde{C}_{k,1}(\cdot) \) for short codes are defined as
\[ \hat{C}_{k,1}(i) = \begin{cases} \sum_{j=0}^{N_{k}-1} a_{k,j} a_{1,j+i}, & 0 \leq i < N_{1} \text{ and } N_{k} < N_{1}, \\ \sum_{j=0}^{N_{k}-1} a_{k,j+i} a_{1,j}, & 0 \leq i \leq N_{k} \text{ and } N_{k} > N_{1}, \\ 0, & \text{otherwise}, \end{cases} \] (18)

and
\[ \tilde{C}_{k,1}(i) = \begin{cases} \sum_{j=0}^{N_{k}-1+i} a_{k,j-i} a_{1,j}, & -N_{k} \leq i \leq 0, \\ 0, & \text{otherwise}, \end{cases} \] (19)

and
\[ \tilde{C}_{k,1}(i) = \begin{cases} \sum_{j=0}^{N_{k}-1-i} a_{k,j} a_{1,j+i}, & N_{k} \leq i \leq N_{1}, \\ 0, & \text{otherwise}. \end{cases} \] (20)

If \( N_{k} = N_{1} \), \( \beta_{k,1}^{(n)} = n - 1 \) and \( \alpha_{k,1}^{(n)} = N_{1} - l_{k} - 1 \). Then, \( \hat{C}_{k,1}(\cdot) \), \( \tilde{C}_{k,1}(\cdot) \), and \( \tilde{C}_{k,1}(\cdot) \) become the discrete aperiodic crosscorrelation function \( C_{k,1}(\cdot) \) of the conventional DS/CDMA system defined in [4].
4. Evaluation of the Average Bit Error Rate

4.1 Gaussian Approximation

Using the Gaussian approximation [4], we can obtain the BER for $b_{1,n}$:

$$
P_{e,1}^{(n)} = Q \left( \left[ \frac{N_o}{2E_b} + \text{Var} \left\{ I_1^{(n)} \left( b_k^{(n)}, \tau_k, \phi_k \right) \right\} \right]^{-1/2} \right),
$$

(21)

where $Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-\frac{u^2}{2}} dv$, $E_b = P_i T_1$ is the bit energy of the first user, and the normalized total MAI is

$$
I_1^{(n)} \left( b_k^{(n)}, \tau_k, \phi_k \right) = \sum_{k=2}^{K} \sqrt{\frac{P_k}{P_i}} I_k^{(n)} \left( b_k^{(n)}, \tau_k, \phi_k \right).
$$

(22)

For convenience, we denote $I_1^{(n)} \left( b_k^{(n)}, \tau_k, \phi_k \right)$ by $I_1^{(n)}$. By taking an expectation with respect to the mutually independent random variables $b_{k,m}$, $\tau_k$, and $\phi_k$, the variance of $I_1^{(n)}$ can be expressed in terms of the discrete partial crosscorrelation functions defined in Sect. 3: for random codes, the variance of $I_1^{(n)}$ can be shown to be

$$
\text{Var} \left\{ I_1^{(n)} \right\} = \frac{1}{3P_i N_1} \sum_{k=2}^{K} P_k,
$$

(23)

which is identical to that [4] for a conventional DS/CDMA system with different transmitted powers. An implication of (23) is that the BER of a desired user does not depend on the processing gain of interfering users which results from the fact that the sequences of random codes are independent and identically distributed random variables.

Let $L$ denote the least common multiple of $N_1, N_2, \cdots, N_K, L_1, L_2, \cdots, L_K$. Then, $a_{k,i L+j} = a_{k,j}$ for $1 \leq k \leq K$ and $P_{e,1}^{(n+L/N_1)} = P_{e,1}^{(n)}$. The average BER for the first user becomes

$$
P_{e,1} = \frac{N_1}{L} \sum_{n=0}^{L/N_1-1} P_{e,1}^{(n)}.
$$

(24)

4.2 The Upper and Lower Bounds on the Average Bit Error Rate

Once we have the density function $f_1^{(n)}(\cdot)$ of $I_1^{(n)}$ in (22), the exact BER for $b_{1,n}$ can be evaluated as [5]

$$
P_{e,1}^{(n)} = \int_{-\infty}^{\infty} f_1^{(n)}(v) Q \left( \sqrt{\frac{2E_b}{N_o}} \left( 1 + v \right) \right) dv.
$$

(25)

Determining $f_1^{(n)}(\cdot)$ and carrying out the above integration are, however, mathematically intractable, particularly when $K$ is large. Therefore, we obtain the upper and lower bounds on (25). In [6] and [7], the bounds for a conventional DS/CDMA system are obtained by playing a few tricks on $f_1^{(n)}(\cdot)$, which can be made arbitrarily tight at the expense of computational burden. By using the technique described in [6] and [7], we evaluate the upper and lower bounds on the average BER for the VPG-DS/CDMA system by replacing the discrete aperiodic crosscorrelation function $C_{k,1}(\cdot)$ with the discrete partial crosscorrelation functions defined in (11)–(13) and (18)–(20) and accommodating unequal transmitted power.

4.3 Numerical Results

In this subsection, we evaluate the performance of the VPG-DS/CDMA system when long, short, and random codes are used. It is well known that if the chip rates and the transmitted power of all users are equal, the average BER performance of a DS/CDMA system gets better as the bit duration increases as a result of increase in the bit energy. In this paper, to see the applicability of the VPG-DS/CDMA system, let us investigate the performance of the system when the bit energy is constant irrespective of the bit duration.

We set the number of users $K = 3$, the processing gain $N_1 = 32$, $N_2 = 64$, and $N_3 = 128$ ($r_1 = 4r_2$, $r_2 = 2r_3$, where $r_k$ is the bit rate of the $k$th user), and the transmitted power $P_1 = 4$, $P_2 = 2$, and $P_3 = 1$: thus, we keep the bit energy of all users at the same value, i.e., $E_b = P_i T_1 = P_i T_2 = P_i T_3$. In the investigation, we use extended m-sequences defined in [9] with random phases for both long and short codes. Since, computational burden increases as the period of spreading code increases, we set the period of long codes to be $2^{15}$. The generator polynomials and phases used for long and short codes are described in Table 1. We use the rectangular pulse function as the chip waveform.

In Figs. 1–3, the results of the average BER for the VPG-DS/CDMA system with short, long, and random codes are plotted as functions of $E_b/N_o$, respectively. In

Table 1 The generator polynomials and phases used.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$N_k$</th>
<th>$L_k$</th>
<th>polynomial</th>
<th>phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Long codes.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>32</td>
<td>215</td>
<td>100003(s)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>64</td>
<td>215</td>
<td>102043(s)</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>128</td>
<td>215</td>
<td>110013(s)</td>
<td>200</td>
</tr>
<tr>
<td>(b) Short codes.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>32</td>
<td>32</td>
<td>45(s)</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>64</td>
<td>64</td>
<td>103(s)</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>128</td>
<td>128</td>
<td>211(s)</td>
<td>100</td>
</tr>
</tbody>
</table>
evaluating the bounds on the average BER for the system, we set the number of subintervals per unit defined in [6] and [7] to be 10. It is observed that Gaussian approximation is not accurate at high $E_b/N_0$. According to (23), in the case of equal bit energy for all users, the variance of $I_k^{(n)}$ ($I_k^{(n)}$, $k = 2, 3, \ldots, K$, are similarly defined as $I_1^{(n)}$) for random codes is proportional to the sum of the transmitted power of interfering users which decreases as $P_k$ decreases (or, as $T_k$ increases). Thus, the average BER $P_{e,k}$ ($P_{e,k}$, $k = 2, 3, \ldots, K$, are similarly defined as $P_{e,1}$) gets worse as the bit rate $r_k$ decreases (or, as $T_k$ increases) in the case of equal bit energy for all users. Similar results can be observed from the numerical results of the variance of $I_k^{(n)}$ for long and short codes (see Table 2). To compare the performance of several codes, we illustrate the upper bound $f_k^U(\cdot)$ of the density function $f_k^{(0)}(\cdot)$ in Fig. 4. Since, the function $f_k^U(\cdot)$ is symmetric about zero, it is shown for $x \geq 0$ only. The lower bound of the density function $f_k^{(0)}(\cdot)$ is almost identical to $f_k^{LU}(\cdot)$, since the bounds are so much tight. As we can see from Fig. 4, the range of $f_k^{(0)}$ for short codes is smaller than that for long and random codes. We would like to mention that the vertical axis is drawn in log scale and that is can be shown $\int_{-\infty}^{\infty} f_k^U(v) \, dv = 1$ in Fig. 4. Thus, we can conclude that the performance of short codes is better than that of long and random codes; this becomes more evident if we take a close look at Figs. 1–3.

5. Conclusions

We have investigated the multiple access interference of a VPG-DS/CDMA system. Then, we have defined discrete partial crosscorrelation functions for long and short codes, and evaluated the BER performance of the system with long, short, and random codes using Gaussian approximation and bounding technique.

The BER performance of a user is not relevant to
the processing gain of interfering users. It is relevant only to the processing gain of the user, transmitted powers, spreading codes, and PN sequences. In the case of equal bit energy for all users, regardless of the three kinds of spreading codes, the average BER gets better as the bit rate increases, since the sum of the normalized total MAI decreases as the bit rate increases. This characteristic of the VPG-DS/CDMA system is quite useful, since service of higher bit rate generally requires lower (better) BER.

The performance of short codes is better than that of long and random codes when the users have different processing gains, since the normalized total MAI value for short codes is smaller than that for long and random codes. Note that short codes have already been shown to be better when all the users have the same processing gain [1],[8].

References


Appendix: Derivation of Eqs. (14) and (16)

The jth chip a_{k,j} of the first user is transmitted through the time interval [jT_c, (j + 1)T_c]. During this time interval, two chips a_{k,j-l_k-1} and a_{k,j-l_k} of the kth user interfere with a_{1,j} through time interval [jT_c, jT_c + S_k] and [jT_c + S_k, (j + 1)T_c], respectively. Thus, R_{k,1}^{(m,n)}(\tau_k) in (9) can be expressed as

\[ R_{k,1}^{(m,n)}(\tau_k) = \int_{nT_1}^{(n+1)T_1/a_k(t - \tau_k)a_1(t)dt} \]

\[ = \sum_{j=nN_1}^{(n+1)N_1-1} a_{k,j-l_k-1}a_{1,j}R_\psi(S_k) \]

By using the partial correlation functions defined in (11)-(13), we obtain (14).

For the system with short codes, a_{k,j} = nN_1 - (l_k + 1) - \beta_{k,1}N_k and a_{k,j+l_k} = a_{k,j}. Then, R_{k,1}^{(m,n)}(\tau_k) in (14) can be expressed as

\[ R_{k,1}^{(m,n)}(\tau_k) = \sum_{j=0}^{N_1-1} a_{k,j+nN_1-l_k-1}a_{1,j+nN_1}R_\psi(S_k) \]

By using the partial correlation functions defined in (18)-(20), we obtain (16). Following the same procedure, we can derive (15) and (17) from (10).

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