Timing Error Detector for OQPSK Signal

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***Abstract*--A simple algorithm is proposed for detection of timing error of an OQPSK data receiver. Taking square of the receiving data signal and then differentiating it, the timing error will be derived from its difference. This simple algorithm operates separately from detecting carrier phase. Analytical derivation of the S-curve of this timing error shows its effectiveness and illustrates its low complexity.**

Keywords – OQPSK; timing error detector; synchronization

I. INTRODUCTION

QPSK and OQPSK signaling have been used for transmitting high-speed data in satellite communications and in mobile communications. OQPSK signal is similar with QPSK signal but there is a difference between two schemes such as the data of Q channel is delayed from that of I channel by T/2 in OQPSK. This property of OQPSK signal results in smaller amplitude variation in OQPSK signal than one in QPSK signal [1]. Therefore OQPSK signal is less distorted by the nonlinear characteristics of RF devices and has been less affected with adjacent channels than QPSK signal.

Various carrier-phase insensitive timing recovery algorithms for QPSK signal have been proposed [2-4], but only a few one have been done with OQPSK signal. The phase insensitive timing recovery schemes give better performance than the phase sensitive ones. D'amico proposed a symbol timing estimator that is phase insensitive with OQPSK modulation [5]. It has an advantage of operating in feedforward structure but requires too much computation. On the other hand, the symbol timing recovery algorithm proposed by Matsumoto might be applied to wireless packet communications [6].

In this paper, we present a simple timing error detector for detecting OQPSK signal that is separately processed from recovering carrier phase and will render the S-curve that has a simple shape.

II. TIMING ERROR DETECTOR

The received baseband signal with OQPSK modulation can be expressed as follows:

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$$r(t)=e^{j\theta_o}\left\{\sum_k d_k^I g(t-kT-\tau) + j\sum_k d_k^Q g\left(t-kT-\frac{T}{2}-\tau\right)\right\} \tag{1}$$

where θ_o is initial phase, d_k^I and d_k^Q are in-phase and quarature-phase terms of the k^{th} symbol respectively, $g(t)$ is the impulse response of a lowpass filter(LPF) for pulse shaping, T is symbol length and τ is timing error.
As a first step, we make real signal by squaring the magnitude of the received signal for estimating the timing error by

$$\begin{aligned} s_q(t) &= |r(t)|^2 \\ &= \sum_k\sum_l \Big\{d_k^I d_l^I g(t-kT-\tau)g(t-lT-\tau) \\ &\quad + d_k^Q d_l^Q g\left(t-kT-\frac{T}{2}-\tau\right)g\left(t-lT-\frac{T}{2}-\tau\right)\Big\}. \end{aligned} \tag{2}$$

When the signal $s_q(t)$ is sampled at every T_s which is shorter enough than the symbol length, the differentiation of sampled signal $s_d(t)$ is represented by

$$\begin{aligned} s_d(nT_s) &\equiv \frac{1}{2T_s}\left[s_q((n+1)T_s)-s_q((n-1)T_s)\right] \\ &= \frac{1}{2T_s}\sum_k\sum_l \Big\{d_k^I d_l^I g(\tau-T_s+kT)g(\tau-T_s+lT) \\ &\quad + d_k^Q d_l^Q g\left(\tau-T_s+\frac{T}{2}+kT\right)g\left(\tau-T_s+\frac{T}{2}+lT\right) \\ &\quad - d_k^I d_l^I g(\tau+T_s+kT)g(\tau+T_s+lT) \\ &\quad - d_k^Q d_l^Q g\left(\tau+T_s+\frac{T}{2}+kT\right)g\left(\tau+T_s+\frac{T}{2}+lT\right)\Big\}. \end{aligned} \tag{3}$$

To acquire the symbol synchronization, the timing error is defined at given sampling time as a following equation.

$$\varepsilon_n(\tau) \equiv s_d\left(nT_s-\frac{T}{4}\right)\cdot\left[s_d(nT_s)-s_d\left(nT_s-\frac{T}{2}\right)\right] \tag{4}$$

The block diagram of this timing error detector is illustrated in Fig. 1.

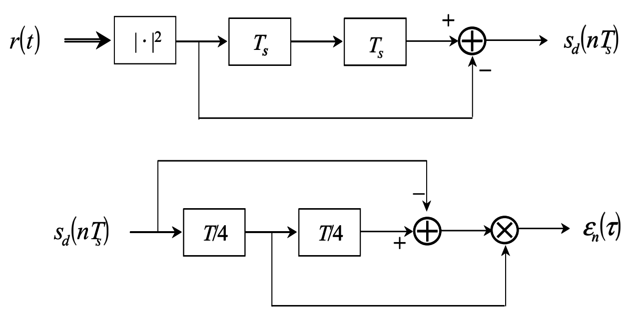

Fig. 1 Block diagram of the timing error estimator

III. Derivation of Algorithm

We define a function to derive the S-curve by taking expectation of the timing error given by in (4).

$$S(\tau, T_s) = E\left\{\ \varepsilon_n(\tau)\ \right\} \tag{5}$$

(5) has to be a single frequency tone in order to obtain information about the symbol timing and can be represented by 32 terms as follows:

$$S(\tau, T_s) = \frac{1}{4T_s^2}\sum_{k=1}^{32} s_k(\tau, T_s). \tag{6}$$

Also $s_k(\tau, T_s)$ in (6) can be expressed by two new variables as follows:

$$s_k(\tau, T_s) = \zeta_k A(\tau, \alpha_k, \beta_k, \gamma_k, \mu_k) + \psi_k D(\tau, \alpha_k, \beta_k, \gamma_k, \mu_k) \tag{7}$$

where ζ_k, ψ_k are constants which can be determined by the values of α_k, β_k, γ_k and μ_k. $A(\tau, \alpha_k, \beta_k, \gamma_k, \mu_k)$ and $D(\tau, \alpha_k, \beta_k, \gamma_k, \mu_k)$ are given by

$$A(\tau, \alpha, \beta, \gamma, \mu) \equiv \sum_k\sum_l\sum_v\sum_w \Big[E\{d_k^I d_l^I d_v^I d_w^I\} g(\tau+\alpha+kT) \cdot g(\tau+\beta+lT) g(\tau+\gamma+vT) g(\tau+\mu+wT)\Big] \tag{8}$$

$$D(\tau, \alpha, \beta, \gamma, \mu) \equiv \sum_k\sum_l\sum_v\sum_w \Big[E\{d_k^I d_l^I d_v^Q d_w^Q\} g(\tau+\alpha+kT) \cdot g(\tau+\beta+lT) g(\tau+\gamma+vT) g(\tau+\mu+wT)\Big]. \tag{9}$$

(8) and (9) can be rewritten as follows:

$$A(\tau, \alpha, \beta, \gamma, \mu) = B(\tau, \alpha, \beta)B(\tau, \gamma, \mu) + B(\tau, \alpha, \gamma)B(\tau, \beta, \mu) + B(\tau, \alpha, \mu)B(\tau, \beta, \gamma) - 2C(\tau, \alpha, \beta, \gamma, \mu). \tag{10}$$

$$D(\tau, \alpha, \beta, \gamma, \mu) = B(\tau, \alpha, \beta)B(\tau, \gamma, \mu) \tag{11}$$

where

$$B(\tau, \alpha, \beta) = \sum_k g(\tau+\alpha+kT) g(\tau+\beta+kT) \tag{12}$$

$$C(\tau, \alpha, \beta, \gamma, \mu) = \sum_k \{g(\tau+\alpha+kT) g(\tau+\beta+kT) g(\tau+\gamma+kT) g(\tau+\mu+kT)\}. \tag{13}$$

(12) and (13) can be rewritten respectively by

$$B(\tau, \alpha, \beta) = \frac{1}{T}\sum_{k=-1}^{1} b_k(\alpha, \beta) e^{j\frac{2\pi k(\tau+\beta)}{T}} \tag{14}$$

$$C(\tau, \alpha, \beta, \gamma, \mu) = \frac{1}{T}\sum_{k=-2}^{2} c_k(\alpha, \beta, \gamma, \mu) e^{\frac{j2\pi k(\tau+\mu)}{T}} \tag{15}$$

where

$$b_k(\alpha, \beta) = \int_{-1/T}^{1/T} e^{j2\pi u(\alpha-\beta)} G(u) G\left(\frac{k}{T} - u\right) du \quad for \quad -1 \le k \le 1 \tag{16a}$$

$$c_k(\alpha, \beta, \gamma, \mu) = \int_{-2/T}^{2/T}\int_{-1/T}^{1/T}\int_{-1/T}^{1/T} \left\{G(u)G(v)G(w-u)G\left(\frac{k}{T} - w - v\right) \cdot e^{j2\pi J(u, v, w)} du dv dw\right\} \quad for \ -2 \le k \le 2 \tag{16b}$$

$$J(u, v, w) = (\alpha - \beta)u + (\gamma - \mu)v + (\beta - \mu)w \tag{16c}$$

$$G(f) \equiv \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt \ . \tag{16d}$$

Using the equations from (10) to (16), $s_k(\tau, T_s)$ in (7) is represented by the sum of 3 terms as follows:

$$s_k(\tau, T_s) = \sum_{l=1}^{3} s_{k,l}(\tau, T_s). \tag{17}$$

Therefore (5) is represented by

$$S(\tau, T_s) \frac{1}{4T_s^2}\sum_{k=1}^{32}\sum_{l=1}^{3} s_{k,l}(\tau, T_s) = \frac{1}{4T_s^2}\left\{\sum_{k=1}^{32} s_{k,1}(\tau, T_s) + \sum_{k=1}^{32} s_{k,2}(\tau, T_s) + \sum_{k=1}^{32} s_{k,3}(\tau, T_s)\right\} \tag{18}$$

Each term of (18) can be presented as follows:

$$s_{k,1}(\tau, T_s) = a_{k,1} B(\tau, \alpha_k, \beta_k) B(\tau, \gamma_k, \mu_k) \tag{19a}$$

$$s_{k,2}(\tau, T_s) = a_{k,2} B(\tau, \alpha_k, \gamma_k) B(\tau, \beta_k, \mu_k) \tag{19b}$$

$$s_{k,3}(\tau, T_s) = a_{k,3} C(\tau, \alpha_k, \beta_k, \gamma_k, \mu_k) \tag{19c}$$

Using (19a) when $\alpha_k = \beta_k$ and $\gamma_k = \mu_k$, the first term in (18) becomes zero.

$$\sigma_1(\tau, T_s) \equiv \sum_{k=1}^{32} s_{k,1} = 0 \tag{20}$$

Also $s_{k,2}(\tau, T_s)$ in (19b) is represented by the sum of 5 terms.

$$s_{k,2}(\tau, T_s) = \sum_{n=-2}^{2} s_{k,2,n} \tag{21}$$

Each term in (21) is expressed as sum of tones by

$$s_{k,2,n}(\tau, T_s) = a_{k,2,n} h_n(\tau, \alpha_k, \beta_k) e^{j\frac{2\pi n\tau}{T}}. \tag{22}$$

Namely, $s_{k,2,n}(\tau, T_s)$ of (22) is composed of a frequency tone $e^{j\frac{2\pi n\tau}{T}}$ and its parameter. Here, $h_n(\tau, \alpha, \beta)$ is satisfied with the following equation.

$$\begin{aligned}\sum_{n=-2}^{2} h_n(\alpha, \beta) e^{j\frac{2\pi n\tau}{T}} \\ = \frac{1}{T^2} \sum_{k=-1}^{1}\sum_{l=-1}^{1} b_k(\alpha, \beta) b_l(\alpha, \beta) e^{j\frac{2\pi(k+l)\beta}{T}} e^{j\frac{2\pi(k+l)\tau}{T}}\end{aligned} \tag{23}$$

We can obtain the following results from (21) and (22).

$$\sigma_{2,1}(\tau, T_s) \equiv \sum_{k=1}^{32} s_{k,2,1}(\tau, T_s) = 0 \tag{24a}$$

$$\sigma_{2,0}(\tau, T_s) \equiv \sum_{k=1}^{32} s_{k,2,0}(\tau, T_s) = 0 \tag{24b}$$

$$\sigma_{2,-1}(\tau, T_s) \equiv \sum_{k=1}^{32} s_{k,2,-1}(\tau, T_s) = 0 \tag{24c}$$

$$\begin{aligned}\sigma_{2,\,2}(\tau, T_s) &\equiv \sum_{k=1}^{32} s_{k,2,2}(\tau, T_s) \\ &= -j8 \cdot e^{j\frac{4\pi\tau}{T}} \operatorname{Im}\left(e^{j\frac{4\pi T_s}{T}} \lambda(T_s)\right)\end{aligned} \tag{24d}$$

where, $\lambda(T_s)$ in (24d) is given by

$$\begin{aligned}\lambda(T_s) = \frac{1}{T^2}\bigg[& b_1^2\left(\frac{T}{4}\right) - b_1^2\left(-\frac{T}{4}\right) \\ & - b_1^2\left(-2T_s + \frac{T}{4}\right) + b_1^2\left(-2T_s - \frac{T}{4}\right)\bigg].\end{aligned} \tag{25}$$

$$b_1(\alpha) = \int_{-1/T}^{1/T} e^{j2\pi u\alpha} G(u) G\left(\frac{1}{T} - u\right) du \tag{26}$$

It is obvious that (21) is satisfied with the following equation because it has real value.

$$\sigma_{2,\,-2}(\tau, T_s) = \sum_{k=1}^{32} s_{k,2,-2}(\tau, T_s) = \sum_{k=1}^{32} s^*_{k,2,2}(\tau, T_s) \tag{27}$$

Using the sum of all terms which is composed of (24d) and (27), (17) turns out as follows:

$$\begin{aligned}S_1(\tau, T_s) &\equiv \frac{1}{4T_s^2} \sum_{k=1}^{32}\sum_{l=-2}^{2} s_{k,2,l}(\tau, T_s) \\ &= \frac{1}{4T_s^2}\left(\sigma_{2,\,2}(\tau, T_s) + \sigma^*_{2,\,2}(\tau, T_s)\right) \\ &= \frac{4}{T_s^2} \sin\frac{4\pi\tau}{T} \cdot \operatorname{Im}\left\{ e^{j\frac{4\pi T_s}{T}} \lambda(T_s)\right\}\end{aligned} \tag{28}$$

$s_{k,3}$ in (19c) can be similarly presented with $s_{k,2}$ in (21) and each term is represented as (29).

$$s_{k,3}(\tau, T_s) = \sum_{n=-2}^{2} s_{k,3,n} \tag{29}$$

$$s_{k,3,n}(\tau, T_s) = a_{k,3,n} c_n(\tau, \alpha_k, \beta_k, \gamma_k, \mu_k) e^{j\frac{2\pi n\tau}{T}} \tag{30}$$

We can obtain 5 terms of (29) using (30) and they are as follows:

$$\sigma_{3,1}(\tau, T_s) = \sum_{k=1}^{32} s_{k,3,1}(\tau, T_s) = 0 \tag{31a}$$

$$\sigma_{3,0}(\tau, T_s) = \sum_{k=1}^{32} s_{k,3,0}(\tau, T_s) = 0 \tag{31b}$$

$$\sigma_{3,-1}(\tau, T_s) = \sum_{k=1}^{32} s_{k,3,-1}(\tau, T_s) = 0 \tag{31c}$$

$$\begin{aligned}\sigma_{3,\,2}(\tau, T_s) &= \sum_{k=1}^{32} s_{k,3,2}(\tau, T_s) \\ &= -j8 \cdot e^{j\frac{4\pi\tau}{T}} \operatorname{Im}\left\{ e^{j\frac{4\pi T_s}{T}} \phi(T_s)\right\}\end{aligned} \tag{31d}$$

$$\sigma_{3,\,-2}(\tau, T_s) = \sum_{k=1}^{32} s^*_{k,3,2}(\tau, T_s) \tag{31e}$$

where

$$\phi(T_s) = \frac{1}{T}\left[c_2\left(\frac{T}{4}\right) - c_2\left(-\frac{T}{4}\right) - c_2\left(-2T_s + \frac{T}{4}\right) + c_2\left(-2T_s - \frac{T}{4}\right)\right] \tag{32}$$

$$\begin{aligned}c_2(\beta) = \int_{-2/T}^{2/T}\int_{-1/T}^{1/T}\int_{-1/T}^{1/T} \{ & G(u)G(v)G(w-u) \\ & \cdot G\left(\frac{2}{T} - w - v\right) e^{j2\pi\beta w} du dv dw \}.\end{aligned} \tag{33}$$

The sum of (31d) and (31e) becomes as

$$S_2(\tau, T_s) = \frac{1}{4T_s^2} \sum_{k=1}^{32} \sum_{l=-2}^{2} s_{k,3,l}(\tau, T_s) = \frac{1}{4T_s^2}\left(\sigma_{3,\,2}(\tau, T_s) + \sigma_{3,\,-2}(\tau, T_s)\right) = \frac{4}{T_s^2} \sin\frac{4\pi\tau}{T} \cdot \mathrm{Im}\left\{ e^{j\frac{4\pi T_s}{T}} \phi(T_s) \right\} \tag{34}$$

As a result of derivation, we get the expression of $S(\tau, T_s)$ with the closed form. This shows a sinusoidal shape with the period of *T/2*.

$$S(\tau, T_s) = S_1(\tau, T_s) + S_2(\tau, T_s) = \frac{4}{T_s^2} \sin\frac{4\pi\tau}{T} \cdot \mathrm{Im}\left\{ e^{j\frac{4\pi T_s}{T}} \left[\lambda(T_s) + \phi(T_s)\right] \right\} \tag{35}$$

Especially, we have simpler form when $T_s = T/4$.

$$S(\tau) = -\frac{64}{T^2} \sin\frac{4\pi\tau}{T} \cdot \mathrm{Im}\left\{ \lambda\left(\frac{T}{4}\right) + \phi\left(\frac{T}{4}\right) \right\} \tag{36}$$

Fig. 2 and Fig. 3 illustrate the S-curve simulation obtained in cases where the roll-off factor of LPF(square-root raised cosine filter for this case) are 1 and 0.35 respectively. The amplitudes of these curves are proportional to the roll-off factor.

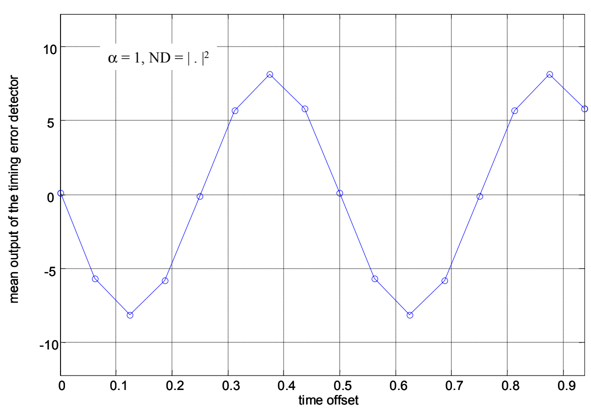

Fig. 2 The S-curve with $\alpha = 1$, $T_s = T/4$

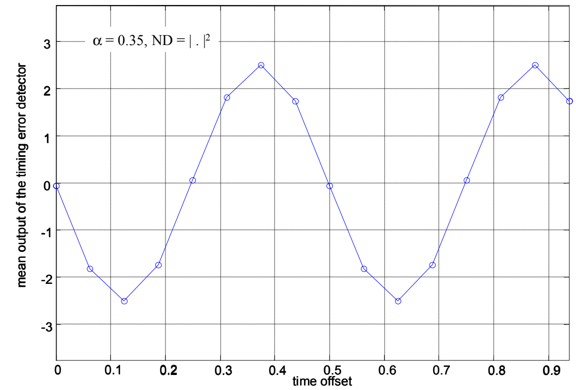

Fig. 3 The S-curve with $\alpha = 0.35$, $T_s = T/4$

IV. CONCLUSIONS

In this paper, a simple timing error detector for OQPSK signal has been presented. This detector is operating with phase insensitive. The S-curve has been also derived analytically in the closed form. This curve is a sinusoidal shape and becomes simpler when the sampling interval is 1/4 of the symbol length. It can be simply implemented with low complexity and is suitable for high-speed digital transmission.

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