AN EFFICIENT RESOURCE ALLOCATION FOR MULTIUSER MIMO-OFDM SYSTEMS WITH ZERO-FORCING BEAMFORMER

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ABSTRACT

In this paper, a low complexity subcarrier allocation scheme is proposed for multiuser MIMO-multiuser OFDM systems with zero-forcing beamforming (ZFBF) to minimize the total transmit power satisfying given target data rates of users. The ZFBF for multiuser MIMO makes multiple spatial channels by removing multiuser interference and the subcarrier allocation can be performed over spatial and frequency domain. Since the optimal method has very high computational complexity, the semi-orthogonal user selection (SUS) is employed to reduce the computational complexity. In numerical results, we show that the total transmit power satisfying the data rates of users is linearly decreased with the number of transmit antennas.

I. INTRODUCTION

Recently, there has been interest in the multi-antenna downlink communication channel. When there is a multi-input and multi-output (MIMO) system with the base station transmitter with $M$ antennas and $K$ mobile receivers with single antenna, the sum capacity can be linearly increased with $\min(M, K)$. In multiuser MIMO systems, it should be serve multiple users simultaneously and then the severe interference problem is occurred. Dirty paper coding (DPC) [1] method is the optimal method to remove the interference signal and achieves the sum capacity, but this method requires high computational complexity and it is difficult to implement. In [3], for a large number of users, it is showed that the zero-forcing beamformer(ZFBF) based on semi-orthogonal user selection achieves the asymptotic sum rate as DPC.

In a multiuser OFDM system, the several resource allocation method were proposed in [4]-[7] to minimize the total required transmit power given QoS or to maximize the total throughput given total transmit power by utilizing the multiuser diversity in frequency domain. Since the optimal method requires high computational complexity, several suboptimal methods are proposed[5, 6]. In the suboptimal methods, the subcarrier allocation and bit allocation are separately performed. In [7], the resource allocation method was extended to MIMO-OFDM systems based on singular value decomposition (SVD). Int this method, the information symbol is transmitted via the maximum singular mode by configuring the antenna weights using the maximum singular vector.

The rest of this paper is organized as follows. In section II, the transmitter and receiver structure for a multiuser MIMO-OFDM system is described. In section III, the optimization problem is formulated and a low complexity subcarrier and bit allocation method is proposed. Section IV shows the performance of the proposed scheme. Finally, the paper is concluded in section V.

II. SYSTEM MODEL

A. Channel Model and Transmitter Structure

We consider a downlink multiuser MIMO-OFDM system with $K$ users equipped with single receive antenna and a base station transmitter with $M$ antennas. The frequency band is divided into $N$ subcarriers. It is considered that channel matrix is not varied during the the coherence interval of $T$. The received signal of the user $k$ on subcarrier $n$ is represented as

$$y_{k,n} = h_{k,n} x_n + z_{k,n}$$  (1)

where $h_{k,n}$ is $1 \times M$ channel gain vector of user $k$ and the entries of $h_{k,n}$ are identically independent distributed with zero mean and unit variance, $x_n$ is $M \times 1$ transmitted symbol vector and $z_{k,n}$ is complex Gaussian noise with zero mean and unit variance of user $k$.

Let $S_n \subset \{1, ..., K\}$ be a subset of user indexes at subcarrier $n$ and $|S_n| \leq M$ where $|S|$ represents the number of elements of set $S$. The information symbol $s_{k,n}$ is multiplied by the $k$th beamforming vector $w_{k,n}$ as follows

$$x_n = \sum_{k \in S_n} \sqrt{T_{k,n}} w_{k,n} s_{k,n}$$  (2)
Then the received signal (1) becomes
\[
y_{k,n} = \sum_{k \in S_n} \sqrt{P_{k,n}} h_{k,n} w_{k,n} s_{k,n} + z_{k,n}
\]  
(3)

In ZFBF[3], the beamforming vector is selected to satisfy the zero-interference condition \(h_{k,n} w_{j,n} = 0\) for \(j \neq k\). Denote \(H_n(S_n)\) and \(W_n(S_n)\) be the corresponding submatrices of \(H_n = [h_{1,n}^T, ..., h_{K,n}^T]^T, W_n = [w_{1,n}, ..., w_{K,n}]\), respectively.

The beamforming matrix \(W_n(S_n)\) satisfying zero-interference condition can be simply implemented using pseudo inverse of \(H(S_n)\) as follows
\[
W_n(S_n) = H_n(S_n)^\dagger = H_n(S_n)^*(H_n(S_n)H_n(S_n)^*)^{-1}
\]  
(4)

Then, the total transmit power for subcarrier \(n\) is
\[
P_{T,n} = \sum_{k \in S_n} P_{k,n}||w_{k,n}||^2
= P_n(S_n) W_n(S_n)^* W_n(S_n)^{\dagger}
= P_n(S_n) (H_n(S_n)H_n(S_n)^*)^{-1}
= \sum_{k \in S_n} \gamma_{k,n}
\]

where
\[
\gamma_{k,n} = \frac{1}{[H_n(S_n)H_n(S_n)^*]_{k,k}}^{\dagger}, \ k \in S_n
\]  
(6)

and \(P_n(S_n)\) be the corresponding submatrix of \(|S_n| \times |S_n|\) diagonal matrix
\[
P_n(S_n) = \begin{bmatrix}
P_{1,n} & 0 & \cdots & 0 \\
0 & P_{2,n} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & P_{K,n}
\end{bmatrix}
\]  
(7)

In equation (6), \([A]_{k,k}\) represents the \(k\)th diagonal element of \(A\) and \(\gamma_{k,n}\) can be interpreted as the effective channel gain at subcarrier \(n\) for user \(k \in S_n\).

B. Problem Formulation

In multiuser MIMO-OFDM systems described in the previous section, the total transmit power for all subcarrier is
\[
P_{tot} = \sum_{n=1}^{N} P_{T,n} = \sum_{n=1}^{N} \sum_{k \in S_n} \frac{P_{k,n}}{\gamma_{k,n}}
\]  
(8)

Denote \(f_k(c)\) be the required transmit power to transmit \(c\) bits satisfying target bit error rate (BER) when channel gain is unity. For example, in the case of Quadrature Amplitude Modulation, \(f_k(c)\) can be represented as
\[
f_k(c) = \frac{N_0}{3} [Q^{-1}(P_o)]^2 (2^c - 1)
\]  
(9)

where \(P_o\) denotes the target BER, \(N_0/2\) denotes the variance of the Additive White Gaussian Noise (AWGN) and \(Q(x)\) is the Q-function [4]. Since \(|H(S_n)W(S_n)|_{k,k} = 1 \forall k \in S_n, n, P_{k,n} = f_k(c_{k,n})\). Since the ZFBF can make \(M\) spatial layer, maximum \(M\) users can be allocated by ZFBF for each subcarrier and only one user is assigned to each layer of a subcarrier. Denote \(\rho_{k,n,i}\) a subcarrier indicator of user \(k\) of subcarrier \(n\) at layer \(i(1 \leq i \leq M)\), i.e. \(\rho_{k,n,i} = 1\) if \(c_{k,n,i} \neq 0\), \(\rho_{k,n,i} = 0\) if \(c_{k,n,i} = 0\).

The first subcarrier constraint (10) means only one user can be assigned to a layer of a subcarrier, the second constraint (11) represents \(M\) users can be assigned to a subcarrier, which is the spatial domain constraint obtained by ZFBF. Let the target rate of user \(k\) be \(R_k\) and then
\[
N \sum_{n=1}^{M} \sum_{i=1}^{N} c_{k,n,i} \rho_{k,n,i} = R_k, \ k = 1, ..., K
\]  
(12)

The optimization problem can be formulated in the sense of the total transmit power satisfying (10), (11) and (12) as follows.
\[
\min_{\rho_{k,n,i}, c_{k,n,i}} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{i=1}^{M} f_k(c_{k,n,i}) \rho_{k,n,i}
\]  
subject to (10), (11), (12)

(13)

Since the effective channel gain \(\gamma_{k,n}\) is dependent on \(S_n\) and the subcarrier allocation process is performed based on \(\gamma_{k,n}\), then the subcarrier allocation process and effective channel gain is mutually dependent. This fact makes the optimization problem more complicated. The optimal user group selection requires an exhaustive search over the entire user set. Since the size of search space becomes \(\binom{K}{M}\) for each subcarrier, the total size of search space becomes \(\binom{K}{M}^N\) and the computational complexity is exponentially increased with the number of subcarriers. Therefore, a low complexity subcarrier allocation algorithm is required and the proposed suboptimal subcarrier allocation is described in the next section.

III. SUBCARRIER AND BIT ALLOCATION

The optimization problem described in the previous section is computationally intractable. In this section, the low complexity subcarrier and bit allocation method considering rate requirement and spatial channel characteristics of users is proposed. The optimization problem (13) is separated into three stages. In the first stage, the number of required subcarriers for each user is roughly determined based on target rate and the average channel gain of each user, which is similar to the method proposed in [5, 6]. In the second stage, a user subset is assigned to each subcarrier to satisfy the required number of subcarriers
obtained in the first stage and the other subcarrier constraints (10), (11). In the third stage, bit allocation for assigned subcarriers to each user is performed.

### A. Resource Allocation Algorithm

In the wireless channel, as the average channel gain of a user is lower, the required transmit power satisfying QoS is increased or vice versa. Similarly, as shown in the optimal method [4, 6], more subcarriers are assigned to the user with lower average channel gain to satisfy the rate constraint of the user. Using this properties, we assume the followings.

**Assumption 1:** each user \( k \) experiences of the identical channel gain for each subcarrier \( \gamma_k = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} ||h_{k,n}||^2 \)

**Assumption 2:** For assigned subcarriers for each user, the same number of bits \( c_k \) is assigned, i.e. \( c_{k,n,i} = c_k \) if \( \rho_{k,n,i} = 1 \) for \( n = 1, \ldots, N, i = 1, \ldots, M \). This assumption is followed by the observation of the results in the optimal solution [4, 6]. Let the number of subcarrier assigned to user \( k \) be \( n_k = \sum_{i=1}^{M} \sum_{n=1}^{N} \rho_{k,n,i} \). In subcarrier constraint (11),

\[
\sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{i=1}^{M} \rho_{k,n,i} = \sum_{k=1}^{K} n_k = MN \tag{14}
\]

In assumption 2, since \( c_{k,n,i} = c_k \) if \( \rho_{k,n,i} = 1 \), the rate constraint (12) is modified as

\[
\sum_{i=1}^{M} \sum_{n=1}^{N} c_k \rho_{k,n,i} = c_k n_k = R_k \tag{15}
\]

Then the original problem (13) is modified as the problem to find \( n_k, k = 1, \ldots, K \).

\[
\min_{n_k} \sum_{k=1}^{K} f_k \left( \frac{R_k}{n_k} \right) \frac{n_k}{\gamma_k}
\]

subject to \( \sum_{k=1}^{K} n_k = MN \tag{16} \)

To find the solution of the above problem, the BABS(bandwidth assignment based on SNR) algorithm [5] similar to greedy descent algorithm can be applied as follows.

**Initialize:** \( n_k = 1, k = 1, \ldots, K \)

while \( \sum_{k=1}^{K} n_k < MN \)

\( V_k \leftarrow f_k \left( \frac{R_k}{n_k} + 1 \right) - f_k \left( \frac{R_k}{n_k} \right) \gamma_k, k = 1, \ldots, K \)

\( l \leftarrow \arg \min_{1 \leq k \leq K} V_k \)

\( n_l \leftarrow n_l + 1 \)

end while

In [5, Appendix 1], it is shown that this algorithm converges to the optimal solution of the problem (16).

### B. Subcarrier Assignment Algorithm

Once the number of subcarriers to each user is determined, the next step is to assign the specific subcarriers to each user. The original problem (13) is modified as the problem to find \( \rho_{k,n,i} \).

\[
\min_{\rho_{k,n,i}} \sum_{i=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} f_k \left( \frac{R_k}{n_k} \right) \frac{\rho_{k,n,i}}{\gamma_k} \gamma_k \tag{17}
\]

subject to

\[
\sum_{k=1}^{K} \rho_{k,n,i} = 1, \quad i = 1, \ldots, M, n = 1, \ldots, N \tag{18}
\]

\[
\sum_{i=1}^{M} \sum_{n=1}^{N} \rho_{k,n,i} = n_k, \quad k = 1, \ldots, K \tag{19}
\]

\[
\sum_{i=1}^{M} \sum_{n=1}^{N} \rho_{k,n,i} = M, \quad n = 1, \ldots, N \tag{20}
\]

In the above problem, the subcarrier constraint (19) is for frequency domain while the subcarrier constraint (20) is for spatial domain. In the conventional subcarrier allocation methods [4]-[6], the subcarriers are assigned to the users with largest channel gains to maximize total throughput or minimize total transmit power. In multiuser MIMO-OFDM with ZFBF, since the channel gain is dependent on the orthogonality of channels the users assigned to a subcarrier, it is efficient to minimize transmit power to assign a user with the channel which is largest magnitude and lowest correlation with the other already assigned users in the subcarrier. Considering this property, the subcarrier allocation algorithm is as follows.

**Step 1) Initialization:**

\[
T_n = \{1, \ldots, K\}, \quad n = 1, \ldots, N
\]

\[
U_i = \{1, \ldots, N\}
\]

\[
S^0_n = \phi
\]

\[
i = 1, \quad \rho_{k,n,i} = 0 \quad \forall k, n \tag{21}
\]

**Step 2) For each user** \( k \in T_n \) for \( n = 1, \ldots, N \), calculate \( g_{k,n,i} \), the component of \( h_{k,n} \) orthogonal to the subspace spanned by \( \{g_{(1),n}, \ldots, g_{(i-1),n}\} \)

\[
g_{k,n,i} = h_{k,n} \left( I - \sum_{j=1}^{i-1} g_{(j),n} \cdot g_{(j),n} \right) \tag{22}
\]

When \( i = 1 \), this implies \( g_{k,n,1} = h_{k,n} \).

**Step 3) Let the cost** \( r_{k,n,i} \) the unit required transmit power when the channel gain is \( ||g_{k,n,i}|| \) as follows.

\[
r_{k,n,i} = f_k \left( \frac{R_k}{n_k} \right) \frac{1}{||g_{k,n,i}||^2} \tag{23}
\]
For the $i$th layer, subcarriers are allocated to users. Repeat the following operations until $U_i = \phi$.

$$
\hat{k} = \arg \min_{k \in T_n} r_{k,n,i}, \quad n = 1, ..., N \quad (24)
$$

$$
\hat{n} = \arg \min_{n \in U_i} r_{\hat{k},n,i} \quad (25)
$$

$$
g_{(i),\hat{n}} = g_{k,\hat{n},i}, \quad \hat{h}_{(i),\hat{n}} = h_{k,\hat{n}} \quad (26)
$$

$$
\rho_{k,\hat{n},i} = 1, \quad S_{\hat{n}}^0 = S_{\hat{n}}^0 \cup \{\hat{k}\} \quad (27)
$$

$$
U_i = U_i - \{\hat{n}\}, \quad n_{\hat{k}} = n_{\hat{k}} - 1 \quad (28)
$$

$$
T_{\hat{n}} = T_{\hat{n}} - \{\hat{k}\} \quad (29)
$$

If $n_{\hat{k}} = 0$, then $T_n = T_n - \{\hat{k}\}, \forall n \quad (30)$

Update $r_{k,n,i} \forall k \in T_n \quad (31)$

If $U_i = \phi$ and $i < M$, then $i \leftarrow i + 1$ and go Step 2. If $i = M$, the algorithm is finished.

In Step 1), $T_n$ is user set of $n$th subcarrier and $U_i$ is subcarrier set at the $i$th layer. In Step 2), $g_{k,n,i}$ is the orthogonal component of $h_{k,n}$ spanned by $\{g_{(1),n}, ..., g_{(i-1),n}\}$. In Step 3), we select a user $k$ and a subcarrier $\hat{n}$ with minimum transmit power among users in $T_n$ and subcarriers in $U_i$. Since a user can be assigned to a subcarrier only once, the assigned user $k$ cannot be assigned to the other layers (eq. (29)), which satisfies the constraint (19). If the assigned user $k$ satisfy the required number of subcarriers, the subcarriers are not assigned to the $\hat{k}$th user any more (eq. (30)), which satisfies the constraint (20).

As a result of the subcarrier assignment, the effective channel gain is obtained as follows.

$$
\gamma_{k,n} = 1/[[H_{n}(S_{n}^0)H_{n}(S_{n}^0)^*]^{-1}]_{k,k}, \quad k \in S_{n}^0 \quad (32)
$$

C. Bit Loading Algorithm

In this subsection, bits are assigned to subcarriers assigned to each user satisfying the target data rates of each user. As a result of the subcarrier allocation in the previous section, the optimization problem can be simplified as the bit allocation problem as follows.

$$
\min_{c_{k,n,i}} \sum_{i=1}^{M} \sum_{n \in Z_{k,i}} f_k(c_{k,n,i}) \quad \text{s.t.} \quad \sum_{i=1}^{M} \sum_{n \in Z_{k,i}} c_{k,n,i} = R_k \forall k \quad (33)
$$

where $Z_{k,i}$ be the set of indices of the subcarriers assigned to the $i$th user at the $k$th layer, i.e. $Z_{k,i} = \{n | \rho_{k,n,i} = 1\}$.

Bits are assigned to each user over frequency and spatial domain. As in the case of single user OFDM [4], the greedy algorithm can be employed. Let the required additional transmit power for additional one bit $\Delta P_{k,n}(c) = [f_k(c + 1) - f_k(c)]/\gamma_{k,n}$. In each iteration, bits are assigned to the user with minimum required transmit power for one additional bit as follows.

Initialize : $c_{k,n,i} = 0, \quad n \in Z_{k,i}, \forall k, i$

Evaluate $P_{k,n}(0), \quad n \in Z_{k,i}, \forall k$

For each $k$, Repeat the following $R_k$ times

$$
\{\hat{n}, \hat{i}\} = \arg\min_{1 \leq i \leq M, n \in Z_{k,i}} \min_{1 \leq l \leq M} \Delta P_{k,n}(c_{k,n,i})
$$

$$
c_{k,\hat{n},\hat{i}} = c_{k,\hat{n},\hat{i}} + 1
$$

Update $\Delta P_{k,\hat{n}}(c_{k,\hat{n},\hat{i}})$

IV. Numerical Results

We assume that the channel of each antenna of each user is identical and independent and experiences frequency selective fading and the channel of each subcarrier experiences flat fading. Target BER is assumed to be $10^{-4}$ and target rate of each user is identical. In Fig. 1, we compare the optimal method and the suboptimal method for multiuser MIMO-OFDM system. Due to computational complexity, we assume the number of subcarriers is 32 and the sum of target bit rate of users is
200 bits/symbol, i.e. $\sum_{k=1}^{K} R_k = 200$. In Fig. 1, the total transmit power of the suboptimal method is a little higher than that of the optimal method while the computational complexity $O(M^2KN^2)$ is considerably reduced to $O(M^2KN^2)$. From Fig.2 to Fig.4, the number of subcarrier is 128 and the sum of target bit rate of users is 500 bits/symbol. Fig. 2 shows that the required transmit power when the number of transmit antenna is varied. Fig. 2 shows that the required total transmit power is linearly decreased with the number of transmit antennas. It represents zero-forcing beamforming and subcarrier allocation based on semi-orthogonal user selection makes the spatial channels as much as the number of transmit antennas. In cellular environment, each user has different average channel gain due to pathloss. Denote $\beta$ (in dB) the difference of the maximum average channel gain and the minimum average channel gain of users in a cell. Fig. 3 shows the required transmit power is increased with $\beta$ and the required transmit power is decreased with the number of users since multiuser diversity gain by subcarrier allocation is increased with the number of users.

The proposed scheme is compared with the proposed scheme to SVD-based MIMO-OFDM scheme[7]. Since in SVD-based method, the base station(BS) transmitter selects only one user with the maximum singular mode, it can utilize the selection diversity gain. Since The total required transmit power of the proposed scheme is linearly decreased with the number of transmit antennas, the proposed scheme outperforms the SVD-based method. In Fig. 4, the total required transmit power of the proposed scheme at $M = 4$ is similar to the total required transmit power of the SVD-based method at $M = 2$.

V. CONCLUSION

In this paper, the subcarrier and bit allocation method for multiuser MIMO-multiuser OFDM systems with ZFBF. Since ZFBF removes among the interference of the users in the same subcarrier and the proposed subcarrier allocation method assigns user set into each subcarrier to minimize total transmit power for given users rate constraint, the required transmit power satisfying target rates of users is linearly decreased with the number of transmit antennas. To reduce the computational complexity, we propose the suboptimal subcarrier allocation based on semi-orthogonal user selection. The suboptimal method based on the semi-orthogonal user selection achieves similar performance with the optimal method despite the low complexity. The proposed scheme can also extend to the case of the multiple receive antennas.

REFERENCES